Composite Mechanical Properties For Use In Structural Analysis

by

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Summary

In order to be able to undertake an analysis of a component the designer will need to know the properties of the material being used. The aim of this work is help the design engineer such that the mechanical properties of continuous glass fibre reinforced composite material can be determined and used in the design analysis of components manufactured from this material.

The literature survey has shown that for the material considered here, then given the constituent properties, the fibre arrangement and the fibre volume fraction, the composite mechanical properties may be determined mathematically by the use of micromechanical equations. The micromechanical prediction of the mechanical properties of uni–directional, random and woven fibre reinforced composites has been examined. The variation of these mechanical properties that may occur in a composite component due to the manufacturing process has been highlighted as being of importance. This has been studied to determine whether such a variation is significant by analysing examples of composite components and plates. The results from these analyses have been correlated with experimental results and investigated to study the importance of such variations in properties.

Many micromechanical equations have been found in the literature for the prediction of the mechanical properties of continuous fibre reinforced composite materials. An accuracy of the predicted properties to within 10% of the experimental data was concluded to be acceptable and good enough for initial design purposes as design engineers are not usually able to design to such tight tolerances. This work has shown that further development of the micromechanical theories is not the most important problem concerning the prediction of the mechanical properties. These properties can currently be predicted with acceptable accuracy from the micromechanical equations already available in the literature. However, the design engineer is unlikely to have knowledge of the micromechanical equations necessary to determine the required properties. It is only by undertaking a large literature survey that the designer would be able to find this information. Many of the micromechanical equations require the use of an empirical factor. The knowledge of a value for such a factor is again something that would not be readily available to the designer. Rather than concentrating upon improving the micromechanical predictions, this work shows that effort should be made to understand the influence of other factors upon the mechanical properties of composite materials. In particular, the behaviour and flow of the material during the manufacturing process has been highlighted as being of importance as it can cause a significant variation in the properties. Thus, analyses of composite components cannot assume that the mechanical properties are constant throughout, and it is therefore necessary to first model the manufacturing process to determine the mechanical properties before undertaking a structural analysis.

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Declaration

The research described in this thesis is entirely the work of the author except where indicated otherwise by a reference. None of the work contained in this thesis has previously been submitted for any other academic award.

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Notation

The following notation has been adopted in this thesis. Note that the references quoted use a variety of different terms and letters to describe the same item. These terms may have been altered from those given in the references to the ones adopted in this thesis to ensure consistency.

( ) = references.

[ ] = equation numbers.

E = Young's Modulus.

G = shear modulus.

ν = Poisson's Ratio.

ϱ = density.

V = volume fraction.

W = weight fraction.

K = bulk modulus / plane strain bulk modulus of elasticity.

P = general mechanical property.

σ = stress.

ε = strain.

τ = shear stress.

γ = shear strain.

Q = stiffness terms.

A = extensional stiffnesses.

Z = fibre misalignment factor.

C = contiguity factor.

h = material thickness.

ξ = geometrical factor in Halpin Tsai equations.

Subscripts

L = 11 = X = longitudinal direction of fibres in a uni–directional or woven material.

T = 22 = Y = transverse direction in a uni–directional or woven material.

33 = Z = through thickness direction in a uni–directional or woven material.

c = composite properties.

m = matrix properties.

f = fibre properties.

2T = transverse properties in a random material.

2D = in plane properties in a random material.

– = lower bound of property.

+ = upper bound of property.

Superscript

* = denotes bound of a property.

Abbreviations

CCA = concentric cylinder assemblage.

CLT = classical lamination theory.

FE = finite element (analysis).

GMT = glass mat thermoplastic.

GRP = glass fibre reinforced plastic.

IACFA = International Advisory Committee on Fracture Avoidance.

NPL = National Physics Laboratory.

RTM = resin transfer moulding.

SAM = slice array model.

SMC = sheet moulding compound.

SPATE = stress pattern analysis by thermal emission.

SRIM = structural resin injection moulding.

Chapter 1 : Introduction

In applications where weight saving and parts integration can be achieved the automotive industry have been investigating the design and manufacture of components from composite materials. Whether a component is designed from steel or a composite material the same design-to-manufacture processes should be followed. The methods used in the different steps in the design to manufacture process in the high volume automotive industry are relatively well known for a steel component, but are not so well established for a composite component. One of those steps is usually that of undertaking a mathematical analysis of the component to determine how it behaves under the service loading conditions. Unlike metal, composite materials are often designed to be anisotropic such that their mechanical properties may vary with direction hence compounding the design analysis problem.

Sol and de Wilde (1) state that "composite materials have been used increasingly as structural materials. A reason for this ... is that composite materials have high strength to weight and high stiffness to weight ratios which can significantly reduce the weight of a structure....... Perhaps the most important feature of composite materials is that their mechanical properties can be 'tailored' to meet a specific criterion." However Johnson et al (2) suggest that "composite design, analysis and fabrication technology must undergo major developments and successful demonstrations before significant structural components will be incorporated in production automobiles and trucks".

Chen and Cheng (3) point out that "in recent years fibre reinforced materials have been paid considerable attention due to the search for materials of light weight, great strength and stiffness. Consequently the characterisation of their mechanical properties becomes important, especially that of uni−directional composites." These mechanical properties which are essential for analysis purposes need to be determined. When a component is designed from a composite material the designer will need to know "not only the principal moduli and strengths of the material, measured in the direction of the fibres, but also the variation of these properties at any other angle relative to the principal axis" (4).

Dropek (5), states that the required mechanical properties may be obtained from material tests or from the use of micromechanics. However, Hashin (6,7), reports that the "experimental determination of all of the effective anisotropic properties of interest is impossible" and thus analytical methods must be developed to determine those properties. The analytical methods should be based on fibre and matrix properties, volume fractions of each, and also perhaps fibre distribution. The problem may be stated as follows; given the mechanical properties of the constituent materials, the proportion of each and the geometrical arrangement, how can the mechanical properties of the overall composite be determined. Hashin (8) states that the prediction of the mechanical properties of a composite in terms of its geometry, relative quantities and elastic properties of its constituents is "a basic problem in the theory of reinforced materials."

1.1 Aims And Objectives

The designer of a component from a composite material will need to know the mechanical properties of that material. The aim of this work is to provide guidelines to the designer such that the mechanical properties of a composite can be determined and used with confidence in any analysis. This may require the use of mathematical equations which require the knowledge of the material properties of the fibre and the matrix, and the volume or weight fractions of each. Such equations could avoid the need for any experimental work required on the composite material before an analysis such as a finite element analysis is undertaken. Hence, this will save time required for such an analysis, in addition to saving the cost of undertaking mechanical property tests on test plates that have to be manufactured for each different type of material, and mix of constituents used.

However accurate the mathematical prediction of the composite mechanical properties may be, they are only predicted properties. When a composite component is manufactured it is possible that the resulting mechanical properties of the component will not be consistent throughout the structure. A variation of properties may be caused by a variation in the constituent properties, or be caused by the manufacturing process. Thus, any analysis that assumes that the properties are consistent throughout the structure may not represent the real life conditions and hence be subject to possible error. Therefore, it is also the aim of this work to study the variation of these mechanical properties that may occur in a composite component and whether such a variation is significant.

The potentially vast amount of information concerning composite mechanical properties available in the literature has prompted the work for this thesis to become focused upon the specific materials of interest, rather than considering composite materials in general. Thus, for example, only resin matrix composites have been considered. It has become apparent from the literature studied that much research work has, and is, being undertaken upon a wide variety of composite materials with resin matrices. The literature survey has discovered work on composites consisting of a variety of different resin systems reinforced with a variety of different fibres including hybrid composites reinforced with more than one type of fibre. The fibres that are studied are in a variety of different fibre orientations, ie, uni–directional, woven in two or three dimensions, or randomly arranged either in two or three dimensions, although the literature usually considers uni–directional fibres. The fibres studied by researchers are considered to be either solid or hollow, and either circular or elliptical. The fibres may also be considered to be either long or short, and either plain or coated. This work will only concentrate on composites reinforced with solid circular fibres as these are the types of reinforcement most commonly in use.

This work will concentrate on composites which consist of glass fibre reinforced resin as, due to the cost, these are the materials that are most widely used. In this work the fibres will only be considered to be continuous. The composites reinforced with continuous fibres can be classified into three categories: uni–directional, random, and woven. With these types of composites the glass mat will be formed into the required shape and then the resin will be injected (known as the structural resin injection moulding, SRIM, or the resin transfer moulding RTM process), or compression moulded with, for example, sheet moulding compound (SMC) or glass mat thermoplastic (GMT).

Chapter 2 : Basic Concepts

2.1 Composite Material

A composite material may be defined as "a macroscopic combination of two or more distinct materials" (9). These materials contain a reinforcement, such as glass fibres, surrounded by a binder or matrix. Hashin and Rosen (10), define a composite material as one consisting of "a relatively soft binder in which much stiffer fibres are embedded."

According to Reinhart and Clements (9) and Datoo (11) composite materials can be broadly classified into three areas; namely particulate reinforced, fibre reinforced, and laminates. When composites contain reinforcements whose lengths are much greater than their cross sectional dimensions they are considered to be 'fibre reinforced'. A report from the National Physics Laboratory, (NPL), (12) further separates fibre reinforced materials into two classes – continuous and discontinuous or short. Laminates are those composite materials composed of two or more layers, or laminae of material.

For a fibre reinforced material many authors (13–22) state that the length to diameter (aspect) ratio of the fibres is very important. Many of these authors (15,18–20) report that if this aspect ratio is greater than 100 then the fibres may be considered continuous, although two authors (17,22) state that this is the case if the aspect ratio is greater than 300. Other authors (16,21,23,24) state that the critical value of the aspect ratio where the material behaves as a continuous fibre reinforced one also depends upon the modulus ratio E_f/E_m of the constituent materials. However, Datoo (11) states that fibres of less than 50mm in length are accepted as short fibres whilst those of lengths greater than 50mm are regarded as continuous. Despite the differing opinions seen above, this work will concentrate upon composites whose reinforcing fibres have an aspect ratio such that they may be considered to be continuous by the majority of researchers.

Composite materials have been developed such that best use is made "of the more favourable properties of the components whilst simultaneously mitigating the effects of some of their less desirable characteristics" (25). It is only when strong very brittle solids are converted into fine fibres that the strength limiting defects normally present in brittle solids are eliminated, thus producing strong, rigid materials (25). However these strong fibres are of limited value in structural applications and they are usually combined with a matrix of, for example, polymer, such that their "high tensile strength and rigidity can be utilised in stress systems more complex than simple tension" (25). The matrix material has several purposes. It is used to bind the fibres together, and yet keeps them separated so that any cracks in the fibres cannot propagate through other fibres which may be in contact with the cracked ones (9,25). The matrix also keeps the reinforcing fibres in the correct position and provides the medium through which the load is transferred into the strong reinforcing fibres, and "it is its inherent ability to resist shear and compressive forces that permits the composite to sustain stresses other than pure tension" (25–27). In addition the matrix protects the fibres from damage through either handling or the environment. The overall properties of a composite material are determined from those of its constituent materials and the relative arrangement of those materials.

The properties that may be improved by creating a composite material include the following (28,29):–

1. Strength.
2. Stiffness.
3. Corrosion resistance.
4. Wear resistance.
5. Weight.
6. Temperature dependant behaviour.
7. Fatigue life.

8. Thermal insulation.

9. Thermal conductivity.

10. Acoustical insulation.

Although other fibre reinforced materials, are widely used, this report will concentrate on glass fibre as representative of the fibre in a reinforced composite. These are the materials that are most widely used due to their lower cost.

The glass fibres within a composite material may be orientated in a variety of manners depending upon the application, and required properties of the component, such that the high strength and moduli of the material "can be tailored to the high load directions, with little material wasted on needless reinforcement" (9). This 'tailoring' of material properties means that composite materials have tremendous potential advantages over conventional materials in terms of strength to weight ratios. For example, one advantage of using composite materials is that they are approximately 5 times lighter than steel and twice as light as aluminium , and thus the stiffness per unit weight of composites compares favourably with metals (30). Also, this ability to tailor a composite material to its particular job is a significant advantage as the stiffness and strength can be put into the areas and directions required for the component.

Long fibres of material tend to be much stiffer than the same material in bulk form (28). This is due to the more perfect structure of the fibre as it has the crystals aligned along its axis, and contains fewer internal defects than a bulk material (28). A fibre is characterised as having a very large length to diameter ratio, and a very small (near crystal sized) diameter, and the name *filamentary composite* is often used (28). A *lamina* is an arrangement of fibres in a uni−directional, random or woven form within a matrix (28).

2.1.1 Uni–Directional Material

A uni–directional composite is one in which the fibres are orientated in one direction such that the material is stronger and stiffer in that direction, but relatively weak in other directions, and is "usually reserved for special applications" (31). A uni–directional composite will give the maximum directional properties, in comparison to other types of composite. The longitudinal properties of a uni–directional composite are determined by the fibres, whilst the transverse properties are determined by the matrix (32).

2.1.2 Continuous Random Fibre Mat

The majority of continuous fibre reinforced composites contain fibres that "are comparable in length to the overall dimensions of the composite" component, (9). With a continuous random fibre mat the fibres are not aligned to give directional strength or stiffness, but as the name implies are arranged randomly. This means that the material properties in a two dimensional random mat can be assumed to be the same in the two dimensions, but are different in the third dimension. Hence, the randomly arranged fibres provide "virtually all of the load carrying characteristics" of the composite component (9).

2.1.3 Woven Roving

A composite material manufactured from woven roving usually has two sets of fibres interwoven at right angles. The 'y' axis represents the warp threads and is the long axis of the fabric roll, and the 'x' axis is the fill, or weft, direction and is the width of the roll (31). The woven material will often have the same properties and be very strong or stiff in the warp and fill directions, but may be relatively weak in the third dimension. Also, the properties of the composite at an angle between 0° and 90° will be different to the properties at those angles, and in fact the properties of the material can be altered by altering the weave pattern, thus providing the design engineer with a wide range of available material properties to choose from. It is also possible to have combinations of different types and sizes of threads hence creating a large number of different materials (31).

2.1.4 Laminates

A laminated composite material is one in which layers, plies, or laminae of fibre reinforced material are built up typically with the fibre direction of each layer orientated differently to give different strengths in each direction (28). This means that the strength or stiffness of a laminate can be tailored to the specific requirements of the components. The layers of the laminate are usually bound together with the same matrix material that is used in the laminae (28). Lamination is undertaken so that the directional properties can be tailored to suit the particular loading conditions of the component. However, a potential problem with laminates is the presence of shear stresses between layers arising from the "tendency of each layer to deform independently" (28) as each layer may have different properties. These shearing stresses are the largest at the edges of a laminate and thus may cause delamination (28). The material properties of the laminate are related to the properties of the laminae by the laminae thickness, stacking sequence and orientation. The material properties of each uni−directional laminae are determined by the properties of the fibre, the matrix, their relative volume fractions, and the fibre orientation. Thus, uni−directional laminae form the basic building block of a laminated material and considerable effort has been spent in investigating uni−directional composites (33).

2.2 Mechanical Properties

The designer and design analyst of a composite component will not usually be interested in how the constituents of the composite interact. Instead engineers will be interested in the overall properties of the composite, and its response to the applied loading conditions. It is thus necessary to try to determine the overall mechanical properties of the composite either from exhaustive measurement or from those of its constituent materials.

2.2.1 Definition Of Terminology

Most common engineering materials are considered to be *homogeneous* and *isotropic* (11,28). A body that is homogeneous has properties that are uniform throughout and not dependent upon the position in the body. A material that is *heterogeneous* has properties that change from point to point in the same direction, ie the properties are a function of position (11). A body that is isotropic has material properties that are the same in every direction at a point in the body, and thus not a function of orientation (11,28,34,35). Johnson (30) states that in an isotropic material planes that pass through a particular point are planes of material symmetry. Composite materials however are often *inhomogeneous* and *orthotropic* or *anisotropic* (28). A body that is orthotropic has material properties that are different in three mutually perpendicular directions at a point in the body, and also have three mutually perpendicular planes of material symmetry (11,28,30,34,35). In an anisotropic material there are no planes of material symmetry that pass through a point (11,35). Bodies that are isotropic, anisotropic or orthotropic can be homogeneous or heterogeneous. Although in an anisotropic material the material properties will vary with direction, some of these anisotropic materials will have some symmetry in their microstructure thus reducing the number of different material properties needed to be measured. Composite materials that have been fabricated from uni−directionally aligned fibres "may be isotropic in the plane transverse to the fibres and are thus known as *transversely isotropic* materials" (30).

2.2.2 Modulus Of Elasticity Or Young's Modulus

The modulus of elasticity is defined as "the rate of change of unit tensile or compressive stress with respect to unit tensile or compressive strain for the condition of uniaxial stress within the proportional limit" (36). For a bar in tension the linear relationship between stress and strain can be expressed as follows (34,37):–

$$\sigma = E\,\varepsilon \qquad [1]$$

Where 'σ' is the stress and 'ε' the strain. The term 'E' is a measure of the stiffness of the material and "is a constant of proportionality known as the *modulus of elasticity* for the material" (37). The modulus of elasticity is the slope of the stress–strain curve in its linear elastic region (see Figure 1), and is different for different materials. The modulus of elasticity is often referred to as *Young's modulus* (37). A typical stress–strain curve for a glass fibre reinforced epoxy composite is shown in Figure 2.

Griffin et al (38) state that composite materials exhibit a number of non–lineararities such that the stress–strain curve in the elastic region is non–linear. When loading composite materials, if the loading is perpendicular to the fibres non–linear behaviour is produced, but with loading in the fibre direction the behaviour is linear. However, Tohlen (39) states that although a lot of laminate material data is non–linear, ie, stress does not vary linearly with strain, the data can be assumed linear such that, for example, a constant, ie, Young's modulus, relates stress and strain, and this assumption is evident in the majority of the work found, such that a linear analysis of a composite material is performed.

With isotropic materials a single value for Young's modulus may "characterise the material for all uniaxial tensile loads" (30), and be the same in both tension and compression. However, for anisotropic materials the modulus of elasticity may be different in different directions (30,40). The modulus may also be different in flexure, tension, and compression although they are often not assumed to be so for the purposes of calculation (30,40,41). The differences between tensile and compressive moduli "are the result of the stiffening effect of the fibres being different in tension and compression", and it is common to measure Young's modulus in each of these loading situations and to quote different values (30).

Datoo (11) notes that the flexural modulus is not a basic mechanical property as it depends upon the geometry and load conditions of the test used.

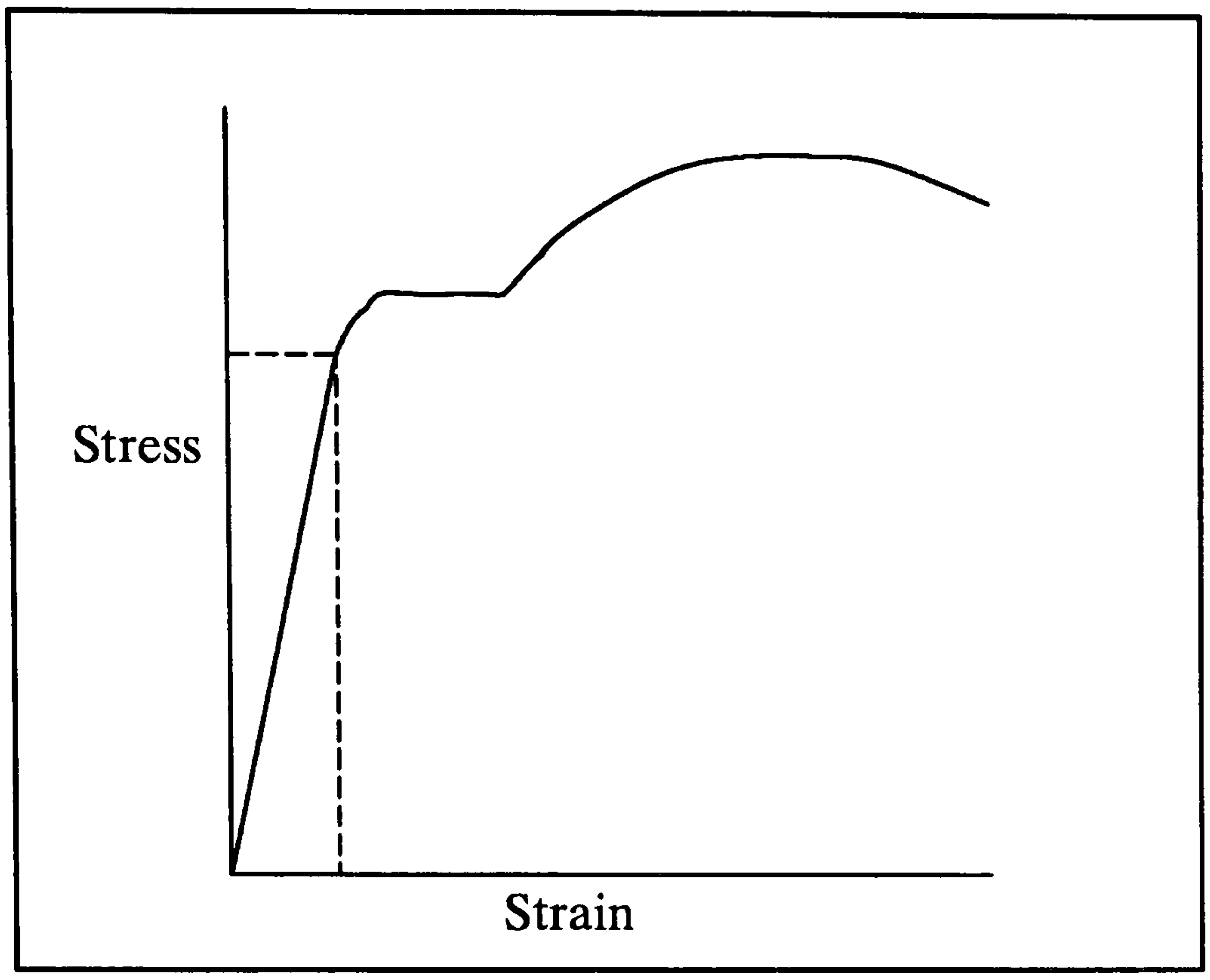


Figure 1 : Typical Stress – Strain Curve For A Metal (37).

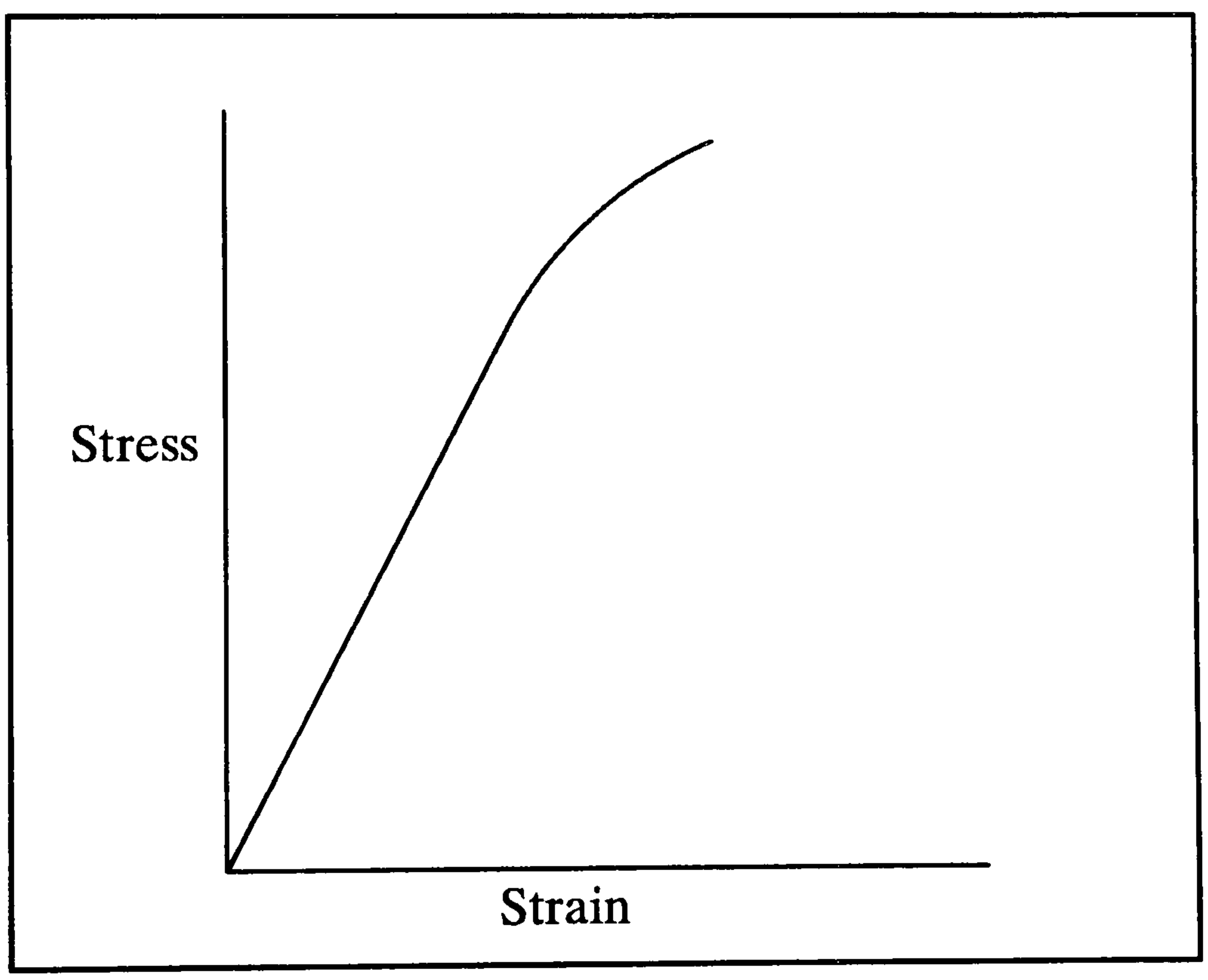


Figure 2 : Typical Stress – Strain Curve For A Glass Fibre Reinforced Epoxy (28).

2.2.3 Poisson's Ratio

When a bar is loaded in tension, there is an axial elongation of that bar. This axial elongation is accompanied by a lateral contraction (37). The ratio of the strain in the lateral direction to that in the axial, or longitudinal, direction is constant within the elastic range of the material, and is known as Poisson's ratio 'ν'. Thus, Poisson's ratio is defined as (37):–

$$\nu = \text{lateral strain} / \text{axial strain} \qquad [2]$$

2.2.4 Modulus Of Rigidity Or Shear Modulus

The shear stress – shear strain diagram will be a straight line if a material has a linear elastic region (37). Thus the shear stress and shear strain are proportional and their relationship can be expressed such that (34,37):–

$$\tau = G\gamma \qquad [3]$$

Where 'τ' is the shear stress and 'γ' the shear strain. The term 'G' is known as the *shear modulus of elasticity* (29,37), or the *modulus of rigidity* of the material (36).

For isotropic materials the equation below is stated to be true in a British Plastics Federation report (40), and also by other authors (7,11,26,29,34,42–44):–

$$G = E / 2(1 + \nu) \qquad [4],$$

where, ν is the Poisson's ratio.

For anisotropic materials the shear modulus may be different in different directions (36). In addition to this, the shear modulus of an orthotropic material, unlike that of an isotropic material, is not dependent upon other material properties such as Young's modulus and Poisson's ratio (28). However, the fibre and matrix materials may be isotropic (45,46), and if so the above relationship is true for these constituent materials.

2.2.5 Bulk Modulus Of Elasticity

The bulk modulus of elasticity is defined as "the ratio of a tensile or compressive stress, triaxial and equal in all directions, . . . to the relative change it produces in volume" (36). The bulk modulus 'K' is can be found from the following equations (26,29,37):–

$$K = E / 3(1-2\nu) \qquad [5]$$

Or, $$K = EG / (9G-3E) \qquad [6]$$

Chapter 3 : Property Determination

It is stated by Griffin (47) that obtaining accurate mechanical properties for an analysis such as finite element analysis is one of the major problems when analysing composites. The through thickness properties in particular are usually not available. Griffin adds that if the unknown out of plane properties are given an assumed value then this will result in errors in stress predictions. Other authors (26,41,47−52) also state that the through thickness properties are important. It is stated that the through thickness stresses are important as the composite strength through the thickness is usually small and therefore these stresses can be "a major source of concern" (52).

Sol and de Wilde (1) state that "the finite element analysis of laminated fibre reinforced polymeric structures requires the knowledge of anisotropic elastic properties for each element. Because these material properties depend on the production methods, it is generally impossible to find these properties in tables or data bases. Sometimes reasonable values can be found using micromechanical models or using the data given by composite material suppliers, but generally the safest way to establish the properties is to measure experimentally on test specimens." A lot of researchers have followed this advice and use experimentally determined properties for their analyses of composite materials.

A paper by Autio et al (53) analyses a three dimensional test structure and compares the FE results to measured ones using experimentally determined material properties. The differences between these results was approximately 20%. Autio et al point out that this error is not necessarily due to errors within the FE analysis as the accuracy of strain gauge measurement depends upon many factors. A 5 − 10% error is usually caused by the strain gauge itself and the measurement system, whilst further inaccuracies may be caused by orientation, location and bonding of the gauge (53). Also, Autio et al state that comparison of

FE results with test data for manually laminated parts is not very meaningful as ply thicknesses and fibre orientations can differ considerably from those specified. The paper concludes that although FE is well suited to the design of composite structures it is important that the composition of the laminate is known accurately, and that the mechanical properties and boundary conditions must be carefully specified.

There are three ways in which the mechanical properties required for an analysis of a composite material may be found. The first, and most obvious method, is to ask the material supplier for the information. However, the properties provided by some material suppliers are often inadequate for analysis purposes. Often only a single value of Young's modulus and Poisson's ratio are available. Only occasionally is data available for all three axes.

Another method of determining the mechanical properties required is to manufacture a sample of the material and to determine the properties required experimentally. This however is time consuming as test specimens first have to be manufactured with the correct fibre volume fraction and fibre orientation. Further tests will then be required if either the fibre volume fraction or the fibre orientation are changed. When the mechanical properties of an anisotropic composite material are to be determined experimentally large test programmes are usually required. As Johnson (30) discusses, most testing for mechanical properties of laminates at present is performed by the material manufacturers themselves whose main concern is to evaluate typical properties for comparison with other materials, rather than the particular properties of that material. As noted by Griffin (47) the lack of reliable mechanical property data is "a major difficulty in performing accurate analysis". Datoo (11) adds that the mechanical properties are dependent upon the manufacturing method and hence test coupons must be obtained from the same manufacturing method as for the finished composite component. Thus composite samples manufactured for test purposes may not be subject to the same processing conditions as the final component, and the properties obtained from such tests are therefore only predicted properties. Accurate properties will only be obtained from performing tests upon the finished component.

The third option is to determine the required mechanical properties mathematically given the constituent properties, the fibre orientation and the required fibre volume fraction. Once the properties of the laminae are known then the overall properties of a laminate may be determined.

Composite materials are often studied from two points of view, namely, *micromechanics* and *macromechanics*. The study of the interaction of the constituent materials of the composite on a microscopic scale is termed 'micromechanics'. The objective of micromechanics is to predict the behaviour of the individual laminae from their constituent properties and volume fractions (12,33,54). To obtain the effective mechanical properties of a composite a mathematical model of the material based on the microgeometry and constituent properties is required (55). An advantage of these micromechanical models is that the mechanical properties determined are a function of the constituent properties which should be obtainable from the material manufacturer. Micromechanical analysis can be used to predict the effective composite properties such as Young's modulus, Poisson's ratio and shear modulus. However, Chamis (56) states that although a considerable amount of research has been conducted in the area of composite micromechanics, only partial success has been achieved.

The micromechanical analysis will determine the local stress and strain fields within the composite. This leads on to the creation of a 'macromechanical' model, which will also contain information on the microstructure of the composite (55). The overall composite material at this point is considered to be statistically homogeneous (28). The effects of the constituent materials are only dealt with as average, or effective apparent properties of the composite as the properties of the laminate are determined from those of the constituent laminae (12,28,29,57). In other words, in order to determine the overall mechanical properties of a composite, micromechanics is first used to determine the properties of each lamina, and then macromechanics is used to combine these lamina properties to determine the overall properties of the laminate. Whilst the macromechanics equations appear to be well defined – see quoted references, no definitive micromechanical equations appear to exist for all of the mechanical properties of interest.

The static strength of a composite is not included in the above list of predicted properties as it is related to the mechanics of individual fibre matrix interaction (30). The static strength "cannot be treated in terms of the overall response of the material" (29), and therefore the prediction of it is less well understood than that of the elastic properties and is "difficult" (54,58,59). The micromechanical analysis requires a single fibre strength whereas in a composite there will be a range of fibre strengths due to imperfections in their structure. The subsequent effect upon the local stress values makes the problem very complex as the process of failure implies fundamental changes in material behaviour and phase geometry (7,26,59). Hashin (7), writing in 1974, stated that the prediction of composite strength is of such difficulty that with the present state of knowledge "it does not seem possible to predict a coherent, reasonably rigorous development". It is reported that at present no satisfactory models exist to predict the strength of a composite from the properties of the constituent materials (41,48,60,61). The models that do exist give results that are at least 50% too high (41,48). Thus, most composite structures are certified by test rather than by analysis, with matters unlikely to change unless more realistic failure theories are developed (60–62).

It is recommended by Johnson and Houston (63) that the mechanical properties of composites used for design purposes be 'adjusted' to take account of likely reductions of the properties by the in service exposure to environmental effects such as temperature and moisture. The effect of such environmental effects are beyond the bounds of this report, and thus will not be considered.

It can be seen that if composite components are to be analysed with any accuracy it is of the utmost importance that all of the mechanical properties required in the analysis are known accurately.

3.1 Computer Packages To Determine Mechanical Properties

A microcomputer package has been developed by Cirese (64) to analyse laminated composite structures. Some of the inputs to the package are the thermo−mechanical properties of the material and the package will then calculate the strength of symmetric and unsymmetric laminate plates, and also predict the material properties of the laminate from micromechanics equations. The mechanical properties predicted were then compared to test results and showed in some cases a fairly small error. According to the paper the mathematics behind the programme are "well known".

The National Physics Laboratory (NPL) have also developed a computer package to analyse laminates in their project entitled "Predictive Modelling of Fibre Reinforced Polymer Composites" (12,65). Their package starts at a point before that of Cirese (64) by allowing the user to input the constituent material properties and fibre lay−up. The software will then use micromechanics theories to calculate the lamina or layer properties and then macromechanics to calculate the overall laminate properties. The properties predicted by the software are very broad and include Young's modulus, Poisson's ratio, shear modulus, thermal expansion coefficients and strengths. The constituent materials may be, for example, thermoplastic or thermoset resins, with carbon or glass fibres. The NPL project has found that some of the micromechanics theories predict some of the material properties accurately but other properties less accurately. The project has also found that some theories are more accurate at low fibre volume fractions, whilst some are more accurate at high fibre volume fractions. However, the resulting NPL software uses a common theory to predict all of the material properties, because "otherwise the programming would be too complicated" (41), and thus not all of the properties will be predicted accurately for all volume fractions. Whilst the software is a very useful tool for designers in the first instance it is based on a relatively small literature survey and should be expanded to include more detail on, for example, the empirical factors found in some of the equations. The predictive equations used by NPL have been checked for their accuracy by comparing the results with experimental data for a limited number of different materials such as SMC, GMT, carbon fibre composites, glass fibre composites.

Similar software to that of NPL is produced by SDRC in their "IDEAS Laminate Composites" package (66). The SDRC software package gives the user the choice of which micromechanics theory to use for the property predictions. This implies that the user needs to be familiar with the different theories before using the software. This is something that is not helpful to the ordinary designer who will not have enough knowledge to make a reasoned choice of which theory to use to calculate the properties. In both the NPL and SDRC packages the through thickness properties whilst said to be important are not determined.

Some other researchers (67) have developed a software package called COMPO which uses an FE model to determine the moduli of continuous and short fibre reinforced composites. The software allows the user to select such things as the fibre dimensions, arrangement and spacing and then determines the composite moduli. However, this sort of information would not appear to be readily available to the majority of designers and thus the software would appear to have limited use. Another commercial package called MISTRA (68) analyses the geometry of a unit cell of the material which is input by the user. FE analysis is then performed to determine the mechanical properties of the composite.

3.2 Determining The Mechanical Properties From Resonant Frequencies

A quite different method of predicting composite properties is presented in a Ph.D thesis by Sol (69) and a number of papers (1,70–73) by which the elastic properties of a composite material plate are determined using experimentally measured resonant frequencies. The measured frequencies from the test plate are compared to those obtained from the numerically computed resonant frequencies of a Rayleigh–Ritz model where the parameters within the numerical model are the unknown elastic properties. These parameters within the numerical model are initially guessed and are then "tuned iteratively" until the computed resonant frequencies match the measured ones. The paper states that "in most practical calculations, linear material behaviour is assumed." The main reasons for this being (1,69):

1. A lot of materials can be approximated in a satisfactory way by a linear model.
2. Non–lineararities are difficult to model mathematically and to measure experimentally.
3. The computer execution time for a non–linear analysis is several orders of magnitude higher than for a linear analysis."

The above method does have the following advantages (1,69–71):–

1. It is a non destructive test.
2. Averaged results are obtained over the domain of the plate.
3. Several elastic constants are found with one experiment.
4. The method is capable of producing very accurate results and a good error estimation.
5. There is no need to machine test specimens therefore damaging the fibre structure.

Disadvantages are stated as follows (1,69,70,72):–

1. Linear material behaviour is assumed.

2. No failure information is obtained.

3. The experimentalist must provide starting values for the plate rigidities.

4. Small variations in thickness of the test specimens can cause errors in the moduli determined

Chapter 4 : Micromechanical Models For Uni–Directional Composites

Micromechanics deals with the determination of the properties of the composite material from those of its constituents, ie the fibre and the matrix, and their relative volume fractions. In this type of analysis it is "usual to consider a composite where the cylindrical fibres are accurately aligned in one direction and separated from one another by a resin matrix" (40). In order to calculate the mechanical properties of a composite material the basic fibre and matrix properties in terms of axial stiffness, strength, Poisson's ratio and relative volume fractions are required (4).

A uni–directional composite material will exhibit different properties along and perpendicular to the fibre direction. Thus, it is necessary to define the various directions of the material in terms of a set of co–ordinate axes. The '1' or 'longitudinal' axis is taken parallel to the fibres, the '2' or 'transverse' axis is taken within the plane of the ply and perpendicular to the fibre direction, the '3' axis is taken perpendicular to the plane of the ply (29) – see Figure 3.

A number of researchers (49,59,61,75–79) report that as in most structural composites the fibres are circular and arranged randomly in a plane, the overall composite in a macroscopic sense is transversely isotropic. This will mean that the properties in the fibre direction tend towards those of the fibre, whilst those perpendicular to the fibres will tend towards those of the matrix (29). If the material is transversely isotropic then the properties perpendicular to the ply are assumed to be the same as those transverse to the ply such that (7,11,25,26,29, 56,61,75,77,80–86):–

$$E_{22} = E_{33}\,, \qquad G_{12} = G_{13}\,, \qquad \nu_{12} = \nu_{13}$$

Thus a uni–directional composite material can be characterised by six elastic constants, namely: E_{11}, E_{22}, G_{12}, ν_{12}, G_{23} and ν_{23}.

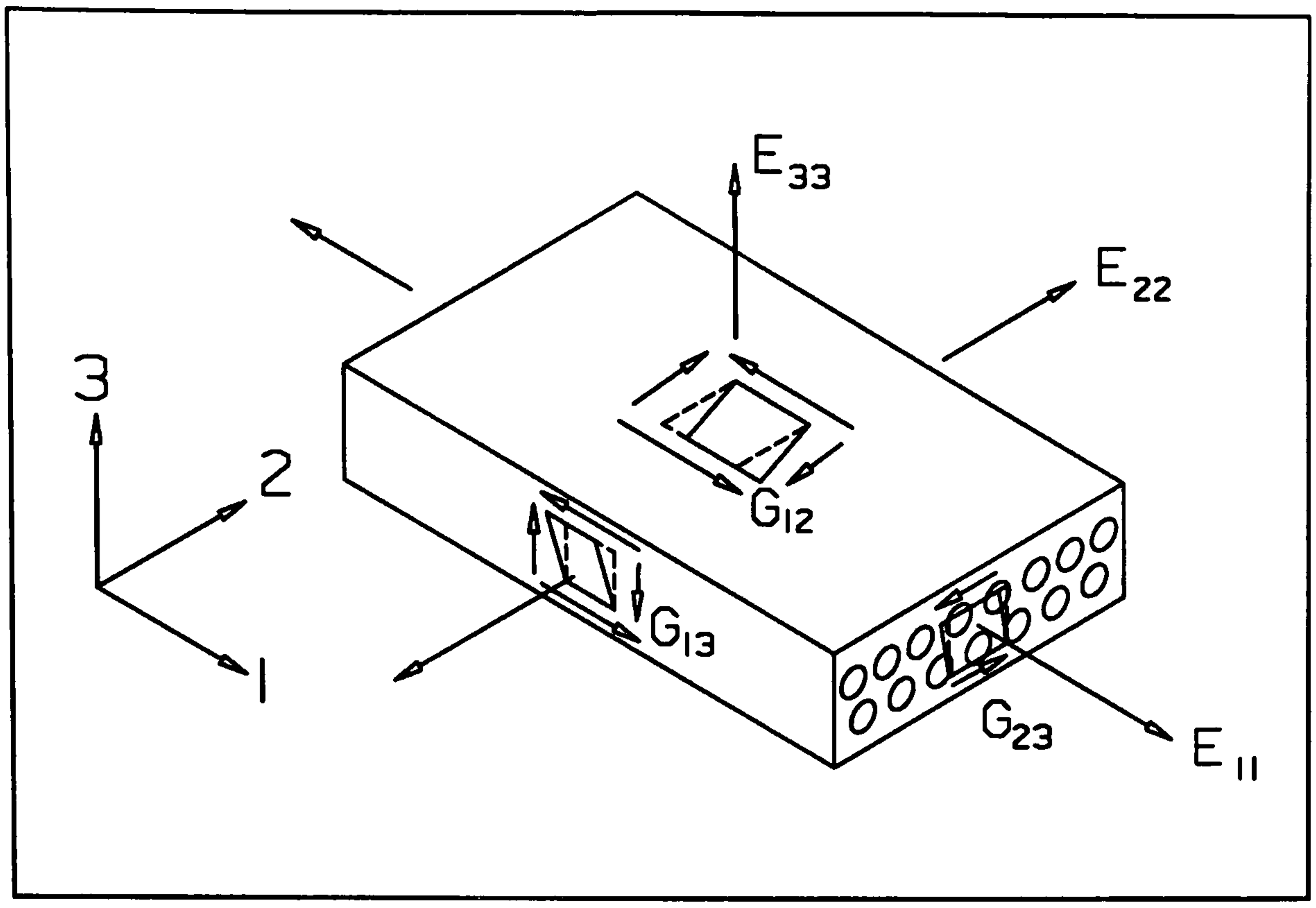


Figure 3 : Cartesian Co–ordinate System For A Uni–directional Composite (25)

4.1 Constituent Materials

A report by the Industrial Advisory Committee on Fracture Avoidance (IACFA) (75), and other references (12,27,29,82,87–90) state that if the fibre material is glass then the fibre is assumed to be isotropic. However, if the fibre is carbon or kevlar then the fibre is assumed to be transversely isotropic, or even anisotropic in the case of boron fibres (29,89–91). The matrix phase is usually considered to be isotropic (29,89,90,92). This thesis is only concerned with glass fibre reinforced composites and thus will concentrate on equations which predict the properties of composites with isotropic constituents.

When the phases, or constituent materials, can be considered isotropic the expression for the bulk modulus is replaced by the expression for the plane strain bulk modulus such that (7,26,29,33,45,93,94):–

$$K = E / 2(1-\nu-2\nu^2) = G / (1-2\nu) \qquad [7]$$

However, for transversely isotropic constituents then the bulk modulus and plane strain bulk modulus are equivalent. Thus, as this work only considers isotropic phases then equation *[7]* for 'K' will be used.

In order to calculate the overall material properties of a composite a geometrical model of the composite – usually a uni–directional composite – is assumed. The model of one fibre and its surrounding matrix is known as a representative volume element (28,78). To predict the mechanical properties of the composite a representative volume element "which is small enough to show the microscopic detail, yet large enough to represent the overall behaviour of the composite" is used by researchers (54). Once this representative volume element has been defined then some boundary conditions must be described. The boundary conditions must be representative of the in situ state of stress and strain within the composite (54). The composite mechanical properties can then be predicted by solving the boundary value problem (54,75). Tsai and Hahn (54) state that although the procedure involved in solving this problem is "conceptually simple" the actual solution is "rather difficult". Thus, as a result of this many assumptions and approximations have been made by the various researchers with consequently many solutions available. However, as Kardos (24) states "any workable prediction method must be straight forward enough to use ..... without consuming huge amounts of expensive computing time".

The behaviour of the composite may therefore be predicted from the behaviour of the representative volume element. However, for a model of the composite to be realistic it must take into account the irregular fibre spacing, variation of fibre radii, and the range of elastic moduli (95). It is not usual for the problem to be treated in this complexity and simplifying assumptions are often made. An idealised representative volume element which does not necessarily represent circular cross section fibres can be seen in Figure 4.

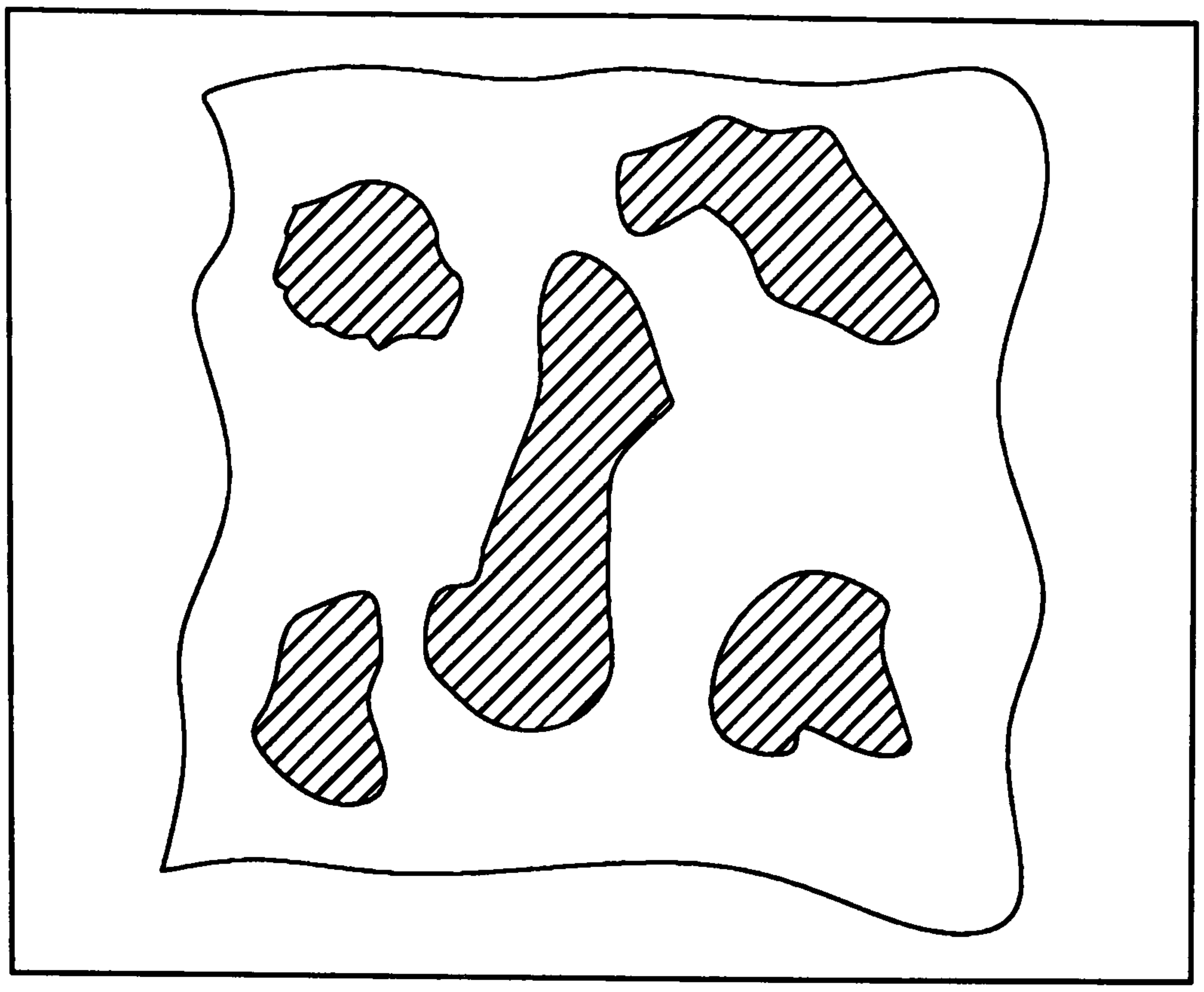


Figure 4 : An Idealised Representative Volume Element (13).

4.2 Assumptions

A micromechanical analysis of a uni–directional composite material is based on a number of simplifying assumptions, and hence an analysis of a composite material through the use of micromechanics does have limitations and may contain inaccuracies. These assumptions are as follows (3,12,27,28,33–35,46, 75,87,96):–

a) That fibres are circular, and have equal and uniform size, with a very large ratio of length to diameter such that they can be considered to be of infinite length and fibre end effects ignored.

b) That the composites/laminae are homogeneous in the macroscopic sense, linearly elastic, macroscopically transversely isotropic, and initially stress free.

c) The fibres are homogeneous, linearly elastic, isotropic, perfectly aligned, and not twisted.

d) The matrix material is homogeneous, linearly elastic and isotropic. The matrix material will in general have a limited linear stress–strain range with a much higher elastic range that is non–linear, however, we are considering short term loading and neglecting viscoelasticity effects.

e) Perfect bonding is assumed between the fibres and the resin. On one side of the interface the properties of the fibre are used, on the other the properties of the resin are used. The interface between the fibre and matrix constituents of a composite is very important as the load transfer between the constituents "strongly depends on and is controlled by the degree of contact and cohesive forces at the interface" (46). Modelling the interface as a third homogeneous region having a finite but very small thickness is said to have a negligible effect on the estimate of the elastic constants for the composite, and it is recommended that this interface is not modelled (46). According to Chamis (27) a perfect bond does not exist between the fibre and matrix and thus this assumption is not valid. However, Kardos (24) reports that the degree of adhesion between the reinforcement and the matrix has a much larger effect upon the strength than the modulus. It is reported (46,47,87) that the nature of the fibre matrix interface is so poorly understood that attempts to model or analyse it suffer disastrously from a lack of accurate material properties.

f) There are no voids present in the composite. Note that in reality the in–situ matrix material is "seldom if ever free of voids" (27).

It is noted by Chamis (27) and Griffin (47) that an additional assumption made is that the bulk and in–situ properties of the constituent materials are the same. Chamis observes that whilst this is true for the fibres, it has not been verified for the matrix. As the matrix properties are determined from the bulk material then these may not be the actual matrix properties in situ in the composite (29,47,48). Griffin (47) writing in 1990 notes that there are no means currently available for measuring the properties of the resin in the composite. Some of the factors influencing the in–situ properties are (27,29,48):–

i) The geometry of the in–situ matrix. As the matrix between the fibres is in some places very thin it is reasonable to expect different molecular alignment and thus different resistance to deformation when compared to the bulk properties.

ii) Non uniformity of matrix thickness between fibres.

iii) Environmental effects due to the fabrication process of the composite.

iv) Residual stresses caused by the fabrication of the composite.

v) Voids in the material.

As previously stated, most researchers who attempt to predict the properties of uni–directional composites assume that the fibres are perfectly aligned. However, some researchers (97–100) study the influence of fibre curvature upon the uni–directional composite properties. This work assumes that the fibres are curved in a single plane and that their diameter is small compared to the curvature. The researchers conclude that the "effect of curvature on the elastic properties is very important " as one might expect (97), and that "relatively small curvature of the fibres causes significant changes in the modulus of elasticity" (98). Tarnopolskii et al (98) note that the probability of fibre curvature increases with the dimensions of the moulded part, and that this is one of the reasons why predicted properties obtained from test pieces or calculations may differ considerably from those obtained from the finished product. Tarnopolskii et al propose a formula for determining the longitudinal modulus of a uni–directional composite taking into account the fibre curvature. However, the formula requires the use of a factor related to the fibre curvature which can only be determined from an experiment using the finished part.

In contrast to other researchers, Sideris (101), assumes that perfect bonding between fibre and matrix does not exist, and that the interface between the fibre and the matrix has a significant effect upon the in plane shear modulus of a uni–directional composite. However, the equations for the in plane shear modulus presented by Sideris include a term representing the radius of this interface layer. Thus the modulus cannot be determined from the constituent properties alone, and thus this work has little practical use. Taggart and Muthu (102) also state that although the interface stiffness does not significantly effect the longitudinal Young's modulus or Poisson's ratio, it does effect the longitudinal and transverse shear moduli.

Unlike most authors who analyse composites reinforced by circular fibres, Zhao and Weng (103) determine the elastic constants of an orthotropic composite reinforced with aligned elliptical fibres. Zhao and Weng also determine the elastic moduli of a transversely isotropic composite reinforced with randomly orientated elliptical fibres. The moduli are given in terms of the cross sectional aspect (length/width) ratio of the elliptical fibres, and their volume fractions. Zhao and Weng state that the effective elastic properties of a composite are known to depend on the microgeometry of the reinforcement, and that when the fibres exist in the form of elliptical cylinders, the aspect ratio of the fibres may have a significant effect on the overall moduli of the composite. The objective of the work by Zhao and Weng was to examine the influence of the aspect ratio of elliptical reinforcing fibres on the effective moduli of the composite. Work has also been undertaken in this area by Tandon and Weng (104) where the short fibres are randomly oriented, and the elastic moduli have been established as a function of aspect ratio.

The IACFA report (75) concludes that an ellipsoidal model of the reinforcement is not suitable for representing short fibre composites unless the fibres are needlelike in shape. Such models should only be used in the case of needlelike fibres or particulate reinforced composites, and that there is little point in using ellipsoids to model continuous fibres. Thus, the equations obtained from ellipsoidal models have not been included in this work.

4.3 Types Of Models

It is reported by Jones (28) that there are two basic approaches to determining the material properties of a composite. The first is the mechanics of materials approach. The main assumption made by this approach is that in a uni-directional composite subjected to a load in the fibre direction, the strain in the fibres and the matrix is the same. Thus, by examining the stress-strain relationships in the fibres and the matrix the overall properties of the composite can be determined. The alternative approach is the theory of elasticity. In the elasticity approach the strain energy of the composite is used to derive the overall composite properties. Either bounds on the material properties or exact solutions may be found with the elasticity method. The elasticity approach is thus said by Jones to break down into three areas – 1. Bounding principles, 2. Exact solutions, 3. Approximate or semi-empirical solutions. The different approaches use either derived equations which are sometimes complex or analytical methods such as finite element analysis to investigate the relationships between the stress and the strain in the overall composite.

As this thesis is not concerned with the detailed mathematical derivation of equations for the prediction of the composite mechanical properties, but instead, with the application of these equations, then only the equations themselves will be presented. Readers wanting more information concerning the mathematical modelling of the composite should consult the references listed. Rigorous derivation and discussion of the relative merits of the models used by researchers can be seen in particular in the following references (13,28,29,34,74). The detailed equations given by researchers can be seen in Appendix A.

4.3.1 Mechanics Of Materials

The simplest models available for predicting the mechanical properties of a uni-directional composite material are the 'parallel' and 'series' models from the mechanics of materials approach. The parallel model is used to predict the longitudinal properties, whilst the series model is used to predict the transverse properties (13,29). These models give equations that are known as the longitudinal and transverse law (or rule) of mixtures equations.

An assumption that is made with parallel model is that of isostrain, ie, the strains in the direction of the fibres are the same in the matrix material and the fibre. This is also known as the "Voigt" model (13,29). This assumption means that neither fibre distribution in the ply or the cross sectional shape of the fibres is considered (29). In general terms the longitudinal law of mixtures equation is given as:–

$$P = P_f V_f + P_m V_m \qquad [8].$$

Where, P is the property to be determined, P_f is the fibre property, P_m the matrix property, V_f the fibre volume fraction and V_m the matrix volume fraction.

It is reported by Kempe's Engineers Year Book (25) and Johnson (30) that the law of mixtures equation is "well supported by experimental data", and Whitney and McCullough (29) state that these relationships give "excellent agreement" with experimental properties. However, most of the sources quoted above do not give evidence of this by comparing the equations with experimental data.

Johnson (30), comments that the weight fraction of the constituents is often easier to determine than the volume fraction. Equations relating volume fraction to weight fraction can be found from a number of sources (12,40,105) and are:–

$$V_f = W_f \varrho_c / \varrho_f \qquad [9]$$

$$V_m = (1 - W_f)\,(\varrho_c / \varrho_m) \qquad [10].$$

Where 'W' is the weight fraction, and 'ϱ' the density. Note that some of the subscripts and symbols used have been altered to make them consistent with other equations from other sources.

An alternative version of equation *[9]* is given in a paper by Knibbs and Morris (32), and is as follows:–

$$V_f = W_f / V_c \varrho_f \qquad [11],$$

where 'V_c' is the volume of the final composite.

A close inspection of the two equations, *[9]* and *[11]*, shows that they cannot be equated to one another, thus suggesting that one of them is incorrect. The fibre volume fraction should be a dimensionless quantity if it is to be used in equations such as the law of mixtures. The first of the above two equations is dimensionless when examined, but the second one is not, suggesting that it is the second equation presented by Knibbs and Morris that is incorrect. Thus this equation has not been used in this work.

The series model gives equivalent law of mixtures relationships for the transverse effective properties of a uni–directional composite. This model uses the "isostress relationship", ie, assumes uniform stress, and is also known as the "Reuss" model (13,29,55,105–107). In the general form this relationship is as follows:–

$$P = P_f P_m / (P_f V_m + P_m V_f) \qquad [12]$$

It is reported by Sendeckyj (106) that the values obtained from the above transverse equation are in "reasonable agreement" with experimental data. However, Whitney and McCullough (29) state that the above relationships give a "serious underestimate" of the transverse properties. This is supported by Pagano and Tsai (108) who state that the above equation was "shown experimentally to be quite poor". The transverse law of mixtures equation is also given in the IACFA report (75), but this reports that the equation underestimates the value of E_{22} by an amount that depends upon the connectivity of the fibres which increases with increasing V_f. Jones (28) observes that from the transverse law of mixtures equation it can be seen that the fibres do not make much contribution to E_{22} unless the percentage of fibres is high, ie around 0.5. Consequently Whitney and McCullough (29) conclude that the major difficulty in predicting the behaviour of a continuous uni–directional fibre reinforced composite concerns the prediction of the transverse properties. The authors state that improvements can be made to both the longitudinal and transverse law of mixtures relationships by considering the packing geometry of the fibres, and assuming either a square or hexagonal array. However, these assumptions lead to expressions that include the fibre spacing dimension which, of course, varies with V_f and is difficult to determine and hence has little practical use.

In a laminated composite there are three shear moduli to be considered (75). These are the in–plane, interlaminar, and transverse shear moduli – G_{12}, G_{13}, G_{23}. For a single layer of uni–directional composite material as the material is transversely isotropic $G_{12} = G_{13}$ (29,56,75,77,85,86), and the transverse shear modulus G_{23} is stated to be always slightly larger than the in plane shear modulus G_{12} (85).

It is reported by Clyne (80) that the transverse law of mixtures equation significantly underestimates the value of G_{23}. A better equation, for a composite which can be classed as transversely isotropic is obtained by assuming that the material is isotropic in the 2–3 plane, and thus the isotropic relationship equation *[4]* can be used. However, this relationship does first require the determination of E_{22} and ν_{23} and as can be seen in Appendix A a number of different equations have been found in the literature for the prediction of these two properties. Thus the determination of G_{23} from this relationship is open to interpretation, and many different values may be obtained. It is therefore felt that equations which predict G_{23} directly from the constituent properties, hence leaving no room for interpretation, are more useful unless consensus can be reached on which equations should be used to predict E_{22} and ν_{23}. For this reason the isotropic relationship between E_{22}, G_{23} and ν_{23} will not be considered further.

The rule of mixtures relationships for the transverse properties would appear to give an underestimate of the actual values of the composite. This discrepancy is said to be a direct consequence of the assumption of uniform stress (28,29). Thus improved models are required to predict these transverse properties and also the shear moduli.

4.3.2 Bounding Techniques

Much work has been undertaken in the area of predicting the upper and lower bounds of the material properties of composites by authors such as Hashin (7,8,45,109,110), Hashin and Rosen (10), Paul (111), and Hill (42,112). This work concentrates on the prediction of the upper and lower bounds of the moduli rather than determining formulae for the exact calculation of the moduli. In these papers experimental values of the material properties are compared with the calculated bounds, and are shown to lie within these bounds. However, the approach generally leads to bounds that are not sufficiently close for practical use (28). The mathematical techniques used are the 'variational energy principles of classical elasticity theory', or simply the 'variational approach'. For a linearly elastic material the stored elastic energy can be expressed in alternate forms, and the bounding methods make use of this (29). Bounds on strain energy (the principle of minimum potential energy) give an upper bound for the effective elastic modulus, whilst bounds on stress energy (the principle of minimum complementary energy) give a lower bound on the effective modulus (28,34,61,74).

An estimate of the bounds obtainable are the relationships for the longitudinal and transverse law of mixtures (28,74). These relationships give the upper and lower bounds respectively. A good approximation to the property being calculated can generally be obtained by averaging these upper and lower bounds unless they are too far apart (113). However, bounds estimated from the rule of mixtures relationships are said to be too far apart to be useful, and that the values of properties given by upper and lower bounds only provide practical guides if these bounds are reasonably close together such that the actual properties are bracketed to within experimental error (7,28,29). Improved bounds which are closer together can be determined using complex mathematical techniques, (variational calculus), and yield results which are said to be identical to those reported by Hill (112) for the self consistent model (29). In order to obtain improved bounds greater knowledge concerning the details of the microstructure are required. Since this sort of detailed knowledge may not readily be available the improved bounding techniques have concentrated on establishing the best possible bounds based on information relating only to the volume fraction and constituent properties (29,74). As a consequence some of these improved bounds still remain too far apart to be of practical value (29,95). However, for the sake of completeness information concerning the bounds of the moduli will be included here.

Such improved bounds are derived by Hashin and Rosen (10) for materials reinforced by parallel hollow (or solid) circular fibres. They state that "exact results have been obtained for hexagonal arrays of identical fibres, and approximate results for random arrays of fibres...." The work assumes that both the fibre and matrix materials are linearly elastic, isotropic and homogeneous. Their analyses use the concentric cylinder assemblage (CCA) type of model. For the random array of fibres the model consists of the fibre with the matrix material around it in proportion to the volume fraction of the matrix in the overall composite. The bounds found for the hexagonal array of fibres are said to be rather far apart (28). Also, the model of the random array of fibres is stated to be an inaccurate representation of a composite as most practical composites do not contain fibres of varying sizes (28).

Hill (42,112) also presents work which determines the bounds of the moduli of aligned and continuous solid cylindrical fibres embedded in a matrix. Hill however assumes that both the matrix and fibre material are homogeneous, and elastically transversely isotropic about the fibre direction. Hill states that his predictions for the bounds of the moduli can be unreliable when the "ratio of fibre and matrix rigidities is extreme." Thus, Hill concludes that the predictions are most useful at intermediate concentrations, but does not give a range of values for which the predictions can be used.

In two papers (114,115) Heaton presents a technique for calculating the "elastic constants of a uniaxial fibre reinforced composite" in which the long fibres are arranged in a square array rather than the hexagonal one used by other authors. Heaton states that the bounds on moduli determined by Hashin and Rosen (10) are "too far apart to be of value in practice" when there are extreme ratios of fibre to matrix properties. The overall elastic moduli of the composite are determined by imposing strains which are "reflected at the microscopic level as constraints" (114) on a small unit cell of material consisting of one fibre and its surrounding matrix. The stiffness constants of the material can then be evaluated by "considering the composite to be in a state of transverse plane strain" (114). Heaton concludes that the predicted longitudinal Young's modulus agrees to within 1% with the value calculated by the law of mixtures method. The second paper presented by Heaton (115) extends the technique to include transversely isotropic fibres in an elastically isotropic matrix.

4.3.2.1 The Composite Cylinder Assemblage Model

The only existing model that permits exact analytical determination of effective elastic moduli is said to be the composite or concentric cylinder assemblage (CCA), as the assumptions made with this model satisfy the equations of elasticity (6,74,75). The CCA type of model consists of a circular fibre core and a concentric matrix shell, see Figure 5. From such a model the analytical results can "predict effective elastic properties with sufficient engineering accuracy" (6). Rosen and Hashin (6), go on to state that the CCA type of model is of considerable importance as it permits "easy determination of effective properties for a variety of matrix properties, fibre properties and volume fractions." However, Adams and Tsai (116) report that the properties predicted by the CCA model are in "poor agreement with experimental data". The predictions made by the CCA model should be good for fibre volume fractions (V_f) of less than 0.35 but the predictions will become increasingly less accurate as V_f increases to the commercial standard of 0.6 (116).

As can be seen from Figure 5, the CCA type of model assumes that there is a number of different sized cross sections of fibres present in the composite, and thus is not an accurate representation of most practical fibre reinforced composites (28). Each cylinder within the CCA model is assumed to have the same ratio of fibre radius to outer matrix radius. This ratio corresponds to the fibre volume fraction of the composite. Hashin (74) states that a comparison of results for the effective elastic moduli from the CCA model and a hexagonal array model are extremely close up to $V_f = 0.7$. This leads Hashin to conclude that as long as the fibres are circular and not in contact, then their actual locations and variations in diameter do not have a significant effect on the results for the effective moduli. However, for a V_f of greater than 0.6 some contact between fibres would be expected.

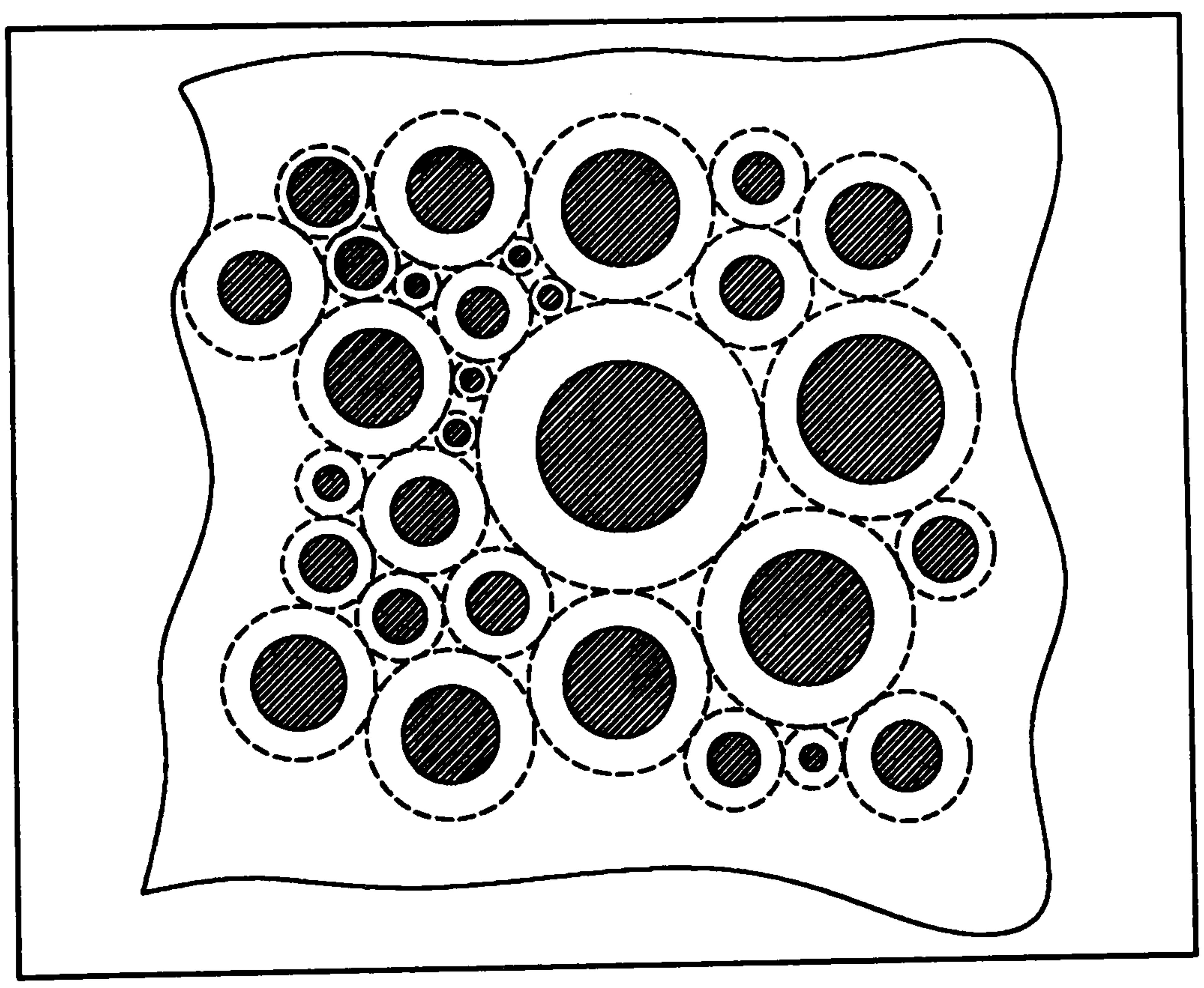


Figure 5 : A Composite Cylinder Assemblage Model (7,59,74).

The CCA model is a commonly used one for analysing uni-directional composites. The IACFA report (75) recommends that E_{11}, ν_{12}, K_{23} and G_{12} are estimated from the results given by the CCA model. Various references (6,7,26,42,45,57,59,110,112,117) give equations relating the constituent mechanical properties to the effective mechanical properties of the composite based on the CCA model. The derivation of these equations assumes that both the fibre and matrix material are isotropic. Rosen and Hashin (6) report that results from the equations determined from this model were compared with experimental values and that the "agreement is excellent." However, Rosen in his paper (117) adds that the value of the shear modulus as determined by this model is lower than measured values, and that some correction factor is desirable. Indeed, Rosen reports that all of the equations determined by the CCA model are susceptible to empirical modification.

The CCA model has also been used by Qiu and Weng (118), and Pagano and Tandon (43) to analyse coated fibre and particle reinforced composites where the model consists of three rather than two distinct phases. Although coatings may occur as a result of the processing conditions during manufacture it is not proposed to include coated fibres as a part of this study.

4.3.4 Exact Solutions

The idealised representative volume element does not lend itself readily to analysis by mathematical methods, and thus regular models of fibres are usually assumed in order to simplify the mathematics. One such model is to assume a representative volume element consisting of "periodic arrays of identical circular fibres, for example, square and hexagonal periodic arrays" (6). These models are often analysed by finite element analysis. By using either stress or displacement functions, and by requiring the continuities of stresses or displacements at the boundaries between the fibre and the resin, the stresses in the composite can be related to the imposed strains, and thus the elastic constants can be obtained from the relationship between the stresses and strains in the material (3,75). The square array model is not suitable for most uni−directional composite analyses, because it is not transversely isotropic, however the hexagonal model is transversely isotropic and is thus more suitable (28,74,75). Also the square array model gives higher moduli values for a given fibre volume fraction than the hexagonal model (95,116). The representative volume element presented by such arrays is that of a circular fibre surrounded by either a square of matrix material, or a hexagon of matrix material, see Figures 6 and 7. It is reported by Jones (28) that the results obtained from a hexagonal array model "agree better" with experiments than those obtained from the square array model.

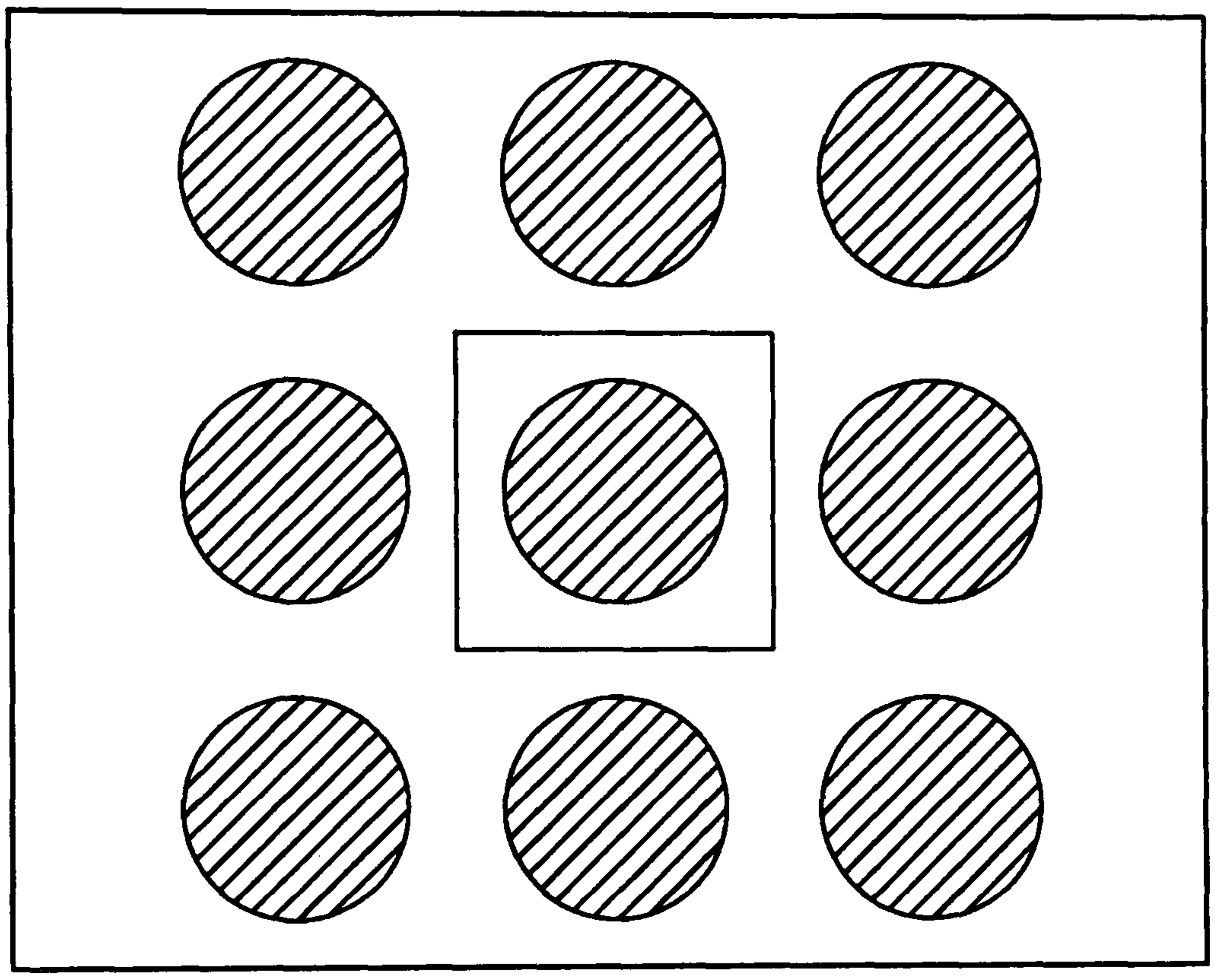


Figure 6 : A Square Array Model (6).

Griffin and Vidussoni (119) state that "at the microscopic, or micromechanics level, both elasticity solutions and finite element models have been used to predict the response of very small quantities of material, normally one fibre and the surrounding matrix. It is apparent that while such analyses lead to understanding of the materials and how stresses are distributed internally they are useless as a tool for designing composite structural components, since many degrees of freedom are required to model a part of the material whose dimensions are only several microns."

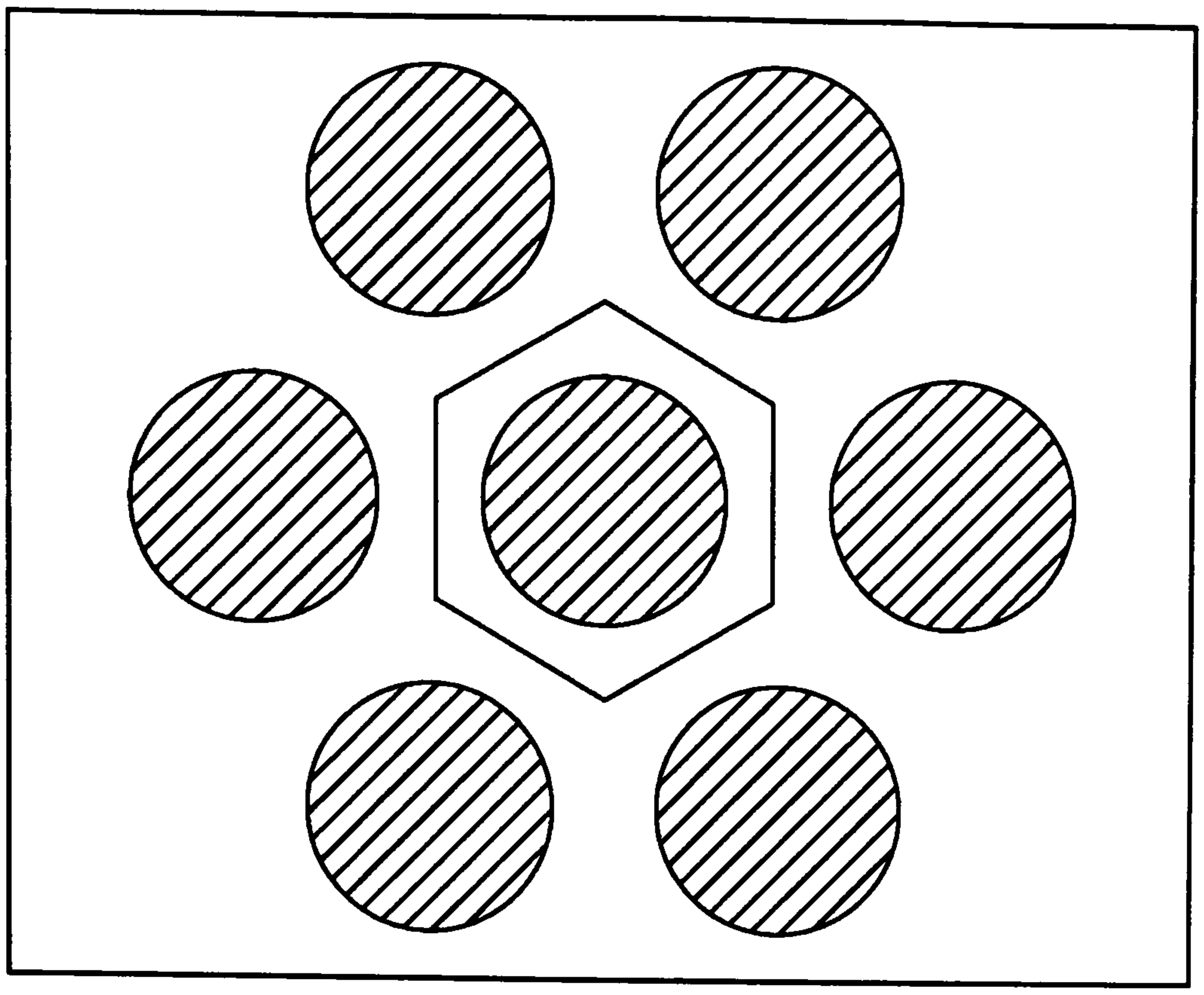


Figure 7 : A Hexagonal Array Model (6).

An alternative method of analysing composites termed the 'method of cells' is presented by Aboudi (46,87). The theory analyses a repeating cell of a uni−directional composite. The repeating cell consists of four subcells, one of which is occupied by the fibre and the other three by the matrix material. The micromechanical analysis of this repeating cell leads to two relationships between the average stress and strain from which the effective elastic moduli can be determined.

In the various mathematical models adopted by researchers the maximum volume fraction that the fibres can have because of packing difficulties due to fibre − fibre contact is given by the maximum packing fraction. The maximum packing fractions possible for uni−directional fibrous composites are given as (20):−

Hexagonal packing $V_f = 0.907$

Square packing $V_f = 0.785$

Random packing $V_f = 0.82$

A number of authors (28,29,34,74,75,116) report that representative volume elements that have a non−circular cross section, such as the square or hexagonal array models, require the use of either a series development or a numerical technique such as finite element analysis in order to solve the problem. If FE analysis is used then there is the problem of defining the boundary conditions. The boundary conditions on a repeating element of the array can be determined by symmetry considerations and thus the numerical analyses can be confined to a single repeating element (29,74). The effective properties can then found by numerical averaging (34,74). Whitney and McCullough (29) report that results from such numerical analyses are specific to the geometry and materials analysed, and thus have limited use. Jones (28) adds that the assumptions made in such analyses regarding the fibre–matrix interaction are not "entirely realistic". However, Hashin (74) reports that these exact methods are of "sufficient accuracy and reliability to render any approximate methods obsolete".

It is reported by Courage et al (55) that a disadvantage of these types of numerical solutions is that they have been calculated for a given set of constituent properties, such that a new analysis needs to take place when different properties are used. However, it would appear that this is the case for any method of calculating the effective properties of a composite from those of its constituents, and is not necessarily a disadvantage.

4.3.4.1 Finite Element Analysis

The rapid growth of composites as structural materials has resulted in the development of methods of analytically predicting their behaviour. The finite element method is emerging as a method of analysing the micromechanical behaviour of composite materials (120), as the theoretical analyses of composite materials have generally been semi–empirical or mathematically rigorous (121). Zybert et al (121) state that this latter approach has often resulted in complicated equations which required computer solutions, thus promoting the use of finite element techniques.

A finite element method for the determination of the material properties of composites is presented by a number of authors (50,55,79,85,116,121–126). Many of these authors such as Zahlan and Guild (50), Courage et al (55), Kelkar et al (124) and Caruso and Chamis (125), generally analyse uni–directional material and use FE analysis to create two models. One model is of the overall homogeneous material modelled with an overall material law. FE analysis is then used to calculate displacement values for an applied load as a function of the material parameters of the composite. The displacements in the actual composite are then analysed for the same boundary conditions by means of a detailed microstructural analysis. In this analysis different elements with corresponding material behaviour and properties for the different constituents of the composite are used. Both displacement fields are then compared and the material parameter values for the overall material law can be calculated. Most of the analyses assume that the constituents of the composite material are linear elastic and isotropic. The results of the analysis are reported to "agree well" with experimental data. Some authors such as Adams and Tsai (116) and Zybert et al (121) after undertaking an FE analysis of the microstructural model using constituent material data simply compare the results obtained with either the available experimental data (116), or values obtained from equations such as the law of mixtures (121).

An alternative method of obtaining the required material properties by using the finite element technique is that proposed by one researcher (122) who uses an iterative technique. The stiffness coefficients of the material are initially estimated, an FE analysis performed and the results compared to test results. The stiffness coefficients are then altered accordingly and the FE analysis performed again. This iterative loop is repeated until convergence of the test and FE results is achieved. This method is obviously not practical for use on a real component as the FE analysis needs to be performed before the component is manufactured and tested. In addition such an iterative process may be expensive in terms of computer time for a real component consisting of a large number of elements. A similar iterative technique is used by Mota–Soares et al (85) and Wearing and Patterson (123) to determine the mechanical properties of a composite laminate. The method used by the authors compares the measured and analytical results for the natural frequencies of a composite plate. The error between the two sets of results is then minimised and the resultant mechanical properties determined. This technique requires an initial 'guess' as to the composite properties which are 'fine tuned' by comparing the test and analytical results until a converged solution is achieved.

The effect of the fibre matrix interface stiffness on the overall effective composite properties of a uni–directional composite is studied using finite element analysis by Taggart and Muthu (79) and Sahu and Broutman (126). The analysis recognises that perfect bonding rarely exists at this interface. The authors use the composite cylinders geometry and show that varying the interface stiffness can significantly change the overall effective composite properties. The authors conclude that E_{11} and ν_{12} exhibit only slight dependence upon the interface stiffness, but that K_{23}, G_{12} and G_{23} are strongly dependent upon the interface stiffness. This is due to the reduction in the fibre matrix load transfer properties as a compliant interface significantly reduces these properties. The analysis is said to give excellent agreement with experimental results (126).

Whilst it can be seen that many researchers use the finite element technique to determine the required material properties of the composite, the question remains as to whether this is a practical method of analysis for the ordinary design engineer. When using this method the designer is having to create two models in order to analyse the composite component. One model is required of the overall component, and one is required of the detailed microgeometry, thus increasing the time taken for any analysis. The basic problem faced by the ordinary design engineer is that such an analysis of the microgeometry of the composite requires a large amount of microscopic geometric information to be known about the materials which may not be readily available. Another problem with using the finite element method is that every time any of the input parameters is altered such as the fibre volume fraction, the matrix material, or the fibre layup geometry then a new finite element analysis needs to be undertaken. As a report by Taig (127) states, the scale of the constituent materials is so small compared to the actual components to be analysed that the representation of the constituent materials "will rarely, if ever be undertaken".

If the micromechanical equations that are available in the literature predict the required properties with acceptable accuracy then there is no need to create the sort of detailed analytical models seen here as the available equations will provide sufficient information for the ordinary design engineer. These detailed FE models are useful if more detailed information is required concerning the interaction of the constituent materials and the importance and behaviour of the interface region.

4.3.4.2 The Strain Energy Equivalence Principle

Zhang and Evans (128−130) present a numerical method of predicting the mechanical properties of composite materials with anisotropic constituents. The numerical method used is the "strain energy equivalence principle". The mechanical properties of the composite are determined by equating the composite strain energy to the summation of the constituent strain energies. The composite is assumed to be linearly elastic, although the principle still applies to nonlinearly elastic composites (128,130). It seems that the elastic constants are solved by first performing an FE analysis to determine the strain energies and then using the equations given in the paper to determine the elastic constants. The inputs to the FE model are the material properties of the fibre and matrix, and what is analysed is a representative volume element whose mechanical properties are equal to the average properties of the particulate composite. This element consists of a circular fibre surrounded by a concentric circular matrix of a unit length. A uniform strain rate is imposed on the fibre, the resultant strain energies of the constituents summed, and the elastic modulus determined from equations such as (128,130):

$$E_{11} = 2\, U_{fe} / (\varepsilon^2 V) \qquad [13]$$

where U_{fe} is the summation of the strain energies resulting from the FE analysis, ε is the constant axial strain for a generalised plane strain state, and V is the overall volume of the concentric cylinder. The above method will give the material properties for a uni−directional material. It is reported by Zhang and Evans that the results obtained from this method show "good agreement" with results obtained from other methods.

Further work has been done by Zhang and Evans (129) in which they are able to consider the anisotropy of the constituent materials for a uni−directional composite. The different phases of the composite may all be considered to be transversely isotropic, whereas in most work to date, either all of the constituents are assumed to be isotropic, or only the fibres are taken to be anisotropic. The work is again based on the strain energy equivalence principle, and the expressions derived for the elastic constants of the composite are given in terms of the constituent constants and volume fractions. However, Zhang and Evans do state that the effective longitudinal modulus of the composite is predicted approximately by the rule of mixtures relationship even for fully anisotropic composites. Zhang and Evans (129) also state that their method may also be applied to determine the effective properties of nonlinearly elastic composites, provided that the nonlinear constitutive relationships exist analytically.

4.3.4.3 Other Methods To Determine The Mechanical Properties

Other methods of determining the composite mechanical properties are used by some researchers. One such method is presented by Behrens (131) for determining the elastic constants of "a great variety" of composite materials by "evaluating the dispersion of sound waves for long wavelengths".

Zhen and Brebbia (132) present work on the analysis of composite laminated plates with circular holes using boundary element methods. The origin of the material properties of the isotropic and anisotropic material analysed in the given examples were not given. Presumably they were obtained from the material manufacturers. The results of the analyses were compared with calculated results and showed a good accuracy. The unnamed origin of the material properties is not unusual in the papers studied.

Finite difference analysis is used by Termonia (133) to investigate the longitudinal modulus of aligned short fibre composites and their dependence on the fibre length to diameter ratio and fibre volume fraction. Termonia reports that the results for the longitudinal modulus found are in good quantitative agreement with experiments. The finite difference method is also used by Adams et al (83) to determine the properties of continuous uni–directional fibre reinforced materials.

A method for determining the material properties of uni–directional composites from those of its constituents and the relative volume fractions is presented in a paper by Dimmock and Abrahams (4). The authors state that methods for determining material properties "often require a knowledge of so many factors that it is sometimes simpler to determine composite properties experimentally, while some other theories are poorly substantiated by experimental data". Unfortunately the equations used by Dimmock and Abrahams in their method have been excluded from their paper as they are "mathematically complex". The axial material properties are calculated from the law of mixtures equations, whilst the transverse properties are calculated from other methods which are solved by finite difference analysis assuming that the fibres are arranged in a square array. The material properties at any angle relative to the fibre direction can then be determined using transformation equations.

4.3.4.4 The Self Consistent Model

A different type of model used by some researchers, such as Hill (42,112), is a "self consistent" model. This model consists of a fibre directly embedded in the equivalent transverse isotropic material representing the remainder of the composite whose properties are taken to be equivalent to the overall effective properties of the composite material. In this model there is no phase which represents the matrix material. Hill showed that the equations obtained from this model give reliable values at low fibre volume fractions, reasonable values at intermediate volume fractions, and unreliable ones at high volume fractions. This is supported by Hashin (7,74) who states that at fibre volume fractions of between 0.5 and 0.7 the self consistent model considerably overestimates the actual moduli of the material. Better and more realistic results are obtained from a generalisation of the self consistent model, shown in Figure 8, which consists of a single fibre embedded in a concentric cylinder of matrix material which in turn is embedded in a homogeneous medium (7,29,34,74,87). The homogeneous outer material is assumed to be transversely isotropic and to have properties that are identical to those of the overall effective composite, ie, the macroscopic picture of the composite cylinder is "indistinguishable" from the overall composite (112). The volume fraction of the fibre in the composite cylinder is the same as that in the overall composite material, however such an assumption is not entirely valid since the matrix material may tend to coat the fibres imperfectly and hence leave voids (28,29,74,75,112).

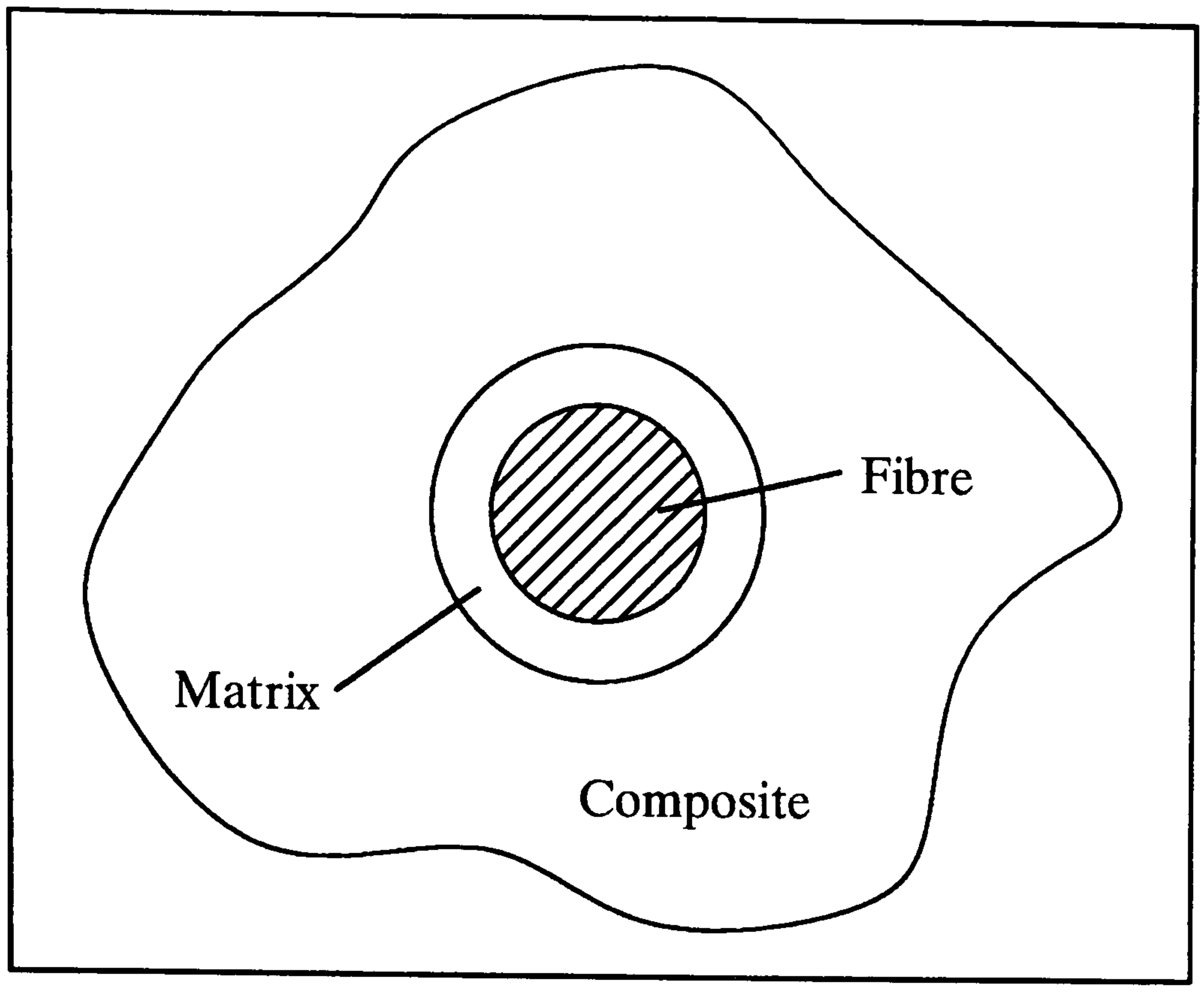


Figure 8 : A Self Consistent Composite Cylinder Model (28).

Whitney and McCullough (29) report that this self consistent model introduces gross simplifications into the micro–structure of the composite in order to deal with the internal distribution of stress and strain. The simplifications tend to submerge the extent to which "neighbouring fibres mutually influence their response characteristics" (29). This results in the approach giving an underestimate of the properties at $V_f > 0.2-0.3$. Although the self consistent approach simplifies the cross sectional geometry of the lamina the method is useful for giving an estimate of the composite properties (29).

The generalised self consistent model is shown by Hashin (7,74) to give the same results for E_{11}, K_{23}, ν_{12}, G_{12} as the CCA model. Hashin further reports that the CCA results are preferable to those of the self consistent scheme as the CCA model is based on a rigorous analysis of a model whilst the self consistent scheme is based on "an approximation of uncertain validity". Hashin concludes that the self consistent scheme is a method of quite limited value.

4.3.5 Semi–Empirical Relationships

The theoretical approaches used by many authors to formulate micromechanical models to determine the material properties of a uni–directional composite tend to be complex. As has already been seen the theories are reliant upon various assumptions. These difficulties have led to the development of semi–empirical methods for determining the overall mechanical properties from those of the constituent materials.

4.3.5.1 Elasticity Solutions With Contiguity

In an actual uni–directional composite material the fibres are randomly packed in a plane normal to the fibres rather than being arranged within a regular array. This means that some fibres will not be surrounded entirely by matrix material, but may instead touch one another, see Figure 9. Thus, the analyses of the moduli of a uni–directional composite must take this into account (28,61).

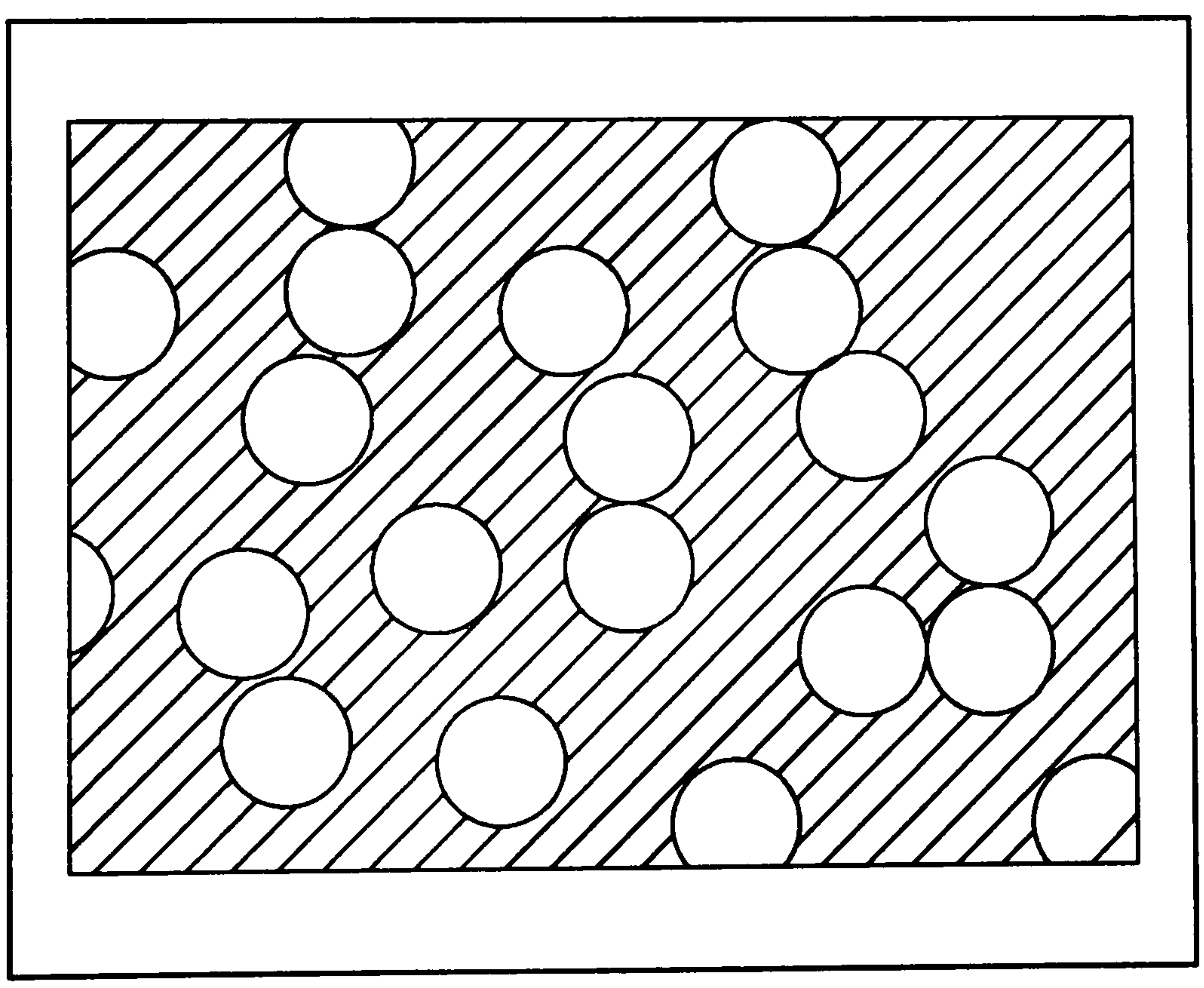


Figure 9 : Actual Random Fibre Arrangement In A Composite (28).

However, in a composite some fibres will touch one another, and some will not. Jones (28) states that from an analytical point of view, a linear combination of a solution in which all fibres are isolated from one another and a solution in which all fibres contact each other will provide the correct modulus values.

4.3.5.2 The Halpin Tsai Semi–Empirical Relationship

The most widely used semi–empirical relationship appears to be the one developed by Halpin and Tsai (134). The Halpin Tsai relationships assume that the actual composite properties lie between the longitudinal and transverse law of mixtures relationships, ie between equations *[8]* and *[12]*. The Halpin Tsai relationship introduces a "scaling parameter", or "geometrical factor" ξ to adjust the properties between these two extremes (28,29). This geometrical factor is associated with the shape and packing of the fibres, and is a measure of the reinforcing efficiency of the fibres (12,29). The geometrical factor can be obtained by comparing the Halpin Tsai expression with experimental data or with exact elasticity solutions using curve fitting techniques (28,29,34). The only difficulty of using the Halpin Tsai equations would thus appear to be in the determination of a suitable value for ξ.

The general form of the Halpin Tsai relationship is given as (29,34,40,61,94,134):–

$$P = P_m(1+\xi\eta V_f)/(1-\eta V_f) \qquad [14]$$

$$\eta = (P_f - P_m)/(P_f + \xi P_m) \qquad [15]$$

Where 'P' is the material property of interest. For continuous uni–directional fibre reinforced composites the fibre aspect (length/diameter) ratio is infinite. Thus for the longitudinal properties E_{11} and ν_{12} the geometrical factor is infinite and the above expression reduces to the law of mixtures result (29).

The value usually given for the geometrical factor ξ for G_{12} is 1.0. However, the IACFA report (75) states that the best fit to experimental data is obtained when the above equation is used with a modification made to the value of ξ. One such modification noted by some authors (75,135) is $\xi = 1 + 40V_f^{10}$. Hewitt and

Malherbe (135) report that this empirical expression has no apparent theoretical justification, but only gives approximately a 10% underestimate of the experimental value compared to a 50% error when using $\xi=1.0$. This is supported by Halpin (34) who states that a value of 1.0 for the geometrical factor gives good correlation for V_f up to 0.65, but that at higher values of V_f the Hewitt and Malherbe modification gives better results. However, one must question the relevance of this as most commercial composites will not have a fibre volume fraction of greater than 0.65. Jones (28) reports that a more refined estimate of ξ could be found but that care needs to be taken not to derive an expression that is too complicated for a simple easy to use design tool which is what the original equation is supposed to be.

A further modification to the Halpin Tsai equation is given by Spencer (136). The aim of the Spencer equation is to improve the accuracy of the original equations by eliminating the arbitrary constant ξ. However, when the Spencer equation as presented is used it gives larger than expected values for G_{12}. For example, for a glass / epoxy composite with the constituent properties as given in Appendix A, for a $V_f = 0.6$, the Spencer equation gives $G_{12} = 33.8$GPa. whilst the experimental data gives $G_{12} = 4.07$GPa. This suggests that there is an error in the paper that the equation was obtained from, and thus the Spencer equation – presented in Appendix A will not be used for any further work. A similar modification of the Halpin Tsai equation for E_{22} is also given by Spencer with similar results.

The IACFA report (75) states that the selection of a value for the geometrical factor ξ is "one of the greatest uncertainties" in using the Halpin Tsai equation to determine E_{22}, but that a value of 2.0 is usually assumed – see also (23,28,29,34,40, 61,94,105,134,135). However, Halpin (34) states that above a V_f of 0.65 better results are obtained by a modification to the geometrical factor similar to that made for G_{12} such that $\xi = 2 + 40V_f^{10}$.

Chapter 5 : Criteria For Comparison

As has been seen in Chapter 4, and Appendix A, a number of different micromechanical equations have been found in the literature for the prediction of the mechanical properties of interest. In order to study the results given by such equations two questions need to be answered. The first is what level of accuracy of predicted values is acceptable? The second is which of the equations can be used to give this level of accuracy?

5.1 Accuracy Of Properties For Analysis

From the literature studied there are two main methods of analysing components manufactured from composite materials. One is to perform calculations by hand with or without the use of a computer to automate the calculations as seen in many references (6,30,40,64,137−146). The majority of these references analyse laminated plates as these are relatively simple shapes to analyse in comparison to many structural components for which composites may be considered. Many of the above references only analyse uni−directional lamina or laminates. These calculations concentrate on the determination of stresses and strains in the material.

The other method of analysing composite components is that of finite element analysis (2,38,53,107,119,147−191), although one paper (132) does use boundary elements, and one (192) the finite difference method. Most of this work tends to study relatively simple problems with idealised load cases. When manufacturing real components not only may the geometry tend to be relatively complex, but the idealised load cases may not be relevant as the composite component will often have to be joined to existing steel components through which the loading and boundary conditions will be applied.

In order to be able to perform calculations to determine how a composite component behaves under the applied loading conditions, the analyst must have the mechanical properties of the composite available. Most of the papers that study the analysis of composite structures do not actually state where the mechanical properties that they use in the analysis have been obtained from, for example, (38, 138, 142, 145, 147–151, 153, 157, 158, 160, 161, 163–167, 169, 172–179, 182–184, 187–189, 192). The analyses that do state where the mechanical properties have been obtained from use one or more of three different ways to obtain these properties. These are often obtained from calculations (6,30,40,107,140,144,186), but also from performing tests (2,30,39,40,53,119,139,140,170), and sometimes from manufacturers of the material (132,137,143,152,154,159). These authors do not comment upon the accuracy of the mechanical property data used in the analysis, or the possibility of the data varying throughout the structure. The authors assume that the properties are constant throughout. Only some of the researchers comment upon the accuracy of the finite element results when compared to test, ie strain gauge results and the analyses are generally shown to give good agreement with these test results (2,155,171,176,189). The literature survey undertaken here highlights that although a lot of work has been done in the area of analysing composite materials by techniques such as FE analysis, the accuracy of the material data used in the analysis does not appear to have been considered.

This raises the question of how accurate ly the relevant mechanical properties should be predicted to obtain acceptable analysis results. As mentioned in Chapter 3 when testing a component the strain gauges and measurement system can give errors of 5% to 10% whilst other errors can be caused by the orientation, location and bonding of the gauges (53). However, any measured mechanical properties may be subject to errors of for example 12% caused by the test methods (193). Thus in reality the analysis results, even using measured mechanical properties, could have even more errors built in. However, as can be seen in papers presented by the author (194–196) the FE analysis results can be reasonably accurate when compared to experimental ones.

In conclusion, to reduce the overall possibility of errors in the analysis the mechanical properties should be predicted as accurately as possible. However, as the measured values are themselves subject to a possible 10% error an accuracy of the predicted properties compared to the experimental properties of 10% will be assumed to be acceptable. This level of accuracy is stated by Johnson and Sims (197) to be acceptable for design calculations as many designers do not usually require greater accuracy.

5.2 Choice Of Micromechanical Equations

As there are many different micromechanical equations for predicting the uni–directional mechanical properties of interest, which equations to use has to be considered. If the answers given by a number of equations are the same does this mean that these equations predict the correct values? Alternatively, should the equations that give the most accurate prediction when compared to experimental data be used?

The rigourous mathematical derivation of many of the micromechanical equations seen in publications would suggest that if equations derived from more than one theory predict the same answer, then these equations predict the accurate theoretical value. Unfortunately, the derivation of these equations is based on a number of simplifying assumptions – see Chapter 4 – and thus may not give accurate results (41). For example, as already stated in Chapter 4, it is a common assumption that the in situ composite matrix properties are the same as the properties of the bulk matrix material. This may not be the case (27,29,47,48) and therefore the matrix constituent properties being used in the calculations may be inaccurate. However, according to Kardos (24) if the fibre volume fractions are high enough then slight differences in the matrix properties used do not significantly effect the composite properties. Kardos does not state how high the fibre volume fraction has to be for this to be the case.

The accuracy of the micromechanical theories is often checked by comparing the results obtained from these equations with results obtained from experimental tests. Often empirical factors are included in the micromechanical equations so that their predictions match more closely the experimental results. However, these empirical factors need to be determined for the material being considered, and may vary from material to material and require the availability of experimental data.

As can be seen from the experimental data available in the literature – see Appendix A – this experimental data is subject to some scatter, partly due to imperfections in the composites, and partly due to inaccuracies and inconsistencies in the measurement techniques used. It is concluded in a report published by the Royal Aircraft Establishment (35) – and supported by Shenoi (51) and Smith (61) – that experimental values of the mechanical properties of composites are subject to greater scatter than in isotropic materials (+/– 15% must be expected). This is stated to be probably due to types and frequencies of faults occuring in the composite (such as pores and air bubbles), the different fibre and matrix properties of the individual specimens, and inadequate testing techniques. The report concludes that the differencies between theoretical predictions and experimental results can, in many cases, be ascribed to the assumptions made in the theoretical analyses which are not always valid.

Most composite components will have a thin resin rich skin at their surface (20,88,198). This thin skin can lead to errors in experimental moduli which would underestimate the actual composite moduli by as much as 10–20% as in torsional or flexural tests the properties at the surface, which are the matrix properties, are emphasized (20,198). Nielsen (20) states that this error can be corrected by using thicker specimens for testing. Also, for the transverse properties of a uni–directional composite there is an effective V_f of fibre inclusions which is larger than the actual V_f due to the "agglomeration of fibres into a bunch" (13).

Mase et al (199) state that this uncertainty in the value of V_f and also the fibre lay–up angles in a composite can also lead to inaccuracies in the prediction of properties. Further the results obtained from measurements on test plaques may not be indicative of the values to be found in a composite component if a different manufacturing method is used to manufacture the test plaques as already discussed in Chapter 3.

Chamis (27,56) notes that even with the simplifying assumptions the micromechanics theories can predict the properties of a uni–directional composite within "acceptable engineering accuracy", even though most of the assumptions upon which the micromechanics theory is based are violated by the real material. Chamis explains that the phrase "acceptable engineering accuracy" is used to mean that the agreement between predicted results and measured data is "considered to be as good as can be expected based on engineering judgement and on considerations of the complexities and uncertainties involved". However, Smith (61) states that for final design purposes theoretical estimates of the moduli should not be regarded as a substitute for reliable test data. Such theoretical estimates may however be useful for evaluating initial designs and exploring the influence of the various parameters, such as constituent properties and V_f, upon the overall component performance.

In this work the micromechanical equations will first be compared to one another to compare the results obtained from the different theories – see Chapter 6. To enable this to be done graphs have been drawn for each property against V_f over the whole range of V_f from 0.0 to 1.0. Although very high fibre volume fractions, such as 0.9, are not practical and cannot be achieved in practice the graphs drawn here for the mechanical properties will go up to $V_f = 1.0$. The properties at this fibre volume fraction in the longitudinal direction should be the properties of the fibre for a uni–directional composite. Similarly, the properties at $V_f = 0$ should be the resin properties. Hence the values given by the equations at those two limits can be checked to see the validity of the equations. In Chapter 7 the predictions obtained from the micromechanical equations are compared against the available experimental data and conclusions drawn as to which equations should be used to predict which property. The constituent properties quoted in Appendix A have been used in the calculations.

Chapter 6 : Comparisons Of Uni–Directional Equations Against Each Other

The equations found in the literature and given in Appendix A for the determination of the six mechanical properties of interest can now be considered for a uni–directional composite. The constituent properties used to evaluate the different equations are for E–glass fibres and an epoxy matrix. These are also presented in Appendix A. As the aim of this work is to provide guidelines to the designer such that the required mechanical properties can be determined and used with confidence, then the theoretical bounds within which the properties will lie are considered to be of less importance than predictions of the actual properties. To enable the equations to be compared, graphs have been drawn for each property to show how it changes with increasing fibre volume fraction. The results obtained from the equations will be compared to the available experimental data in Chapter 7. This Chapter takes the view that the equations should not give results at odds with common sense over the whole range of volume fractions, and must be easy for the design engineer to use. The equations discussed are summarised in section A.7 of Appendix A.

6.1 Longitudinal Young's Modulus

A total of eight equations have been found in the literature for the prediction of E_{11} of a uni–directional composite. A comparison of these equations can be seen in Figure 10 where E_{11} is plotted against fibre volume fraction V_f. From Figure 10 it can be seen that most of the equations predict a linear relationship between E_{11} and V_f. However, three equations – *[54]*, *[62]* and *[63]* all from the bounding technique – predict a non–linear relationship. The predictions made by these three equations show E_{11} increasing up to a maximum at $V_f = 0.5$, and then decreasing until $V_f = 0.7$. This is not what would be expected as E_{11} would be expected to increase up to a maximum at $V_f = 1.0$ where $E_{11} = E_f$, and therefore these equations will not be considered further. Note, that at $V_f = 0.8$ and $V_f = 0.9$, a negative value for E_{11} is given by these equations, and also that equations *[62]* and *[63]* do not give the resin modulus E_m at $V_f = 0$ as would be expected.

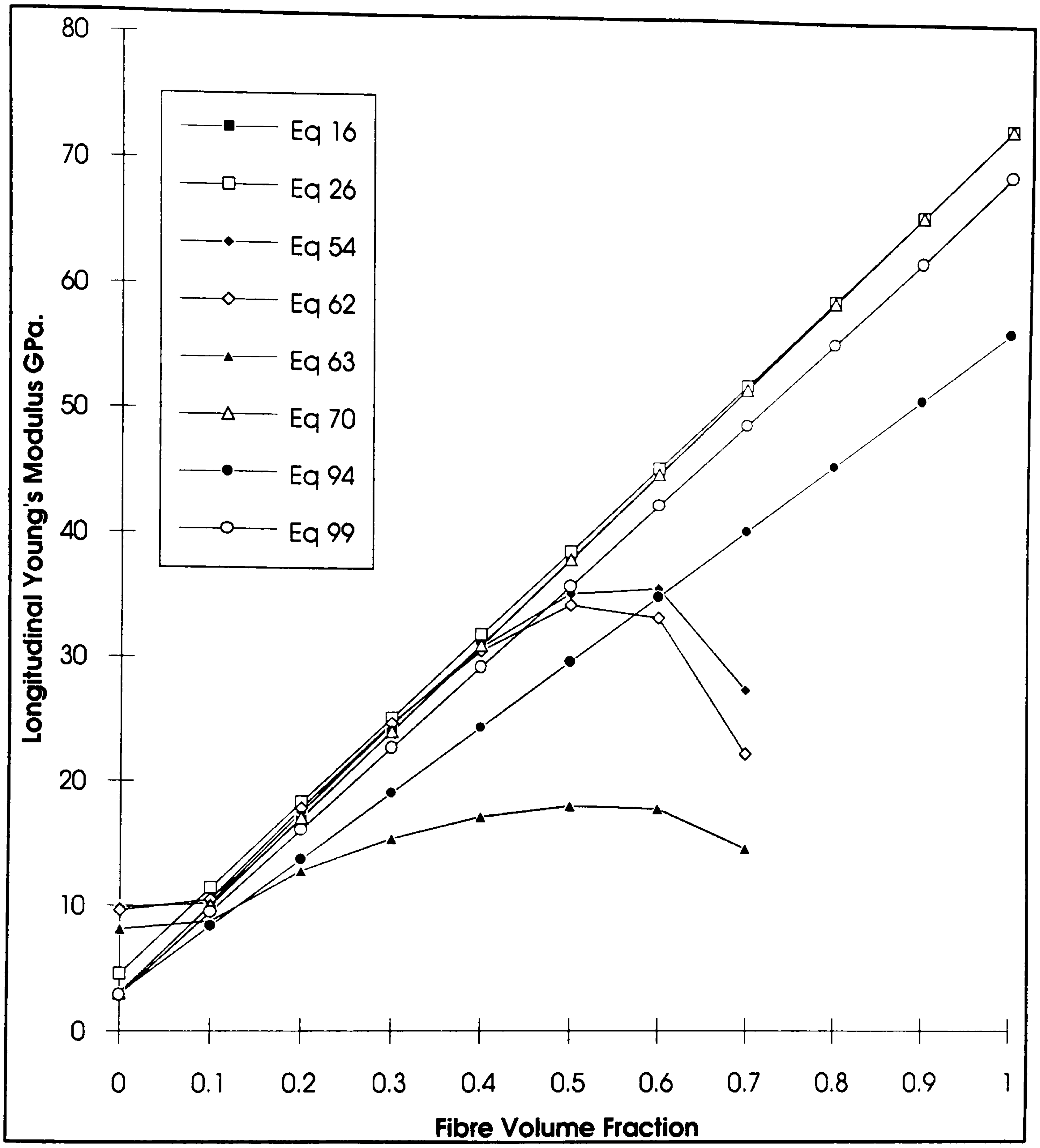


Figure 10 : Calculated E_{11} Against V_f.

Of the other equations given in the literature, equation *[16]* – the law of mixtures – from the mechanics of materials approach, and equation *[70]* from the CCA model give virtually identical results. The results from equation *[99]* – a semi–empirical equation – are slightly lower than the law of mixtures result, and the results obtained from another semi–empirical equation – equation *[94]* – are markedly lower. The mechanics of materials approach also gives equation *[26]* which, whilst not giving E_m at $V_f = 0$, gives virtually the same results as equations *[16]* and *[70]*. For these reasons equation *[26]* will not be considered further as equation *[16]* is simpler to use as data concerning the matrix Poisson's ratio is not required. Equation *[70]* which is based on the CCA model requires knowledge of the constituent shear and bulk moduli as well as the Poisson's ratios thus making it more difficult to use than the law of mixtures. Indeed, it is noted by many authors (6,42,45,74,110) that when the fibres are considerably stiffer than the matrix, as they will be with a glass reinforced resin, then the last term in equation *[70]* becomes negligible leaving the law of mixtures result. Thus equation *[70]* will not be used in any further comparisons.

E_{11} determined from equation *[99]* depends upon a 'fibre misalignment factor, Z' which can vary between 0.9 and 1.0. If the value is 1.0 then the law of mixtures result is observed, ie equation *[16]*. The value of the constant 'Z' in equation *[99]* was taken to be an average of the values between which it is quoted to lie by Jones (28), 0.9 – 1.0, ie, 'Z' was taken to be 0.95.

Only three equations which give slightly different results will thus be considered in the next Chapter when the predicted values are compared to experimental values. These equations are *[16]* from the mechanics of materials approach, and equations *[94]* and *[99]* both of which are semi–empirical equations.

6.2 Longitudinal Poisson's Ratio

Five equations have been found in the literature for the prediction of ν_{12} of a uni–directional composite. A comparison of these equations can be seen in Figure 11 where ν_{12} is plotted against V_f. As can be seen from Figure 11 four of the equations considered, ie, *[17]* from the mechanics of materials approach, *[69]* from the CCA model, and *[96]* and *[101]* both semi–empirical relationships, give very similar results and are virtually the same.

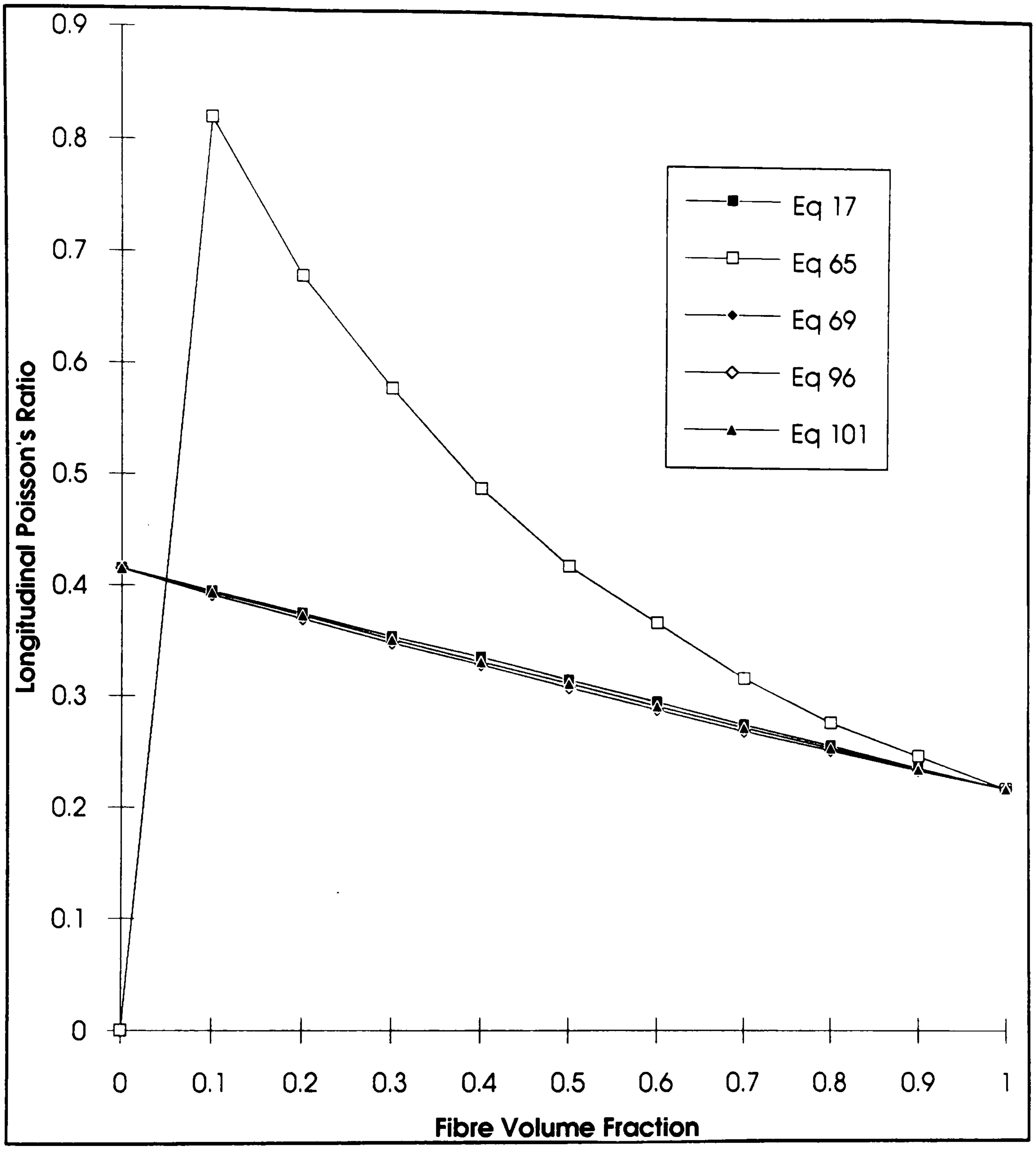


Figure 11 : Calculated ν_{12} Against V_f.

At the two extremes of V_f then all of the equations predict the expected values, ie, ν_m at $V_f = 0$ and ν_f at $V_f = 1$. However, equation *[65]* from the bounding technique gives results that are not what would be expected. The value of ν_{12} predicted by this equation, except for the two extremes at $V_f = 0$ and $V_f = 1$, are far higher than would be expected and are greater than either of the constituent values.

This equation will therefore not be considered further especially as the other four equations predict virtually identical results which are markedly different to that predicted by equation *[65]*. Of the other four equations, equation *[17]* – the law of mixtures relationship, only requires information about the constituent Poisson ratios. However, equation *[69]* from the CCA model also requires knowledge of the constituent bulk and shear moduli thus making it more difficult to use than equation *[17]* as more information of the constituent materials is required. Indeed it is noted by many authors (6,29,42,45,74,110) that when the fibres are considerably stiffer than the matrix, as they will be with a glass reinforced resin, then the last term in equation *[69]* becomes negligible leaving the law of mixtures result, ie equation *[17]*. Thus, equation *[69]* will not be considered further. Similarly, equation *[96]* requires more information to be known about the composite than simply the constituent Poisson ratios, and requires the calculation of the 'areal' modulus and will therefore not be considered further.

Equation *[101]* is also similar to equation *[69]* in that it requires knowledge of the constituent bulk and shear moduli in addition to the Poisson ratios. This equation takes into account the random packing of fibres that will occur in a real composite. The solutions from equation *[101]* depend upon a contiguity factor 'C' which varies between 0 and 1.0 depending upon the amount of contact between the fibres, ie the volume fraction of the fibres. As an example, if 'C' is assumed to be equal to 1.0, ie all of the fibres are in contact, and $V_f = 0.5$, then $\nu_{12} = -68.34$. If for the same conditions the value of 'C' is taken to be 0, $\nu_{12} = 0.314$, and if 'C' is taken to be 0.5, $\nu_{12} = -34.01$. The results from equation *[101]* for C = 1.0 and C = 0.5 are clearly incorrect as Poisson's ratio for the materials considered will lie in the range of 0 to 0.5. However, the result for C = 0.0 lies within this range and thus this value has been used in all of the calculations. This tends to agree with Jones (28) who states that the value of 'C' is usually low. This conclusion also agrees with that of Nielsen and Chen (200) who present the same equation but assume that there is perfect bonding between the fibres and the matrix, ie that the fibres do not touch and 'C' is set to zero. However, the use of this equation does depend upon a choice being made for the value of 'C'. Thus, this equation will also not be considered further, as the results obtained when C = 0.0 are virtually the same as for equation *[17]*.

The only equation to be considered therefore when comparing the predicted and experimental results for ν_{12} in the following Chapter will be equation *[17]* the law of mixtures equation.

6.3 Longitudinal Shear Modulus

A total of thirteen equations have been found in the literature for the prediction of G_{12} of a uni–directional composite. A comparison of these equations can be seen in Figure 12 where G_{12} is plotted against V_f. As can be seen from Figure 12 many of these equations predict the same, or very similar, values of G_{12} at low values of V_f. However, as V_f increases G_{12} predicted by these equations differs.

A number of equations are found to all give practically the same values for G_{12}. These equations are : *[49]* from the bounding technique, *[52]* from both the bounding technique and the CCA model, *[89]* from the self consistent scheme, *[84]* from the CCA model, and *[102]* and *[103]* which are both semi–empirical. The fact that so many different researchers using different approaches give practically the same values for G_{12} would tend to suggest that these are the correct values. However, equation *[53]* for example, from the bounding technique, does give results that differ considerably from these equations and therefore these results do need to be compared to experimental data. Note that all of the equations that give the same results only require knowledge of the constituent shear moduli.

The solutions from equation *[102]* however, depend upon the contiguity factor 'C' which varies between 0 and 1.0. As an example, if 'C' is assumed to be equal to 1.0, and $V_f = 0.5$, then $G_{12} = 10.9$GPa. If for the same conditions the value of 'C' is taken to be 0.0 then G_{12} = 2.91GPa, and if 'C' is taken to be 0.5 then G_{12} = 6.9GPa. The value obtained from equation *[102]* when C = 1.0 is very high compared to the other values and not what would be expected. However the value when C = 0.0 compares better with values obtained from other equations, and thus this value has been used in the calculations made here. The use of this equation does depend therefore upon a choice being made for the value of 'C'.

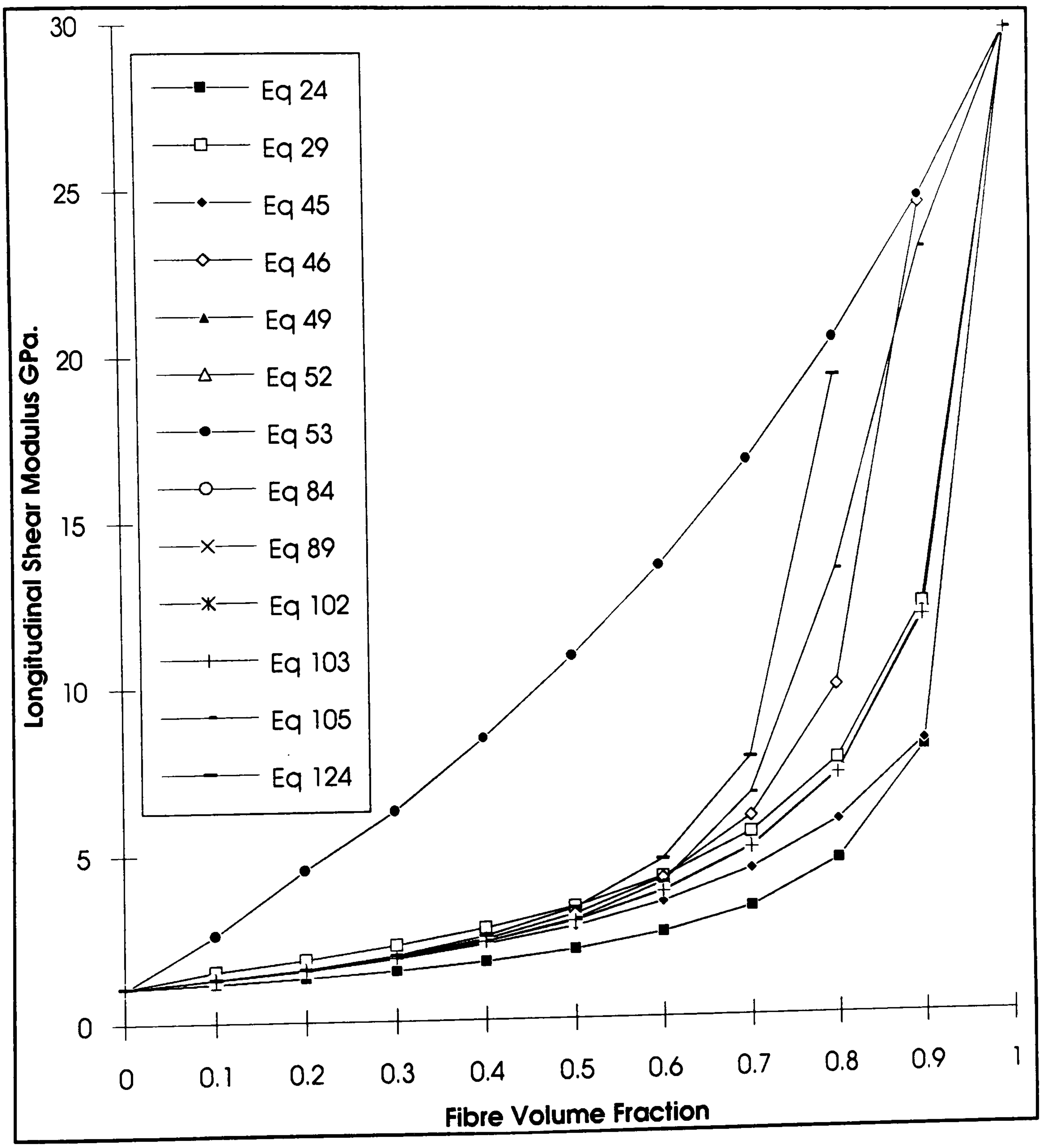


Figure 12 : Calculated G_{12} Against V_f.

Rosen (117) states that the values of the shear modulus determined from equation *[52]* are lower than measured values and that some correction factor is desirable. Thus, if some correction factor is required, it would seem logical to use an equation which gives the same results and has an empirical factor. The Halpin Tsai semi–empirical relationship requires a geometrical factor to be used. Whilst this factor is usually said to be equal to 1.0 – see Appendix A – it is modified by some researchers to give a better fit to experimental data. Thus, as this equation is open to possible empirical modification then this is the equation that will be considered when making comparisons with experimental data rather than the other equations which give the same results.

Hewitt and Malherbe's (135) modification to the Halpin Tsai relationship – equation *[105]* – is only seen to give different values at higher fibre volume fractions, ie above a V_f of 0.5. The Nielsen (20) modification to the Halpin Tsai relationship – equation *[124]* – gives higher values than the Hewitt and Malherbe modification, however this equation gives negative values at a V_f at 0.9 and1.0. Equations *[45]* and *[46]* also return negative answers at a V_f = 1.0. These extremely high values of V_f cannot be achieved in practice and so a prediction of G_{12} at these values is not critical.

6.4 Transverse Young's Modulus

It can be seen from Figure 13 that a total of twelve equations have been found in the literature for the calculation of E_{22} of a uni–directional composite. Most of these equations predict similar results except for equation *[78]* from the CCA model. This equation predicts a slight decrease in E_{22} as V_f increases which is not what would be expected. Thus, this equation will not be considered further.

The other eleven equations have been developed from different approaches, ie, *[21]*, *[27]*, *[30]*, and *[33/34]* from the mechanics of materials approach, *[80]* from both the CCA model and the self consistent scheme, and *[92]*, *[95]*, *[100]*, *[108]*, *[110]*, and *[121]* which are all semi–empirical relationships. All of these equations will be considered in the next Chapter when comparisons are made with experimental data as no conclusions can be drawn at this stage as to which of these many equations can be used to predict E_{22} with the most accuracy. It is to be noted that equations *[27]* and *[92]* do not predict E_m at $V_f = 0$ as would be expected.

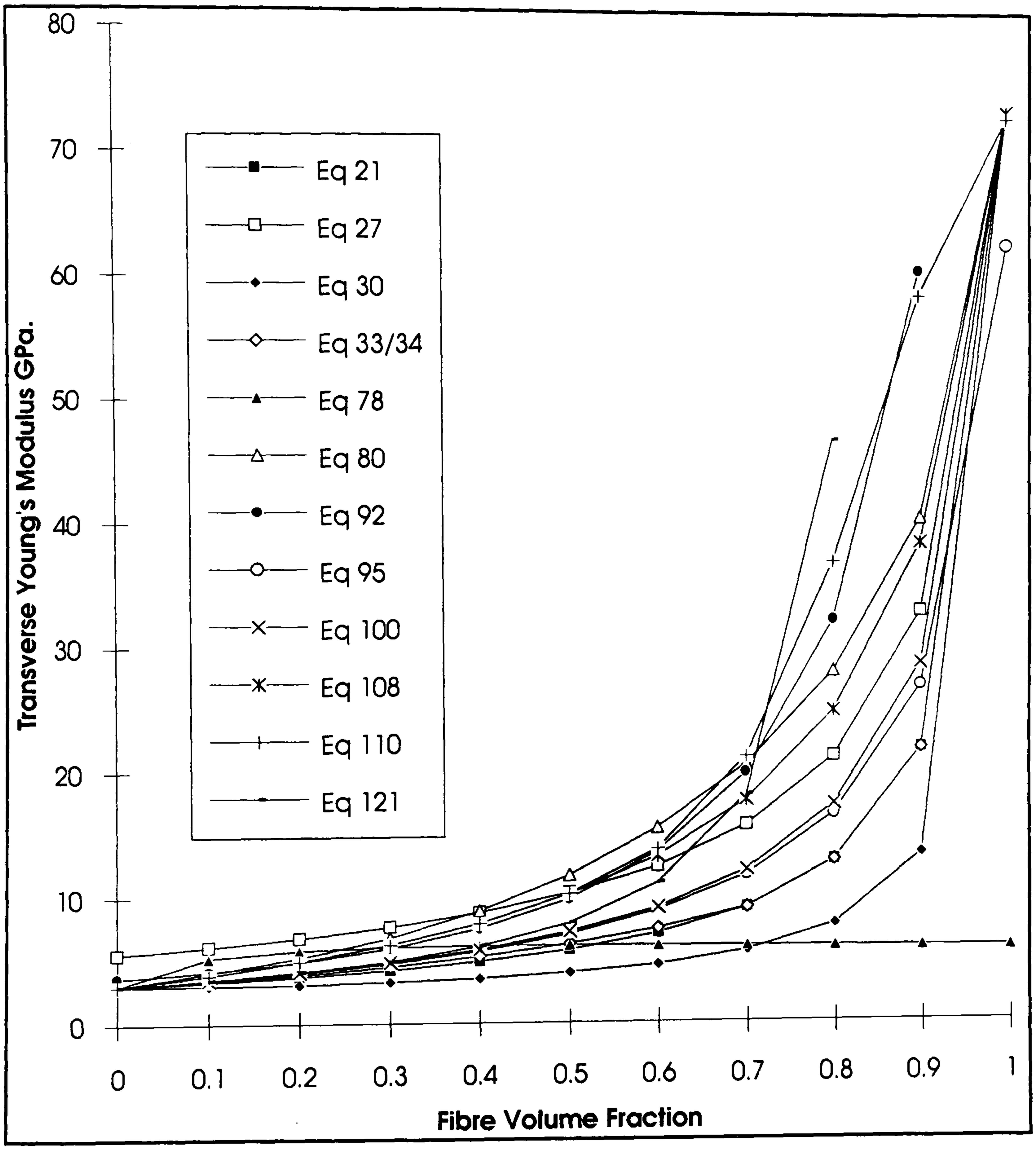


Figure 13 : Calculated E_{22} Against V_f.

Jones (28) presents an equation – equation *[100]* – which takes account of the random packing of fibres within a real composite. The equation contains a contiguity factor 'C' which is discussed in section 6.3. As an example, if 'C' is assumed to be equal to 1.0, and V_f equals 0.5, then E_{22} is 21.5GPa. If for the same conditions the value of 'C' is taken to be 0.0 then E_{22} is 14.5GPa, and if 'C' is taken to be 0.5 then E_{22} is 18.02GPa. The value obtained when C = 1.0 is very high compared to the other values and not what would be expected. However the value when C = 0.0 is similar to predictions made by other equations and thus this value has been used in these calculations. Jones states that the value of 'C' is usually low, and this conclusion agrees with that of Nielsen and Chen (200) who present the same equation but assume that there is perfect bonding between the fibres and the matrix, ie they assume that the fibres do not touch.

6.5 Transverse Poisson's Ratio

Only four equations have been found in the literature for the prediction of ν_{23} of a uni–directional composite. A comparison of these equations can be seen in Figure 14 where ν_{23} is plotted against V_f. All of the equations predict the expected values of ν_{23} at $V_f = 0$ and $V_f = 1$, ie the constituent Poisson ratios. However, between these two extremes three of the equations predict values for ν_{23} that are greater than either of the constituent values. These equations are : *[32]* from the mechanics of materials approach, *[81]* from the CCA model, and *[91]* from the self consistent scheme. Only one equation – equation *[23]* from the mechanics of materials approach – predicts a reduction in ν_{23} between the constituent Poisson ratios as V_f increases.

Equations *[32]*, *[81]*, and *[91]* all rely on the determination of other composite moduli such as G_{23}, and E_{11}. As there are a variety of equations available for the calculation of these moduli then a number of different answers could be achieved for ν_{23}. For the calculations undertaken here then, for example, in equation *[81]* which uses the CCA model, the other composite moduli E_{11}, ν_{12}, and G_{23} have also been determined from the CCA model. Appendix A gives all of the relevant details concerning which equations have been used.

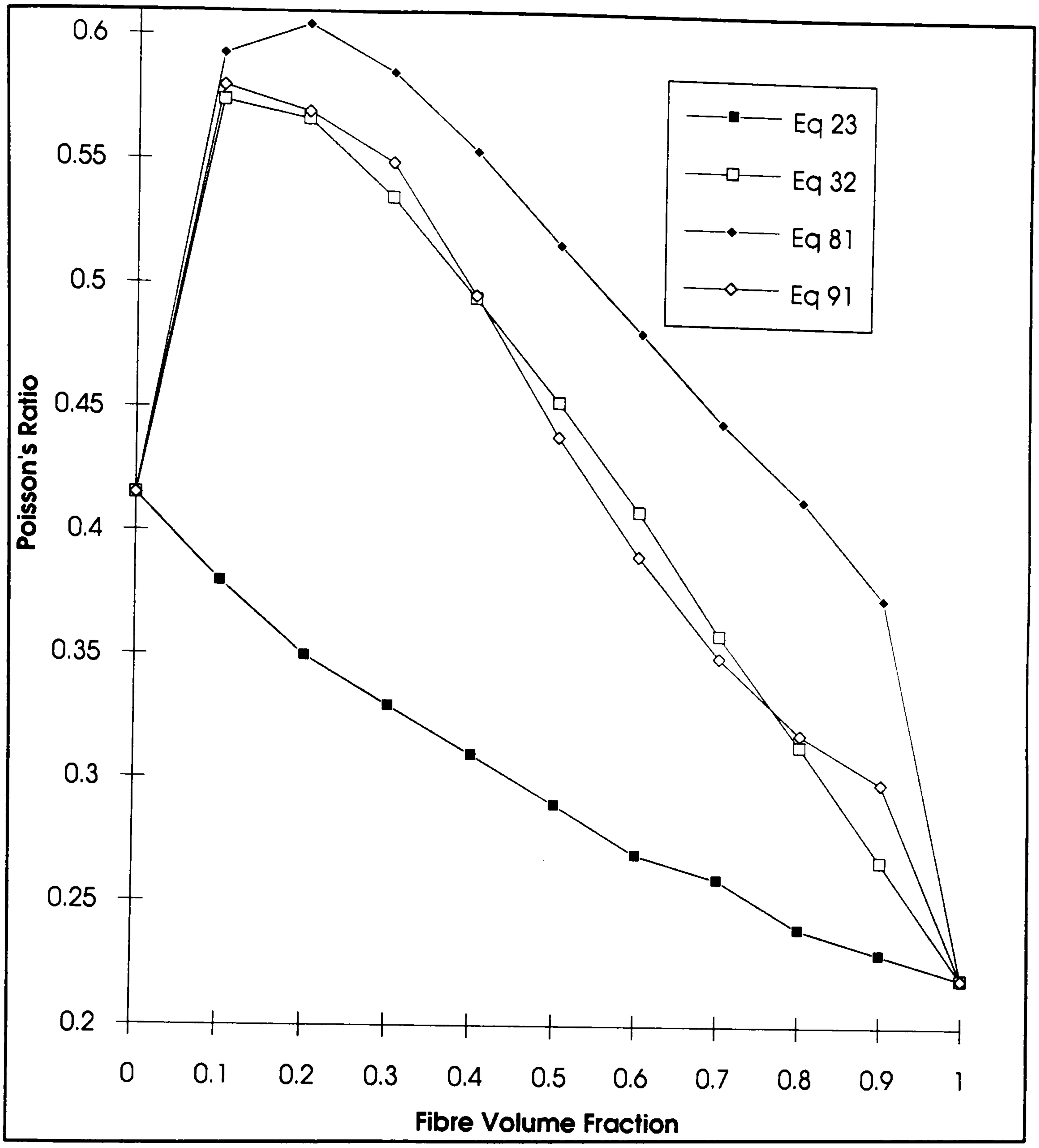


Figure 14 : Calculated ν_{23} Against V_f.

From Figure 14 it can be seen that equations *[32]* and *[91]* predict very similar results. Of these two equations the results predicted by equation *[32]* are more consistent than those predicted by equation *[91]*. Thus equation *[91]* will not be considered further.

6.6 Transverse Shear Modulus

It can be seen from Figure 15 that eight equations have been found in the literature for the prediction of G_{23} for a uni–directional composite. Figure 15 shows that equations *[50]* from the bounding technique, *[90]* from the self consistent scheme, and *[112]* a semi–empirical relationship, all predict the same values although they are derived by different methods. This would tend to suggest that the predictions made by these equations are the correct values. However, some equations, eg, *[51]* from the bounding technique, predict quite different results. Therefore the results given by these equations need to be compared to the available experimental data to confirm the above observation. Of the three equations that predict the same results, equation *[112]* contains an empirical factor that may be able to be adjusted to better suit the material under consideration. For this reason this is the one of the three equations considered in the next Chapter, and equations *[50]* and *[90]* will not be considered further.

At the two extremes of V_f then all of the equations except equation *[125]* predict the expected values of G_{23}, G_m at $V_f = 0$, and G_f at $V_f = 1$. The Nielsen (20) modification to the semi–empirical Halpin Tsai relationship – equation *[125]* gives negative values at $V_f = 0.9$ and $V_f = 1.0$. However, as these are not practical values of V_f then this equation will still be considered in the next Chapter when the predicted results are compared to the available experimental data.

It should be noted that one equation not considered here that is recommended by many authors for the determination of G_{23} is the isotropic relationship – equation *[25]*. As already discussed in Chapter 4, this equation requires the use of E_{22} which has been seen to have many possible variations, and ν_{23} which has quite different predicted values. Thus, unless accurate predictions are able to be made for E_{22} and ν_{23} then equation *[25]* cannot be used with any confidence. It is felt that equations which rely directly upon the constituent properties are more reliable as they are less open to interpretation.

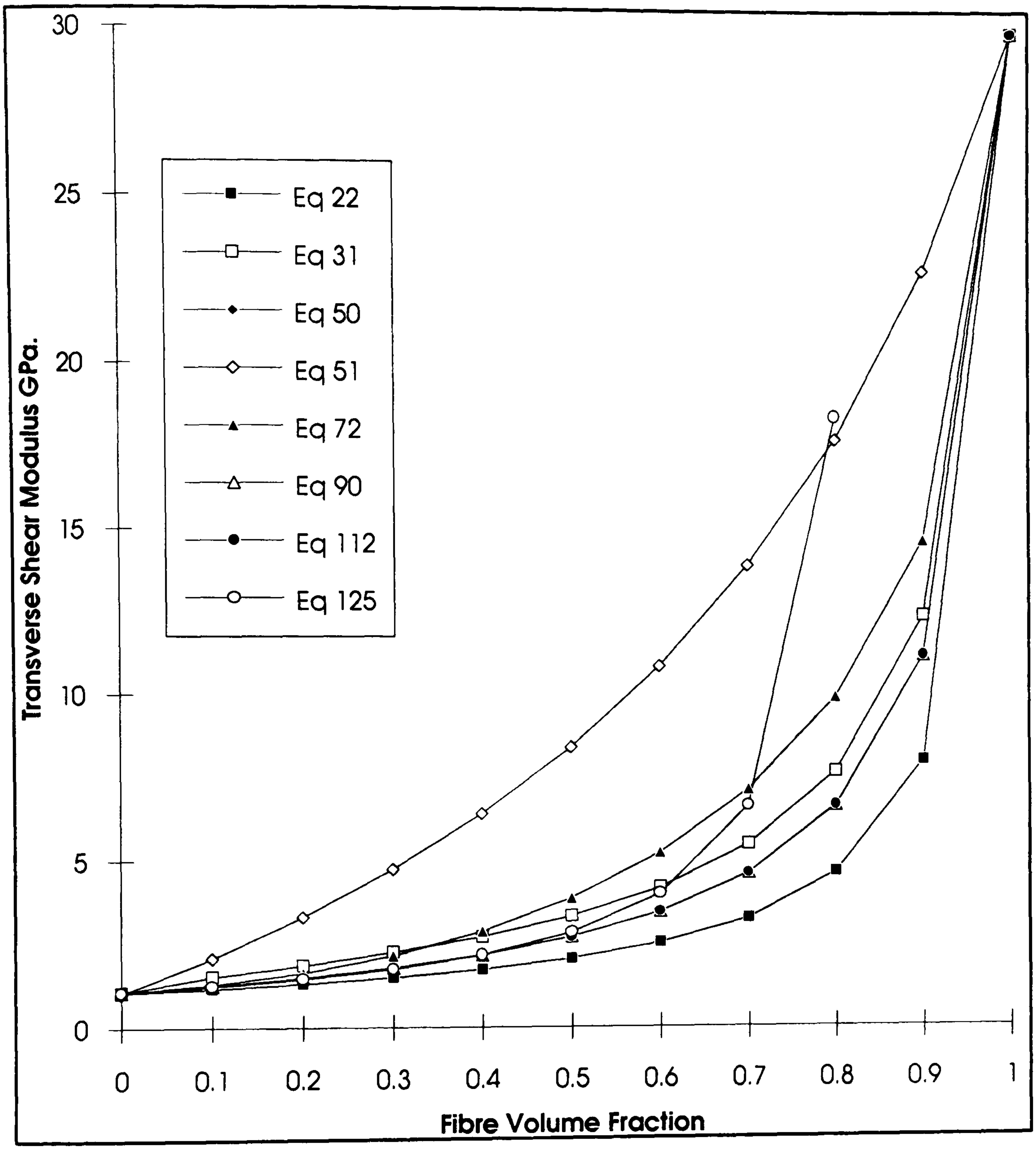


Figure 15 : Calculated G_{23} Against V_f.

Chapter 7 : Comparison Of Uni–Directional Equations With Experimental Data

This Chapter compares the accuracy of the results obtained from the equations found in the literature with experimental properties for E–glass/epoxy composites given in the literature. The detailed results from the equations used can be seen in Appendix A together with the experimental data and the constituent properties used. Presented here is a summary of the results. The accuracy of the predictions made for E–glass/epoxy composites are confirmed by considering also the predictions for the properties of E–glass/polyester composites found in the literature. It should be noted that only the composite properties with these two types of resin have been considered as these materials are normally used in the more structural applications.

7.1 Longitudinal Young's Modulus

From the results obtained for the epoxy resin composite specimens the longitudinal Young's modulus E_{11} can be predicted most accurately with the law of mixtures result, ie equation *[16]*, when compared with experimental data over the range of V_f. This equation gives an average error of approximately 2%.

If the experimental data in Table 12 in Appendix A is studied it can be seen that E_{11} when $V_f = 0.45$ is inconsistent with the other experimental data. This inconsistency may have been due to a different epoxy resin system being used. However, this is unlikely as the resin modulus would have to be more than double the quoted value to obtain this result. As none of the authors who quote this data give the resin properties this cannot be checked. It is more likely that the quoted value of V_f is too low. If the average error given by the law of mixtures equation is determined without considering this inconsistent value then the average error is seen to be approximately 1%. The average error given by the law of mixtures equation for the experimental data using polyester resin is seen to be 8%. This loss in accuracy may be because any small scatter in the epoxy composite results will not have a significant effect as a number of results are considered.

The fact that the law of mixtures gives the most accurate results confirms the conclusions reached by many researchers. The other equations considered here, ie, equations *[94]* and *[99]* give less accurate results. Equation *[99]* which is a semi–empirical equation, uses a fibre misalignment factor to reduce the law of mixtures result. The value of this factor is not clear and could depend upon the method of manufacture of the composite or fibre preform.

In conclusion, the modifications made by some authors to the law of mixtures equation make little difference to the accuracy of the results, but increase the complexity of the calculations. The most accurate results are found with the law of mixtures equation derived from the mechanics of materials approach.

7.2 Longitudinal Poisson's Ratio

Of the equations that attempt to calculate the Poisson's ratio, ν_{12}, only the law of mixtures – equation *[17]* – has been considered. The predictions given by this equation for the epoxy resin composites have an average error of approximately 7%. The law of mixtures equation sometimes overestimates and sometimes underestimates the experimental values. However, these experimental values do show some scatter – see Table 12 in Appendix A.

When the experimental value of the longitudinal Poisson's ratio for a uni–directional composite using polyester resin is compared to the predicted value, it is found that the result is more accurate than those on average found for the epoxy resin composite. However, only one experimental value is available for polyester resin, and thus it is difficult to draw any conclusions from this.

The recommended equation for the Poisson's ratio is the law of mixtures equation derived from the mechanics of materials approach.

7.3 Longitudinal Shear Modulus

If the experimental data for the longitudinal shear modulus in Table 12 in Appendix A is studied it can be seen that G_{12} for $V_f = 0.45$ and also for $V_f = 0.7$ are inconsistent with the other experimental results for an epoxy resin composite. The value for G_{12} at $V_f = 0.7$ is grossly inconsistent, although the other experimental mechanical properties quoted by the relevant author at this V_f appear consistent. This suggests that there is a possible error in the measurement of G_{12} in this case. Therefore, the average error quoted for the equations in Appendix A does not include the values for the inconsistent shear moduli at $V_f = 0.45$ and $V_f = 0.7$ as they would tend to distort the results. There are only two polyester composite results available and thus any scatter will be highlighted resulting in a loss in accuracy in these predictions.

The values calculated for the longitudinal shear modulus vary in accuracy considerably. Of the equations considered, when compared to the experimental values, many are found to give an average error of around 10%. Therefore there are a number of equations which give acceptable predictions according to the criteria considered earlier in Chapter 5. The semi–empirical Halpin Tsai equation – equation *[103]* – appears to give very accurate results for low values of V_f, but gives progressively less accurate results as V_f increases.

The Halpin Tsai equation includes an experimentally determined geometrical factor which is often given to be equal to 1.0. More accurate results are obtained from this equation by taking into account Hewitt and Malherbe's (135) modification to the geometrical factor, ie equation *[105]*. Their modification makes the value of the geometrical factor dependent upon the fibre volume fraction. The IACFA report (75) states that this modification gives a better fit to experimental data, and this statement is confirmed by the results obtained here. Thus the fact that a number of different techniques, as seen in the previous Chapter, all give the same value of G_{12} as the Halpin Tsai equation does not mean that they are the most accurate when compared to the range of experimental data considered here. In fact the Hewitt and Malherbe modification to the Halpin Tsai equation which is supported by many authors provides greater accuracy as V_f increases. A better fit to the experimental data may possibly be obtained from the Hewitt and Malherbe modification to the Halpin Tsai equation by altering the equation for the geometrical factor. However, this has not been undertaken here as relatively accurate results are obtained from this equation, ie with an average error of 7%.

Nielsen (20,201) also modifies the Halpin Tsai equation in equation *[124]*. The Nielsen equation includes a term which represents the 'maximum packing fraction'. This means that an assumption about the packing geometry needs to be made if this equation is to be used. As stated in Appendix A, the value used for this packing fraction in the calculations is that for a random array of fibres, ie 0.82. If the value of the packing fraction is taken to be that for either a cubic array of fibres or a hexagonal array of fibres then the equation becomes on average less accurate. The Nielsen equation obtains an improvement in accuracy over the original Halpin Tsai equation, and also improves upon the Hewitt and Malherbe modification. However, if this equation is used to determine G_{12} at $V_f = 0.7$ it gives a negative value and cannot therefore be used.

The recommended equation for the longitudinal shear modulus is the Halpin Tsai equation with the geometrical factor modified by Hewitt and Malherbe, ie equation *[105]*. This gives relatively accurate results when compared to the available experimental data, and the results at relatively low values of V_f are confirmed by equations derived by other methods – see Chapter 6.

7.4 Transverse Young's Modulus

A number of sources give equation *[21]* for the calculation of E_{22}. However, the results obtained here suggest that this equation is relatively inaccurate when compared with the available experimental data, and that it becomes more inaccurate as V_f increases. These findings agree with IACFA (75) which reports that the equation underestimates the value of E_{22} by an amount that depends upon the connectivity of the fibres which increases with increasing V_f.

E_{22} is predicted on average most accurately by the Nielsen (20,201) modification to the Halpin Tsai equation – equation *[121]* – for the epoxy resin composites up to $V_f = 0.6$. However, at $V_f = 0.7$ this equation returns a negative value for the constituent properties used and cannot therefore be used. The Nielsen equation uses a term representing the maximum packing fraction as discussed in section 7.3. The Nielsen equation sometimes underestimates and sometimes overestimates the modulus.

The Halpin Tsai relationship – equation *[108]* consistently over estimates E_{22} – except for the experimental value at $V_f = 0.5$ – and gives an average error of 20%. If the experimental data for E_{22} of an epoxy resin composite in Table 12 is studied it can be seen that the value obtained from the literature for $V_f = 0.5$ is inconsistent with the other experimental data. If this inconsistent value is not considered then the Halpin Tsai equation predicts the modulus value with an average error of 23%

The value of the geometrical factor 'ξ' in the Halpin Tsai equation is an experimentally determined one, and although usually taken to be 2.0, the IACFA report (75) states that the selection of a value for ξ is "one of the greatest uncertainties" in using the equation. A modification to the geometrical factor is given by Halpin (34) in equation *[110]* who states that the modification gives better results at a V_f of greater than 0.65. However, this is not the case with the one experimental value at $V_f = 0.7$ that is available here.

A number of the equations presented in the literature for the prediction of E_{22} are found to give less than a 20% error when predicting the properties of an E–glass/epoxy resin composite. Thus, any of these equations could have an empirical factor applied to them to improve the accuracy of the predictions. However, as the Halpin Tsai equation uses a geometrical factor in its calculations and that as the selection of a value for the geometrical factor is uncertain then the Halpin Tsai equation may be the best equation to use if a suitable geometrical factor can be determined. A better fit to the experimental epoxy resin composite results is obtained if the geometrical factor is set to 1.0.

If the Halpin Tsai equation is modified such that the value of ξ is taken to be 1.0 then an average error of less than 6% is achieved. If the inconsistent experimental value is not considered then the average error is less than 2%. Thus, the recommended equation for calculating the transverse Young's modulus is as follows:–

$$E_{22}/E_m = (1+\eta V_f)/(1-\eta V_f)$$

where, $\eta = \{(E_f/E_m)-1\}/\{(E_f/E_m)+1\}$

As only one experimental value has been obtained from the literature for the transverse Young's modulus of an E–glass/polyester composite conclusions cannot be drawn from this.

7.5 Other Mechanical Properties

It is stated by Owen (60) that glass reinforced composite structures are usually thin, and therefore as a first approximation can be assumed to be subject to plane stress conditions. The condition of plane stress is obtained if the third direct stress in the material (σ_{33}) can be assumed to be zero, ie that the material is sufficiently thin relative to its other dimensions such that the through thickness effects can be neglected (81,202). This assumption is valid for thin sheets of material such as the skin of an aircraft and would be modelled with thin shell finite elements. If it can be assumed that conditions of plane stress exist in the component only the four elastic constants, E_{11}, E_{22}, G_{12}, ν_{12}, that have already been determined are required to specify the stiffness properties for analysis purposes.

σ_{31}

σ_{32}

However, it is noted by a number of authors (26,41,47–49,60,127) that the through thickness properties are important in an analysis of a composite component as σ_{33} cannot always be assumed to be zero, and that a distinction needs to be recognised between the in–plane and through thickness properties. It should also be noted that Young's modulus in tension, compression and flexure may not be precisely the same although they are often assumed to be so for the purposes of calculation where the value of the tensile modulus is usually used (30,40). If plane stress conditions cannot be assumed in the composite component then the other mechanical properties will need to be determined.

In the previous sections of this Chapter the most commonly quoted mechanical properties have been calculated from equations available in the literature. As an input to an analysis package other values such as E_{33}, ν_{13}, ν_{23}, G_{13}, G_{23} may be required. As has already been stated in Chapter 4, a uni–directional composite is transversely isotropic such that $E_{22} = E_{33}$, $G_{12} = G_{13}$, and $\nu_{12} = \nu_{13}$. Thus, only ν_{23} and G_{23} are still to be determined.

7.5.1 Transverse Poisson's Ratio

For the calculation of ν_{23} there is no experimental data available to investigate the use of the equations presented in the literature. From the graph of ν_{23} against V_f previously presented it can be seen that one equation – equation *[23]* – gives quite a different result to the other three equations. The other three equations give values that are initially quite a bit larger than either of the constituent values. As has already been stated the value of ν_{23} is not considered if a thin structure is to be modelled. Thus, in this case, the value of the ν_{23} may be assumed to be the same as that for ν_{12} with no loss of accuracy in the analysis results. However, if a thicker structure is modelled with, for example, solid finite elements being used then assuming that the transverse and longitudinal Poisson's ratios are the same may well result in a loss in accuracy as noted by Griffin (86).

The Poisson's ratio transverse to the fibres in a uni-directional material should be similar to that transverse to the fibres in a continuous random fibre material. Limited experimental data is available for ν_{23} of a continuous random fibre material – see Appendix B. If the equations found in the literature for the prediction of ν_{23} of a uni-directional material are used to predict these values it is found that equation *[23]* gives the most accurate results. This equation predicts the experimental values to within an average error of 3% – see Chapter 8. Thus, as no other experimental data is available, if ν_{23} is required, then it is recommended that it is calculated from the transverse law of mixtures relationship, ie equation *[23]*.

7.5.2 Transverse Shear Modulus

From the very limited experimental data available the most accurate equation for calculating G_{23} for an epoxy resin composite is equation *[72]* which is based on the CCA model. This equation predicts the value of G_{23} to within 1%. However, this result is based on only one experimental value. Alternatively, as one reference (85) states that G_{23} is always slightly larger than G_{12} – which is confirmed by the one experimental value available – the result for G_{12} could be taken and, for example 15% could be added. Thus $G_{23} = 1.15G_{12}$. If the recommended equation for G_{12} is used to determine G_{23} in this manner then there is only approximately a 10% error in the result when compared to the only available experimental value. Thus, from the limited data available it would seem reasonable to estimate G_{23} in this manner if this value is required when modelling a thicker structure.

In conclusion it is recommended that the value of G_{23} be determined by increasing the longitudinal shear modulus by 15%. It should be noted that this of course gives inaccurate values for the shear modulus at the two extremes of fibre volume fraction, ie at $V_f = 0.0$ and $V_f = 1.0$.

Chapter 8 : Randomly Oriented Continuous Fibre Composites

In this Chapter the micromechanics equations appearing in the literature for the prediction of the mechanical properties of randomly oriented continuous fibre composites will be investigated. As will be seen, far less work has been undertaken by researchers for this type of composite than for uni–directional composites. Note that 3D random continuous fibre composites will not be considered here as Nielsen (20) states it is almost impossible to manufacture a composite in which the fibres are randomly arranged in three dimensions. The random fibre composites of interest here have fibres only in two dimensions. Note that the fibres are assumed to be random in the 1–2 plane and that the in plane properties are given the subscript '2D' to distinguish them from the uni–directional properties.

It is stated by Christensen (203) that formulae similar to the rule of mixtures for uni–directional composites are needed for randomly oriented long fibre composites. However, as for uni–directional composites, these simple formulae whilst suitable for design applications preferably should not be empirical forms established from limited data examination (203). Ideally the formulae should be derived from the principles of mechanics which would "provide forms that invite physical interpretations and understanding, as well as ease of application" (203). The two dimensional random fibre composite micromechanical equations derived by Christensen (13) assume that a state of plane stress exists in the composite and uses the relationships between the stress and strain in the composite to derive the overall mechanical properties. The effective properties for the two dimensional case are defined by the equations given in Appendix B and are reported to compare well with experimental data.

A comparison of the Christensen equation for the Young's modulus with experimental data and the law of mixtures is shown in Figure 16. It is stated by

Christensen and Waals (14) that the comparison is not a close one as in most cases the actual composites contain partially aligned fibres. However, these authors state that that the rule of mixtures prediction for the Young's modulus for this material gives "absurd results" in these cases as it does not take into account the random orientation of the fibres. Christensen (13) also states that his equations can be used when there is a random orientation of short fibres, however, it is noted that the derivation of the equations assumes that the fibres are long enough for the fibre end effects to be ignored.

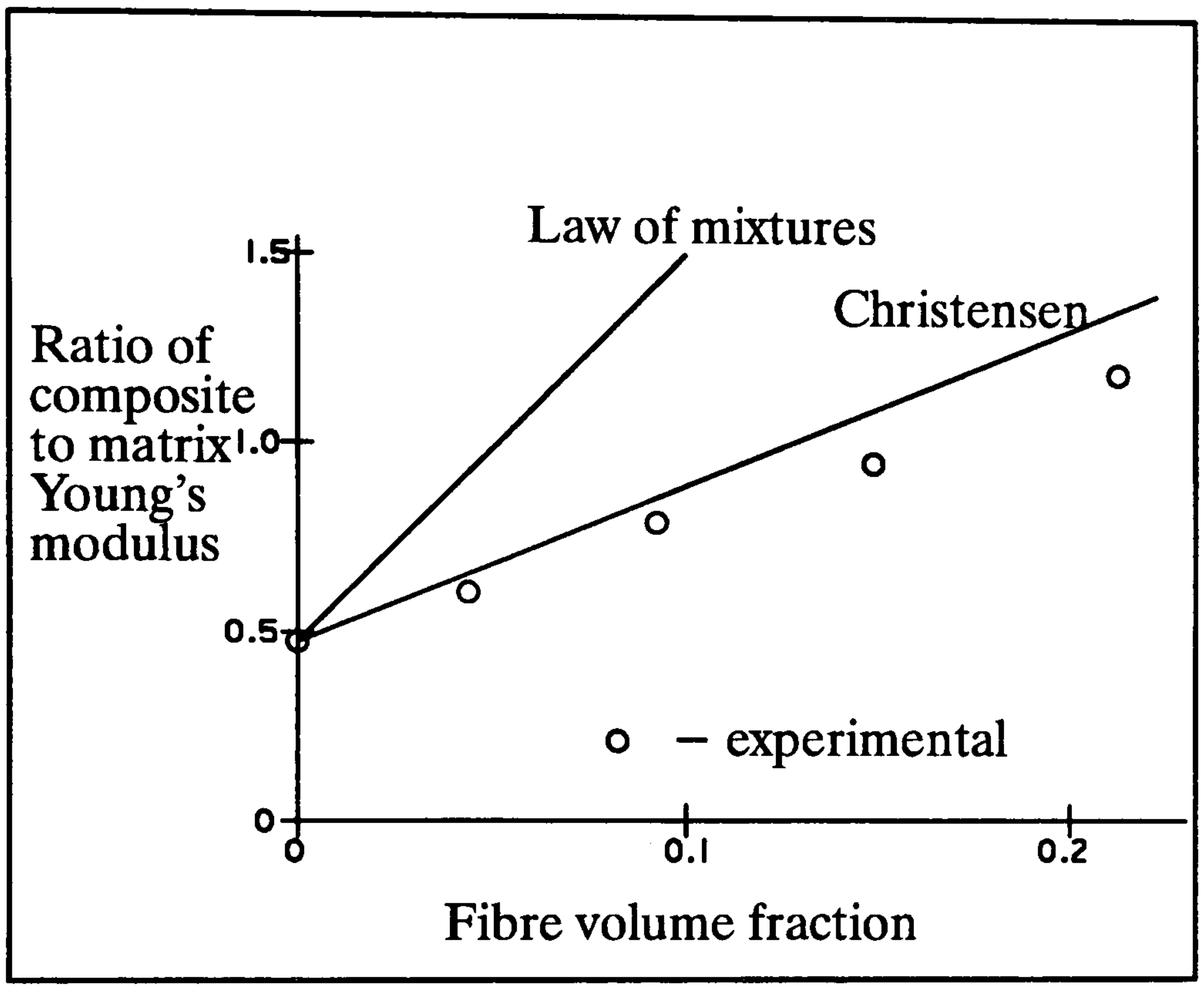


Figure 16 : A Comparison Of Young's Modulus E_{2D} With Experimental Data (14)

A further paper by Christensen (203) simplifies his previously defined expressions for two dimensional random fibre composites by assuming that certain conditions exist such that, for example, the fibre and matrix Poisson's ratios are equal – which is unlikely, and the fibre volume fraction is much less than unity. Whilst these assumptions produce much simplified expressions they have not been included here as the more general results are more useful as the Poisson's ratios of the constituent materials considered here is not equal.

Other authors such as Chou (26) and others (20,61,105) present very simple equations for the determination of the in plane properties of continuous random fibre composites. In the case of Chou these equations are based only on the fibre constituent properties and the fibre volume fraction, in the case of the other equations, as with those of Christensen, they are based on the uni–directional composite properties. Thus, if the properties of a continuous random fibre composite are to be determined then the uni–directional composite properties first need to be determined.

Equations are presented by Manera (17) by which the moduli of randomly oriented discontinuous glass fibre composites may be determined. However, one of the assumptions made by Manera is that the fibre length to diameter ratio is greater than 300. As has been previously discussed, with a fibre aspect ratio of this order the fibres can be considered to be continuous, and hence Shenoi (105) and Smith (61) state that the Manera equations can also be used to determine the moduli of randomly oriented continuous fibre composites. Once again these equations are based on the uni–directional properties.

8.1 Comparison Of 2D Equations Against Each Other

The equations found in the literature and detailed in Appendix B for the determination of the mechanical properties of interest will now be compared. The equations discussed are summarised in section B.3 of Appendix B.

8.1.1 In Plane Young's Modulus

A total of four equations have been found in the literature for the calculation of the in plane Young's modulus of a random continuous fibre composite. A comparison of these equations is shown in Figure 17 where E_{2D} is plotted against V_f. It can be seen in this figure that three of the equations to predict E_{2D} give very similar results. However, equation *[123]* predicts a linear relationship between E_{2D} and V_f. Further, at the extreme value of E_{2D} at $V_f = 0$, then this equation predicts that $E_{2D} = 0$ rather than $E_{2D} = E_m$ as would be expected. This equation will thus not be considered further.

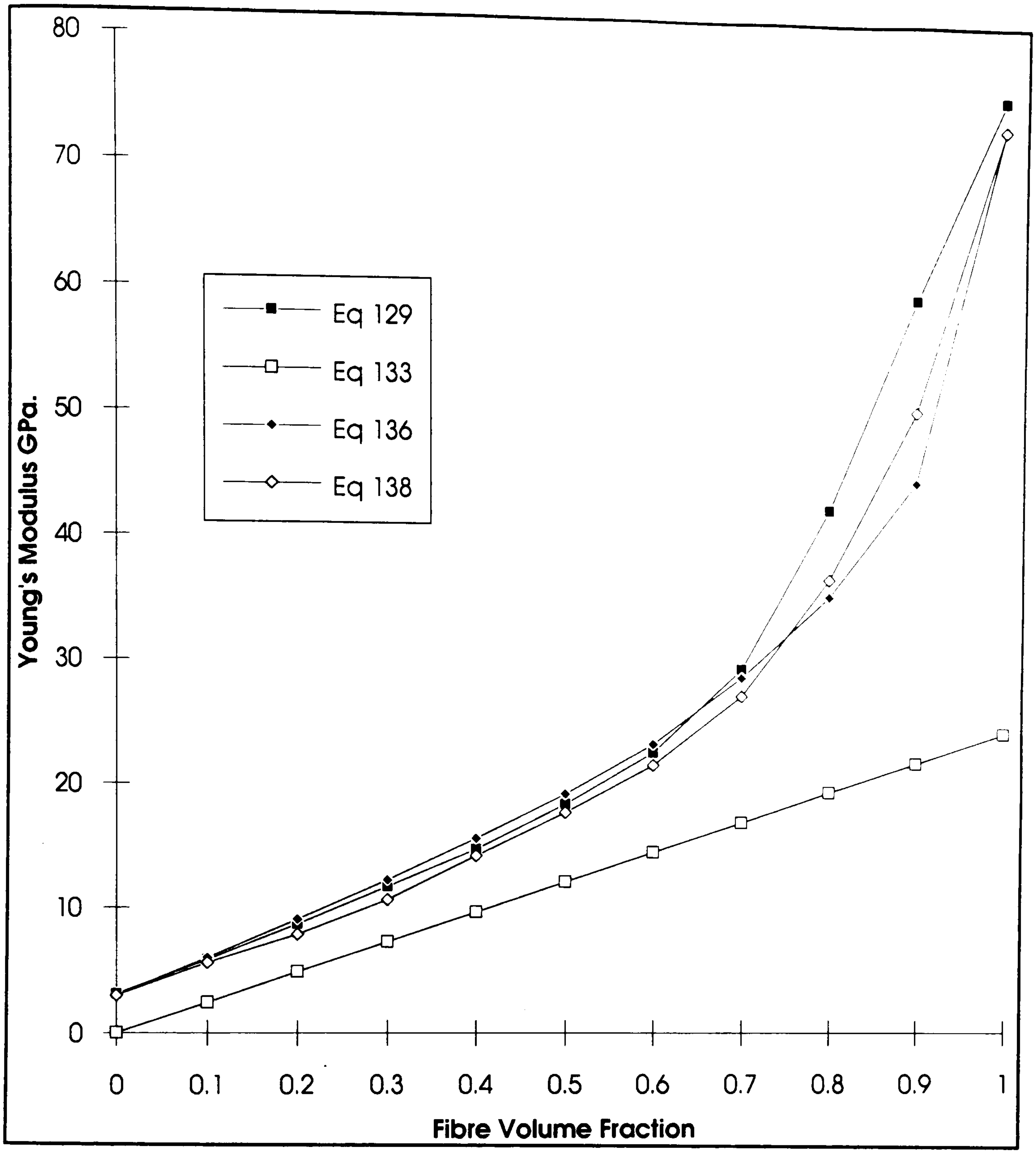


Figure 17 : In Plane Young's Modulus Against V_f.

Of the three equations that predict very similar results, the difference between the predicted results even at a relatively high value of $V_f = 0.7$ is fairly small. This difference reduces as V_f decreases. As practical values of V_f for this type of composite are likely to be below 0.5 then the difference seen between these equations means that only one of them needs to be considered further. Thus the equation that will receive further consideration is *[136]* which relies only upon the longitudinal and transverse Young's moduli of a uni-directional composite, and is therefore a relatively simple calculation to make.

8.1.2 In Plane Poisson's Ratio

A total of four equations have been found in the literature for the calculation of the in plane Poisson's ratio of a continuous random fibre composite. A comparison of these equations is shown in Figure 18 where ν_{2D} is plotted against V_f. It can be seen in figure 18 that the four equations presented in the literature give quite different predictions, although equations *[130]* and *[140]* do give similar results.

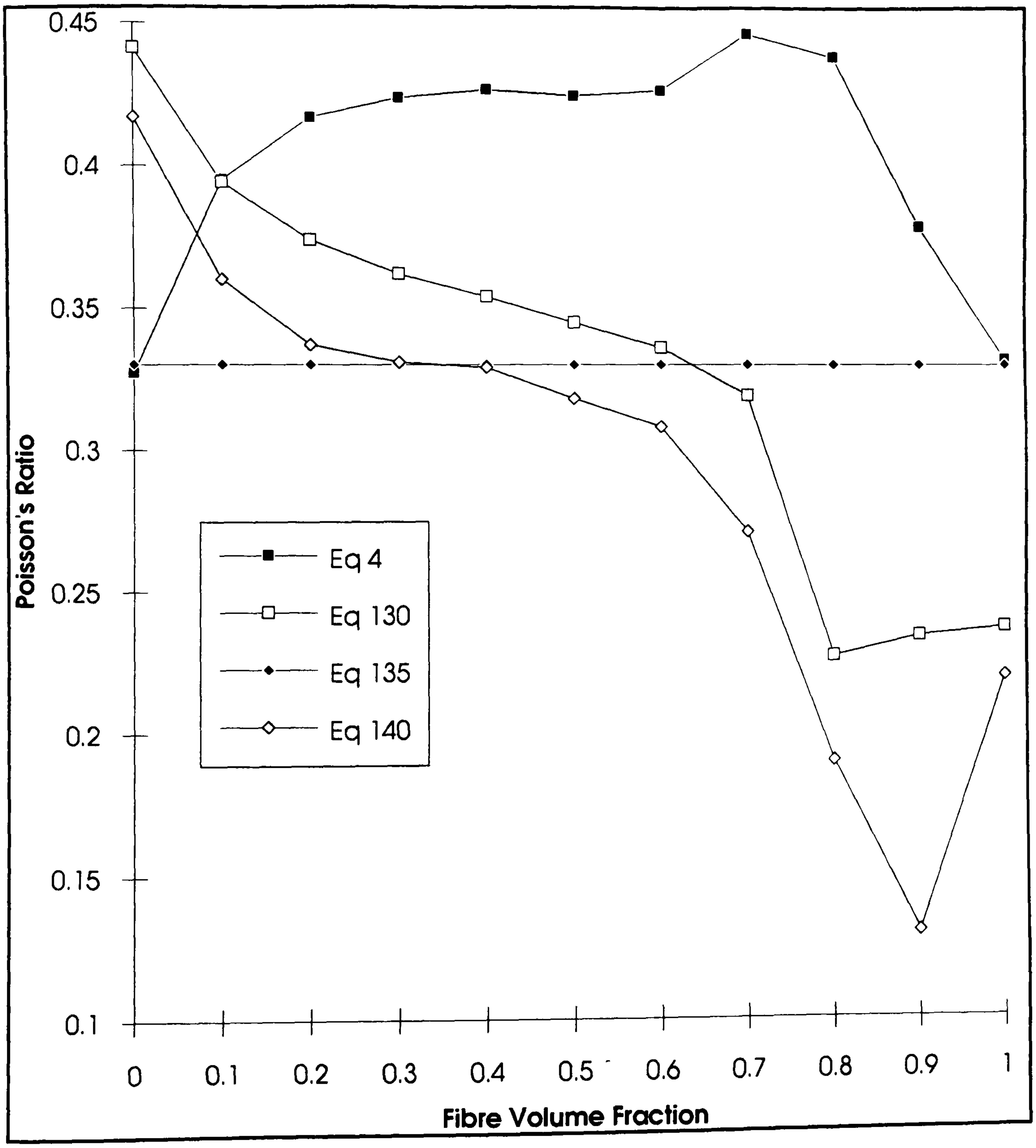


Figure 18 : In Plane Poisson's Ratio Against V_f.

8.1.3 In Plane Shear Modulus

Three equations are found in the literature for the prediction of the in plane shear modulus of a continuous random fibre reinforced composite. A comparison of these equations can be seen in Figure 19 where G_{2D} is plotted against V_f.

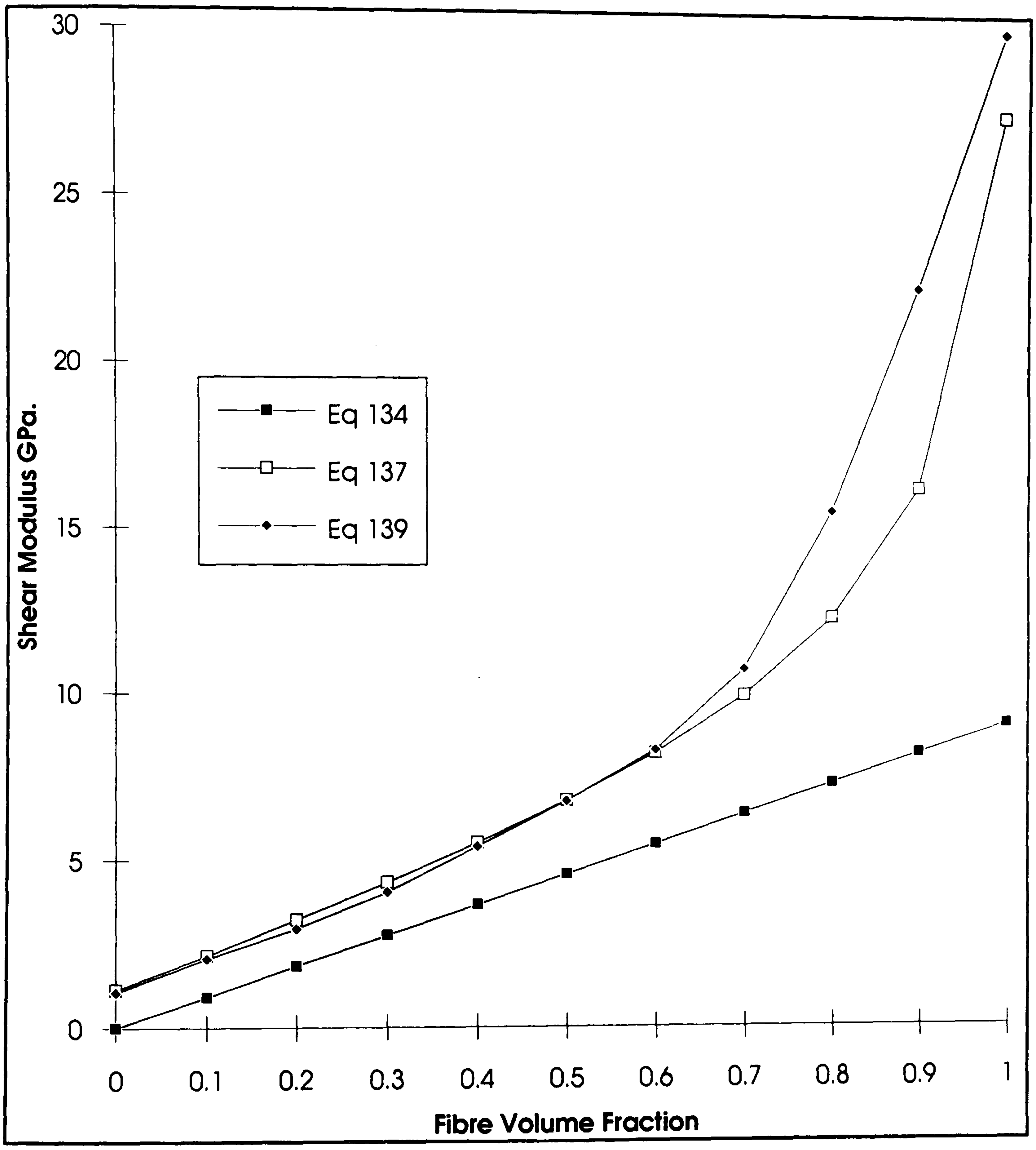


Figure 19 : In Plane Shear Modulus Against V_f.

Of the three equations found, one of them – equation *[134]* is seen to give a linear relationship between G_{2D} and V_f, whilst the other equations predict a non–linear relationship. Further, equation *[134]* predicts a value of $G_{2D} = 0$ at V_f

= 0 rather than the expected value of $G_{2D} = G_m$. This equation will therefore not be considered further. The other two equations predict very similar results up to V_f = 0.7 which is a very high value of V_f for this type of composite. Composites of this type would be expected to have values of V_f lower than this, and there is therefore only need to consider one of these two equations further. Equation *[137]* only requires the determination of the longitudinal and transverse Young's moduli of a uni–directional composite, which are also required for E_{2D}. However, equation *[139]* requires additionally the calculation of G_{12}, ν_{12} and ν_{21} of a uni–directional composite. Thus, this equation requires more calculations to be made, which whilst easy to implement on a computer, requires many more calculations to be made than equation *[137]*. It would therefore seem logical to only consider the use of equation *[137]* when comparing the results against experimental data.

8.2 Comparison Of Equations Against Experimental Data

This section compares the accuracy of the results obtained from the equations found in the literature with experimental properties for E–glass/epoxy composites. The detailed results from the equations used can be seen in Appendix B together with the experimental data and the constituent properties used. Presented here is a summary of the results. The accuracy of the predictions made for E–glass/epoxy composites are confirmed by considering also the predictions for the properties of E–glass/polyester composites found in the literature. It should be noted that only the composite properties with these two types of resin have been considered as these materials are more likely to be used for structural applications.

8.2.1 In Plane Young's Modulus

The use of equation *[136]* is fairly straight forward as it only requires the calculation of the longitudinal and transverse Young's moduli of a uni–directional material. This equation is shown to predict the value of E_{2D} with an average error of 5%, and therefore it is recommended that equation *[136]* be used to determine the in plane Young's modulus.

8.2.2 In Plane Poisson's Ratio

Of the four equations given in the literature for the calculation of ν_{2D} only three give reasonably consistent results when the predicted values are compared to experimental ones. The results obtained from the isotropic relationship – equation *[4]* – are seen to be the most inaccurate. The most accurate equation is equation *[140]* with an average error of 11%. It is interesting to note that if a constant value of 1/3 is used for the in plane Poisson's ratio as recommended by some authors (19,26), then a relatively accurate result with an average error of approximately 13% is observed. The results from equations *[4]*, *[130]* and *[140]* over estimate the experimental results, as can be seen in Appendix B, and thus more accurate results may be obtained if a 'correction factor' was applied to these equations. Results from the equations used to predict both ν_{12} and ν_{23} for a uni–directional material given in Appendix A suggest that those equations would also over estimate the result for a continuous random fibre composite.

When correction factors are applied, then if ν_{2D} determined from equation *[140]* is reduced by 10% then an average error of only 9% is observed. Similarly, if the result from equation *[130]* is reduced by 15% then an average error of 9% is observed. The correction factors are therefore seen to make only a marginal difference to the accuracy of these equations. Of these two equations, equation *[130]* first requires the calculation, or determination, of E_{11}, G_{12}, ν_{12}, G_{23}, and K_{23} for a uni–directional material. Similarly, equation *[140]* requires E_{11}, E_{22}, G_{12}, ν_{12} and ν_{21} to be calculated. As no experimental data is available for K_{23} and very little for G_{23} the equations used to calculate these values must be subject to some doubt. However, experimental data does exist for E_{11}, E_{22}, G_{12}, and ν_{12} – see

Appendix A – and thus the calculation of these values has been verified. The other property required ν_{21} is determined from other mechanical properties.

The experimental values of ν_{2D} given in Appendix B are seen to exhibit a large amount of scatter, and thus more experimental results are required before any definitive conclusions can be drawn. In the absence of more experimental data then equation *[140]* which gave an average error only marginally larger than the criteria adopted in Chapter 5 is recommended for the calculation of the in plane Poisson's ratio.

8.2.3 In Plane Shear Modulus

The results from equation *[137]* can be seen in Appendix B to sometimes overestimate and sometimes underestimate the experimental data. Results from the equations used to predict both G_{12} and G_{23} for a uni–directional material given in Appendix A suggest that most of those equations would also underestimate the results.

An alternative approach could be to use the Halpin Tsai equation for the longitudinal shear modulus of a uni–directional material. This equation contains a geometrical factor to adjust the calculated modulus to between the values predicted by the longitudinal and transverse law of mixtures equations – see Chapter 4. Thus the Halpin Tsai equation could be used with an appropriate geometrical factor being determined from the available experimental data. The Hewitt and Malherbe modification to the Halpin Tsai equation has a geometrical factor of $1+40V_f^{10}$ which could be further modified so that the curve begins at a lower value of V_f. If this equation is curve fitted to the available experimental data for the in plane shear modulus then a geometrical factor of $5+10^5V_f^{10}$ is determined. The Halpin Tsai equation with this geometrical factor results in an average error of 5% when compared to experimental data. Thus the recommended equation for the calculation of the in plane shear modulus is the Halpin Tsai equation with a geometrical factor of $\xi = 5+10^5 V_f^{10}$.

8.2.4 Transverse Properties

The equations obtained from the literature and presented in Appendix B only allow the calculation of the properties in the plane of the material, and not any of the transverse properties. As has already been stated the transverse mechanical properties are also important unless the material can be considered to be in a state of plane stress, and in fact equations *[129]* and *[130]* assume this to be the case. For the material considered for this thesis it has been assumed that the through thickness, or transverse, properties may be required to enable a complete description of the material behaviour. The following sections discuss how these transverse properties may be determined. Note that the transverse properties will be described by the use of the subscript '2T'.

8.2.4.1 Through Thickness Young's Modulus

In a perfect thin sheet of continuous random fibre composite the fibres will be random in two dimensions, in the $1-2$ plane, such that the modulus in the plane of the material in the two in plane directions should be equal and hence E_{2D} in direction '1' will be equal to E_{2D} in direction '2'. The material should not be isotropic and thus the Young's modulus in the third direction, through the thickness, ie E_{2T} will be different.

In practice the third Young's modulus, through the thickness of the material, will be of importance if a relatively thick composite is being considered. However, no formulae are available in the literature for the calculation of this through thickness modulus. It would initially appear that this through thickness modulus should be the same as the transverse Young's modulus in a uni-directional composite as they are both taken across the fibres. However, the fibres in the random mat may not be entirely two dimensional and may contribute somewhat to the stiffness in the through thickness dimension. The most accurate of the equations used to calculate the transverse Young's modulus in a uni–directional composite has an experimentally determined geometrical factor. It is possible that this equation is appropriate for the continuous random fibre material but with a different experimentally determined geometrical factor. However, no experimental data is available to enable the determination of this geometrical factor. The lack of experimental data also means that whether the in plane fibres do increase the through thickness modulus above that of a uni–directional material cannot be investigated. Thus, due to the lack of experimental data to suggest otherwise, it is felt that a reasonable approximation of the through thickness modulus can be found by using the expression for the transverse Young's modulus in a uni–directional composite.

8.2.4.2 Through Thickness Poisson's Ratio

As the glass mat is random in the 1 – 2 plane then the Poisson's ratios would be expected to be the same in the 1–3 and 2–3 directions, and be different to the in plane Poisson's ratio ν_{2D}. In the absence of any equations in the literature to predict the transverse Poisson's ratio the use of the three equations found in the literature to predict ν_{23} for uni–directional materials and recommended for comparison in Chapter 6 has been investigated. These have shown that ν_{2T} can be predicted most accurately by equation *[23]* which predicts the results with an average error of 3%.

8.2.4.3 Through Thickness Shear Modulus

As the glass mat is random in the 1 – 2 plane then the shear moduli would be expected to be the same in the 1–3 and 2–3 directions, and be different to the in plane shear modulus G_{2D}.

No formulae are available in the literature for the calculation of the through thickness shear modulus. It would appear that the through thickness shear modulus should be the same as the transverse shear modulus in a uni–directional composite as they are both taken across the fibres. However, the fibres in the random mat may not be entirely two dimensional. As no experimental data is available to suggest otherwise it is felt that a reasonable approximation of the through thickness shear moduli can be found in the same manner as for the transverse shear modulus in a uni–directional composite, ie by increasing the in plane shear modulus by 15%.

Chapter 9 : Continuous Woven Fibre Composites

All woven fabrics consist of interlaced fibre tows, threads, or yarns, in the warp and fill (or weft) directions. In such a composite the individual glass fibres are wound together to form a larger glass fibre thread which may consist of, for example, two hundred of the smaller fibres. The exact number of smaller fibres making up the thread may be different in the warp and fill directions. The fibres making up the thread may be linearly arranged or may be twisted together. The fibre volume fraction of a thread is dependent upon the number of fibres in the thread and the orientation of the fibres relative to the thread axis, ie the angle of twist (26). It follows from this that the fibre volume fractions of the warp and fill threads may be different (31,82,204), and that the overall fibre volume fraction of the composite may be different to that of the threads (205).

In a traditional woven composite the action of interlacing the threads through the thickness of the material is limited to two or three thread diameters (26). Thus although the moduli through the thickness will be somewhat greater than those for a uni–directional material for example, these woven composites are considered to be two dimensional. However, it is possible to have a weave pattern that creates a significant increase in the through thickness properties by weaving the threads in three dimensions. The two dimensional weaves are the most common, and thus this Chapter will concentrate only upon these types of woven composites.

The different types of woven fabrics can be indentified by the repeating patterns of the interlaced regions in the two directions (206). Many different types of weave pattern exist. These different types are classified by the number of the repeat 'n' (31,207–210). The number of 'n' in the fill direction is defined such that a filling thread is interlaced with every n'th warp thread, ie, the same weave pattern will appear after every 'n' warp threads in the direction of filling. In general weaving structures 'n' for the fill can differ from 'n' for the warp (207). The most common weave pattern is the plain weave – see Figure 20. The basic construction of this pattern requires two warp and two fill threads. The plain weave is the most highly interlaced weave pattern and is thus the most resistant to in–plane shear (31). The satin weaves (with 'n' greater than 4) represent a family of patterns with a minimum of interlacing. In this type of weave the fill threads periodically skip over several warp threads. The satin weaves thus produce a pattern which has low resistance to shear and is easily moulded (131).

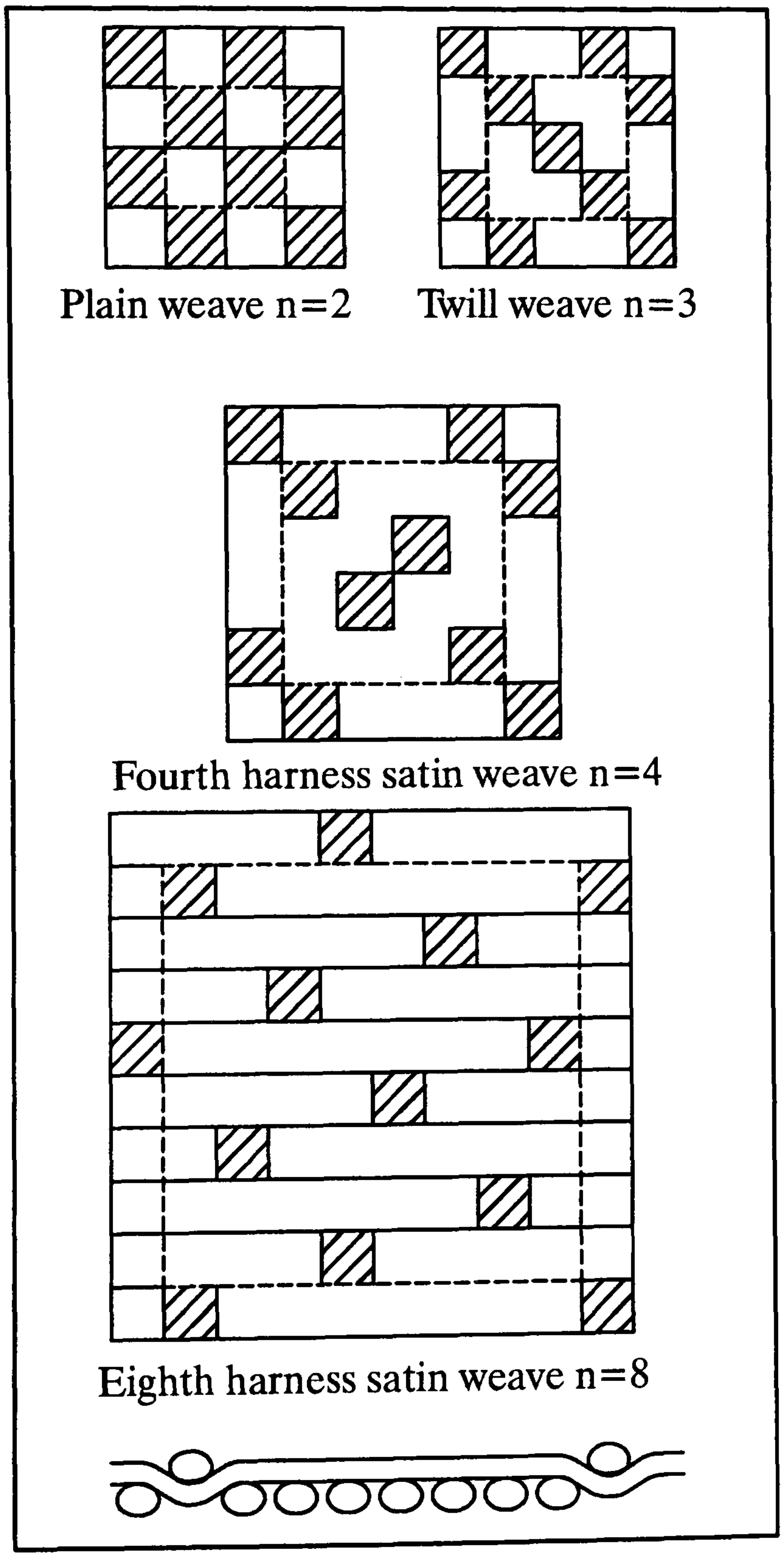


Figure 20 : Different Weave Patterns (84,209,210).

Whitcomb (211) and Naik and Ganesh (205) state that because of the variety of possible weave patterns, analytical models are needed to predict the effect of the various weave parameters upon the mechanical properties of the composite. In a balanced material such that the weave pattern is the same in both the warp and fill directions then, $E_x = E_y$, $G_{xz} = G_{yz}$, $\nu_{xz} = \nu_{yz}$ (136), and the yarn is stated to be transversely isotropic such that (249):– $E_{22} = E_{33}$, $G_{12} = G_{13}$, $\nu_{12} = \nu_{13}$. To distinguish the woven properties from uni–directional properties the subscripts 'x', 'y', and 'z' have been used, where 'x' is in the '11' direction. Although the fibres may be woven in three dimensions the material considered here consists of glass fibres woven in two dimensions only, ie in the 1 – 2 plane, as this is the type of woven material most commonly in use.

Hirokawa et al (204) analyse two and three dimensional carbon fibre woven composites, and estimate modulus values by the rule of mixtures. These estimated values are compared to experimental ones where it was found that although "the rule of mixtures is a reasonable assumption" it overestimated the modulus values by between 12% and 20%. Two other publications (30,40) also attempt to predict the longitudinal Young's modulus of a woven material by modifying the law of mixtures.

Other authors also analyse woven composites, however the micromechanical equations presented by these authors such as Chou and Ishikawa (84) and Naik and Ganesh (205) require parameters that may not readily be available to the analyst (such as the weave angle). Hence, the composite properties may not be able to be determined from the properties of the constituent materials alone. These equations are summarised in Appendix C. These authors create a detailed model of the weave pattern and use classical lamination theory, see Appendix C, to determine the overall composite properties. This section will only briefly describe the analytical models used by these authors to determine the composite properties. It is felt that an analyst who is analysing a component manufactured from such a material requires a quick and easy method of determining the required properties such as the equations used to determine the uni−directional composite properties. In addition, the classical lamination theory used by these authors does assume that plane stress conditions exist, and hence does not consider the through thickness properties. As the through thickness properties may be of importance then other means of predicting the composite properties will be investigated.

9.1 Mosaic Model

A series of papers by Ishikawa et al (84,207−210,212−214), and a book by Chou (26), present three models by which woven fibre composites (mainly consisting of carbon fibres) may be analysed. The 'mosaic' model, see Figure 21, consists of an assemblage of what are basically two layers of cross ply lamina of a width equal to the thread width. This model is combined with the isostress and isostrain assumptions to predict the upper and lower bounds of the elastic moduli by using classical lamination theory. The mosaic model however, does neglect the continuity and undulation of the fibres, and therefore only provides a "rough estimate" of the properties of fabric composites (84).

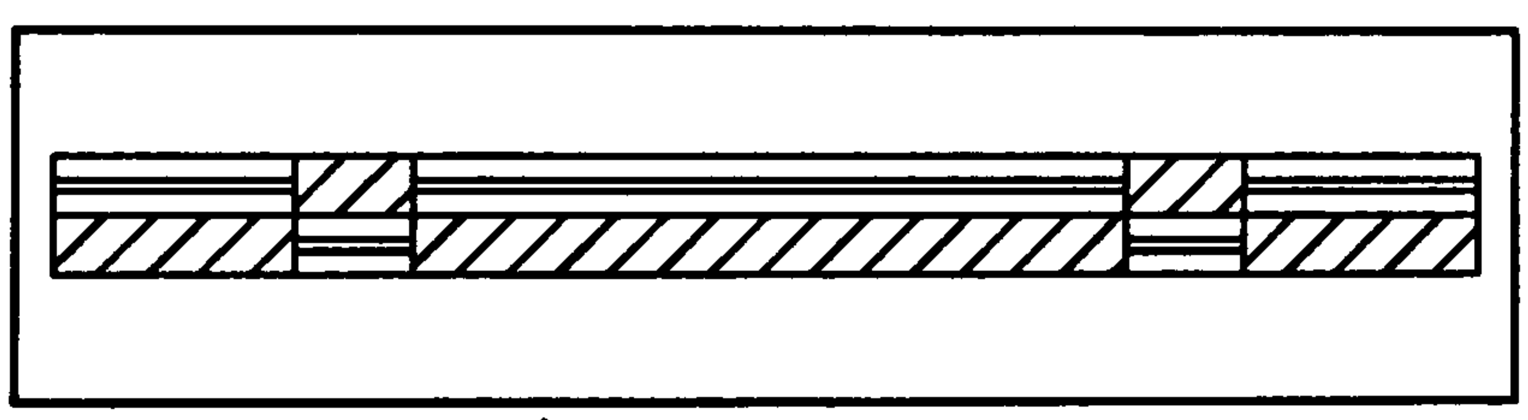


Figure 21 : The Mosaic Model (26,84,208,209).

9.2 Fibre Undulation Model

The second model is termed a fibre undulation or crimp model, see Figure 22, and is suitable for plain weave composites (84,214). This model only considers fibre continuity and undulation in the loading direction. The model also considers the thread thickness 'h', the length of undulation 'a', and the angle of the threads 'θ' as they weave around one another (26,208,209). As a pure matrix region appears in this model the overall fibre volume fraction can be different to the fibre volume fraction in the threads or yarns (26). This model uses the uni-directional transformation equations, see Appendix A, to determine the thread properties due to the angle of undulation. Classical lamination theory is then used to determine the overall properties as with the mosaic model. Whilst modelling the weave more accurately than the mosaic model this model obviously requires a lot of information about the weave pattern which may not be readily available to the design engineer. It is reported that taking into account the existence of the thread angle leads to a reduction of the effective elastic moduli (208,209). Chou (26) reports that the results for the moduli obtained from this model coincide exactly with the upper bound results from the mosaic model. This model is essentially two dimensional and takes no account of fibre undulation in the transverse direction.

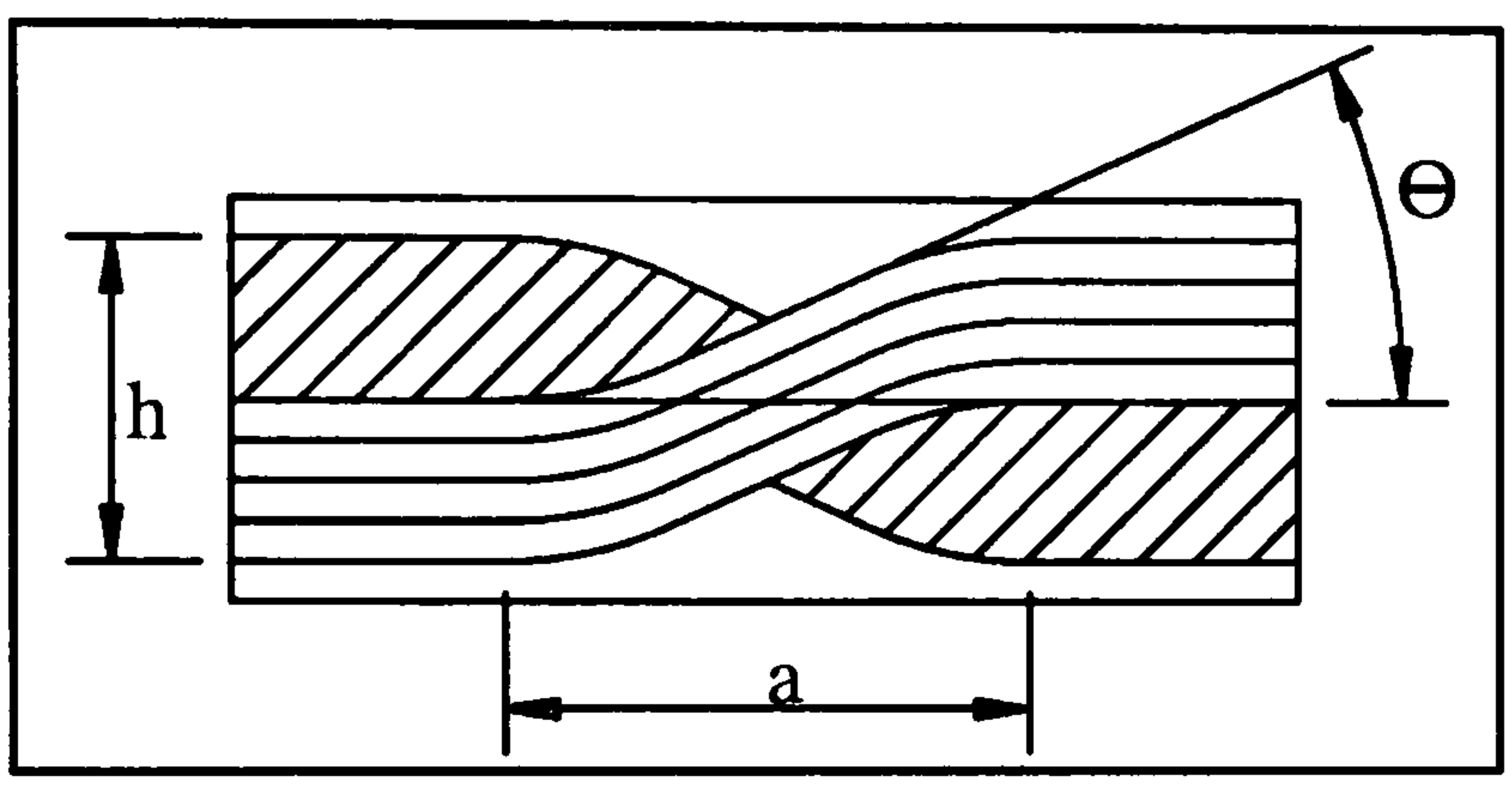


Figure 22 : Fibre Undulation Or Crimp Model (26,84,208,209,214,215).

An analytical model to predict the moduli of a 2D woven fibre composite which takes into account voids in the structure has been developed by Farouk et al (215), and is based on the undulation model. It is stated by Farouk et al that the moduli predicted by this model are within 10% of the experimental values. The matrix properties of this model are modified by Farouk et al to take account of voids.

9.3 Bridging Model

Interlaced regions in a satin weave composite are separated from each other and thus Ishikawa et al (26,84,208,209,214) have developed a 'bridging' model, see Figure 23, to take this into account. The bridging model is valid only for satin weave composites (26,84). The repeating element of this model consists of the central interlaced region where the undulation occurs (the undulation model), and four separate regions making up its surrounding area which is modelled as layers of cross ply lamina (the mosaic model). Regions 2,3 and 4 act as bridges for the load transfer between regions 1 and 5, and again classical lamination theory is used to obtain the overall composite properties.

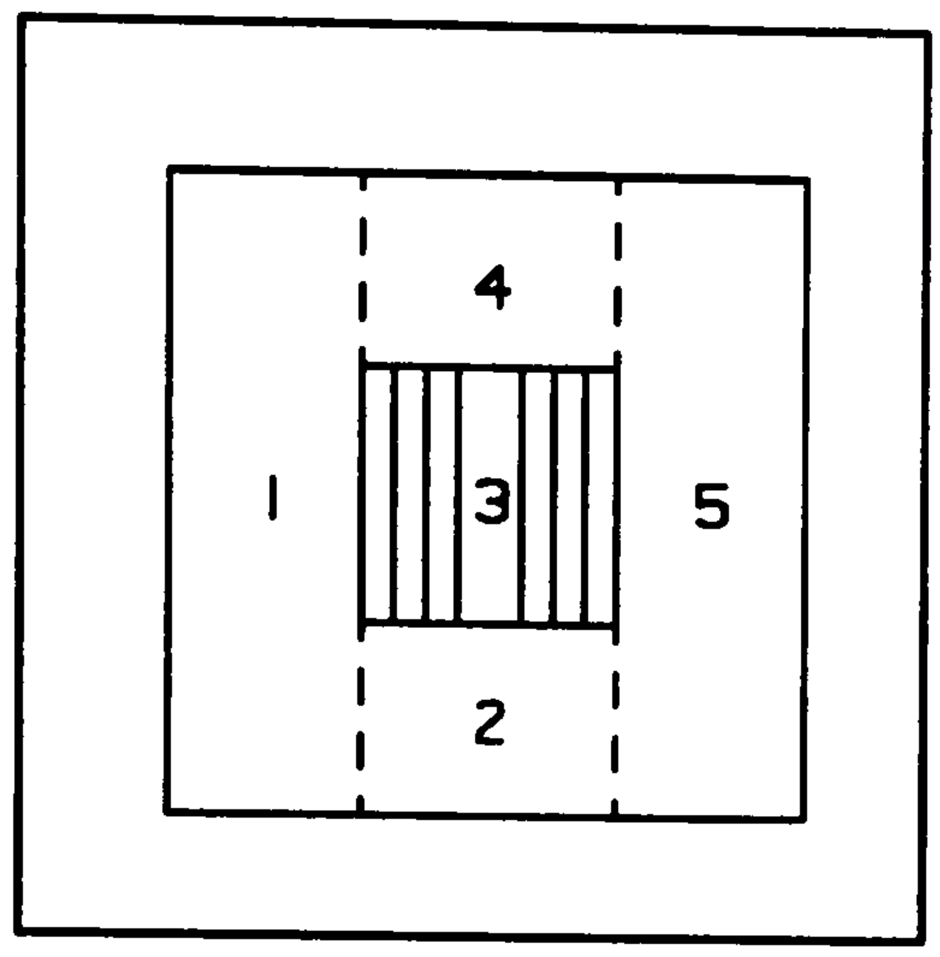


Figure 23 : The Bridging Model (26,84,208,209,214).

In one paper (212) Ishikawa et al compare theoretical and experimental results for the moduli of woven carbon fibre composites based upon the above models. The theoretical results are said to compare well with the experimental data. Ishikawa et al agree with Zhang and Harding (206) by stating the at the fibre undulation ratio (h/a) is a very important parameter which strongly affects the elastic moduli of plain weave composites. The values of the elastic moduli are also dependant upon the laminate ply number 'n' as neighbouring layers in a fabric tend to suppress the warping of one another (26,212). It has been found that as the undulation ratio increases then the value of E_x decreases and ν_{xy} and ν_{xz} both increase slightly (206). It was also found that the in-plane shear modulus G_{xy} is mainly influenced by the fibre volume fraction which decreases linearly with 'n' (212). Ishikawa et al report that the mosaic model "shows particularly large discrepancies in the prediction of the moduli of plain weave composites". Thus, they recommend that the undulation model be used for values of 'n' between 2 and

4, and the bridging model for values of 'n' greater than 4. This is supported by Chou (26,84) who states that this is because there is no bridging effect in plain weave composites as there are no straight yarn regions surrounding an interlaced yarn.

9.4 Laminate Analogy

A different model to those discussed above termed the slice array model (SAM) is analysed in an NPL report (65) and in a paper by Naik and Ganesh (205). In the SAM model the thread is taken to be transversely isotropic and its properties evaluated from the fibre and matrix properties using the CCA model for uni−directionally arranged fibres described earlier. The unit cell, shown in Figure 24, is a quarter of the section of the composite where the warp and fill threads overlap. This unit cell is divided up into a number of vertical slices (50 are used by Naik and Ganesh) which are idealised as four layer laminates of uni−directional material. The laminate analogy is then used to determine the elastic constants of the composite − see Appendix C. Naik and Ganesh note that if the maximum thread angle 'θ' is high then the SAM model would fail to give accurate results.

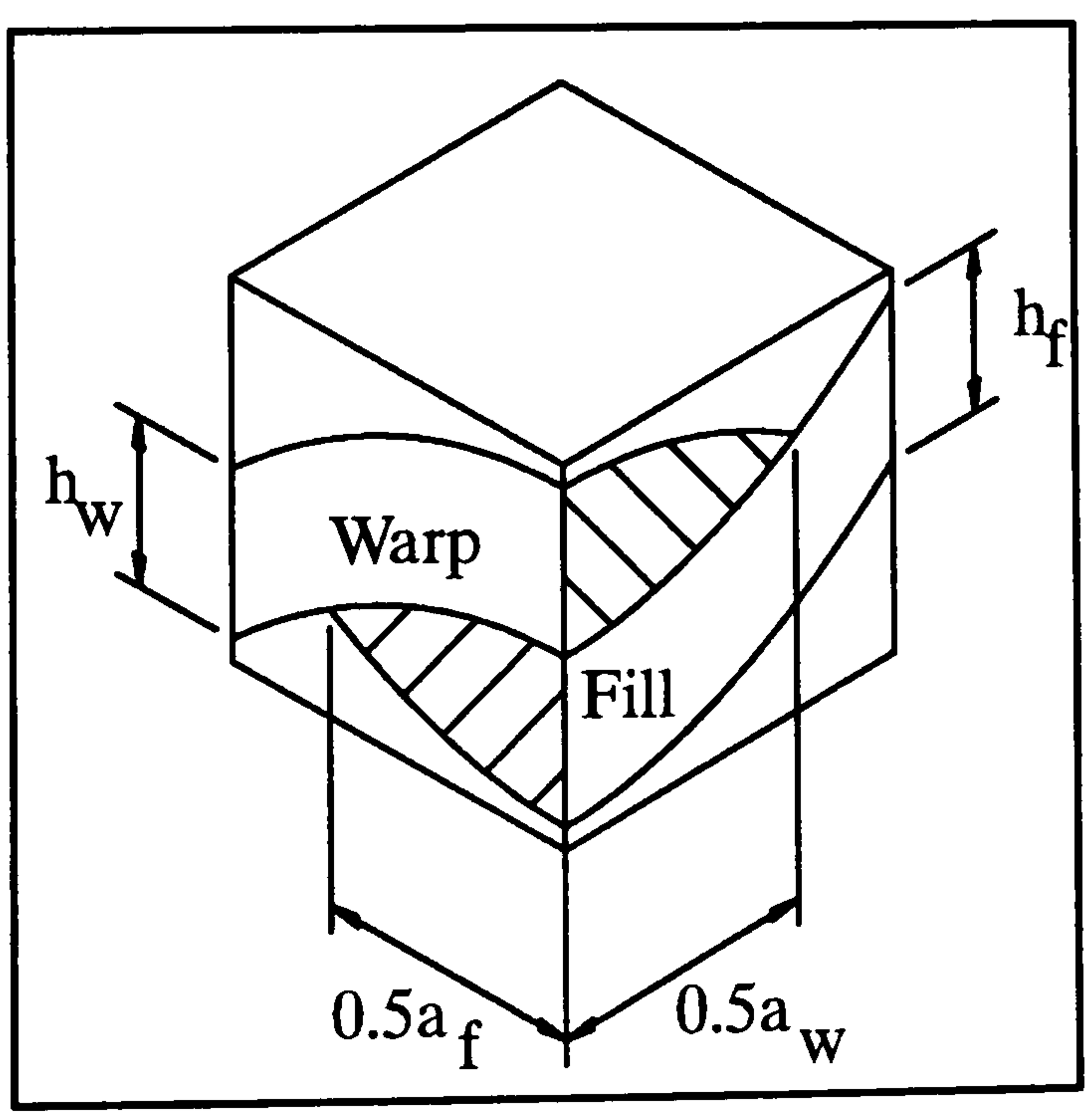


Figure 24 : Unit Cell Of The Slice Array Model (205)

The laminate analogy is also used by Halpin et al (216) to predict the properties of two dimensional and three dimensional woven fabric composites. In their method the 2D plain weave fabric is assumed to behave as a laminate fabricated from layers of oriented fibres. The layers are stacked together to form the laminate which represents the in–plane properties of the composite. The layer properties take into account the horizontal and vertical spacings between fibres in the two directions, and the angles between the fibre axes. No experimental validation of the results is given in the paper, but Halpin et al state that results obtained from their model are "qualitatively correct". The theory assumes that the material is very thin such that only four of the mechanical properties can be determined, ie, E_x, E_y, G_{xy}, and ν_{xy}.

Smith (61) and Shenoi (105) also advocate the use of the Halpin model. They state that a woven cloth can be treated as a number of overlaid uni–directional plies with a ply 'efficiency factor' being introduced. Thus, a bi–directional cloth is treated as a pair of equivalent, superimposed, uni–directional laminae, each occupying the same space as the original ply, and having the same thickness. These equivalent laminae have a reduced ply efficiency by which the stiffnesses are multiplied. The equivalent uni–directional plies of a balanced woven material are said to have a ply efficiency of 0.5. A biased fabric may be represented by adopting different efficiency factors in different directions. Thus a material with a 4:1 fibre distribution would have an efficiency of 0.8 in the main fibre direction and 0.2 in the other direction. These models, however, will not take account of the through thickness effects of the fibres (41).

9.5 Finite Element Analysis

Finite element analysis is used by Paumelle et al (82,217) to determine the overall properties of a woven material. The basic cell analysed of the woven material consists of two warp and two interlacing fill fibres and is modelled with approximately 1300 elements. The inputs to this microscopic analysis are the resin properties, assumed isotropic, and the thread properties, assumed transversely isotropic. The thread properties are calculated by assuming that the thread consists of a uni–directional fibre–resin combination. The paper does not state how these uni–directional properties are calculated. Other inputs to the microscopic analysis are obtained from the material manufacturer and are the fibre volume fraction of the warp and fill threads, the width and thicknesses of the threads, and the distances between threads. The results from the analyses are compared to experimental data and are found to have a "good agreement" for the Young's moduli and shear moduli. The results for the Poisson's ratio are "not as good" but this is stated to be due to the fact that the "uncertainty in the experimental values is large." The authors then vary the input parameters to the analysis to study the effect that these have upon the overall properties of the woven material.

Ishikawa et al (207–209) use finite element analysis to determine the strain distribution in woven composite plates. They model the area of the undulation, ie the fibres, in great detail, and state that the theoretical results from the model compare extremely well with experimental results. Note that the overall thickness of the woven composite plate can differ from that of the thread due to the presence of layers of matrix material at the surface (214). Jortner (100) also uses FE analysis, however, in this case uni–directional composites with curved fibres are studied. Jortner does point out that the waviness of reinforcing tows in a woven material is not well treated by this method. However, this method is used by Whitcomb (211) to study the effect of waviness on the moduli of a plain weave woven fibre composite. Whitcomb uses 768 elements to model the fibre and matrix in a unit cell of the weave to determine the overall composite properties. Leischner and Johnson (218) model a unit cell of a hybrid woven composite with 8400 elements, and report that the predicted moduli are within 2% of test data.

Another paper by Zhang and Harding (206) deals with the micromechanical analysis of a one ply plain weave carbon fibre composite material using the strain energy equivalence principle with the aid of finite element analysis. The model of the plain weave composite only considers undulation along the fill direction, and will therefore only give the composite properties in that direction. The composite properties are assumed to be orthotropic and linearly elastic. Zhang and Harding report that the predictions for both E_x and ν_{xy} agree satisfactorily with experimental data, but that the predictions of G_{xy} and E_y are underestimated by their model. This discrepancy is due to the model only having undulation in one direction.

9.6 Calculation Of Woven Composite Mechanical Properties

The literature survey has highlighted some work by researchers who create analytical models by which the mechanical properties of the woven composite can be predicted. The majority of the equations presented in Appendix C from the work by these researchers assume that some information concerning the microgeometry of the weave pattern is available to the analyst. The equations assume that the analyst is able to obtain various parameters such as 'θ', 'a' and 'h' – see Figures 22 and 24. The paper by Chou and Ishikawa (84) is typical of such work where the elastic constants of the thread with respect to the global axes are derived. The equations presented require an angle 'θ' to be known which relates to the weave pattern. The equations derived by the authors simply take into account that the thread itself is a transversely isotropic composite, and that in effect a correction factor needs to be applied to the thread uni–directional properties because of the weave pattern. However, even though the literature survey undertaken has found some woven mechanical property data, the references do not include details of the weave geometry such that these equations can be used. Instead, the references give a volume fraction – sometimes just an overall V_f, but also often a thread or yarn V_f recognising that the thread is itself a uni–directional composite.

A number of researchers create a detailed model of the weave geometry and then use FE techniques to determine the overall mechanical properties of the woven material. An analyst, who is analysing a component manufactured from a woven material, would not first want to create a detailed FE model of the weave pattern in order to obtain the composite properties. If these composite properties are not available from the material manufacturers then simple, quick methods are required to determine them. The models developed by the various researchers do not provide the type of simple easy to use equations seen previously for the uni–directional, or random fibre, materials in which the composite properties are predicted directly from the constituent properties. Only one such equation has been found that allows the modulus of a woven fibre composite to be determined directly from the constituent properties – equation *[148]*. This equation includes an empirical factor to reduce the effect of the fibre modulus in the law of mixtures equation.

It is stated by Paumelle et al (82) that the threads of a woven composite consist of a fibre–resin combination as the threads are often pre–impregnated with resin before the fabric is woven. This means that the thread V_f will be less than unity, and be different to the overall V_f of the woven composite (26,82,84,205). The properties of the threads may be found by assuming them to be uni–directional and transversely isotropic (82,84). According to Paumelle et al, a typical thread V_f may be 79–81%. However, the fibre volume fractions in the warp and fill directions may be different (31,82,204). Hence, the values of the Young's moduli in the plane of the weave may be different depending upon the type of weave and thus E_x may be different to E_y. Both of these values will be different to the Young's modulus through the thickness of the material, E_z. Thus, V_f may for example be 41% in the warp direction and 59% in the fill direction for an unbalanced weave, and 50% in each direction for a balanced weave.

It would appear therefore that a two stage micromechanical analysis is required. The first stage would consider only the thread as a uni–directional composite with the thread fibre volume fraction being used. Thus the equations found in Chapter 7 could be used to calculate the thread properties. The second stage of the calculation would take the calculated thread properties and use these to determine the overall composite properties by considering the weave pattern and warp and fill fibre volume fractions. However, the thread V_f may not always be available, and any method used to calculate the composite properties should not rely on the availability of this value. Also, the value given for the overall V_f should take into account the thread V_f and thus the value of the thread V_f should not be required in the calculations.

The determination of the mechanical properties of woven materials is thus complicated by the possibility of the presence of different weave patterns and unbalanced weaves. It would therefore appear that the only way to mathematically determine the properties accurately would be to do what a number of researchers have done and create a detailed FE model of the weave pattern. However, even the simpler analytical models developed by researchers require information that does not appear to be readily available such as 'θ', 'a', and 'h'. Thus the best that may be hoped for with only the knowledge of the constituent properties and overall V_f would be to obtain an estimate of the expected mechanical properties and to use these in any analysis. Of course, at some stage the accuracy of these estimated properties would need investigating, and hence a more accurate analysis may then be performed.

There would appear to be two ways of obtaining an estimate of the mechanical properties required. The first would be the laminate analogy used by some researchers, the second would be to adopt an empirical approach. The variety of different woven composites available because of different weave patterns, and the lack of experimental data available, means that the following sections will only consider plain weave composites.

9.6.1 Laminate Analogy

A comparison of the results obtained from the laminate analogy for E_x, G_{xy}, and ν_{xy} with the limited available experimental data can be seen in Appendix C.

Note that for the experimental data given by Paumelle et al (82) where there is more than one value then the first value quoted has been used for comparison in all cases, except for ν_{xy}, as the FE analysis results calculated by Paumelle et al more closely match this data. The first value given by Paumelle et al for ν_{xy} is inconsistent with the other experimental data and also the FE predictions, and thus the other experimental value given has been used in this case.

In summary it can be seen that the laminate analogy predicts E_x to within 52%, G_{xy} to within 42%, and ν_{xy} to within 89% of the experimental values. However, it should be pointed out that very little experimental data is available for the comparison. Due to the inaccuracy of the above results the use of an empirical method based on uni–directional moduli will be investigated.

The inaccuracy of the laminate analogy result suggests that the thread V_f should be used as well as the overall V_f in a two stage micromechanical analysis, ie the threads should be first treated as a uni–directional composite to determine the effective 'fibre' or thread properties, and the effect of the thread is not being taken into account by simply using the overall V_f as suggested earlier. However, as the thread V_f, although available for the experimental data used in Appendix C may not always be available, then it is felt that equations that do not rely upon this data should be used. Hence the use of the laminate analogy will not be investigated further. Also, the thickness of each ply is not known, and therefore the ply thicknesses can only be assumed to be equal.

9.6.2 Empirical Method

As woven material is similar to two sheets of uni–directional material at 90° to each other, but weaving in and out of one another, it would seem reasonable to use the uni–directional equations to determine the woven mechanical properties. However, in a woven material the two uni–directional layers are not merely laid one upon the other. The fibres actually wind in and out of one another in different weave patterns. This means that the fibres are not purely uni–directional as they do bend somewhat into the third dimension thus reducing the longitudinal properties, and increasing the through thickness properties compared to those of a uni–directional material. An empirical factor would thus be required to take account of the effect of the weave upon the directional properties.

The amount of reduction in the longitudinal properties must be related to the number of times 'n' the fibres weave around one another. As can be seen from Figure 20 a number of standard weave patterns exist where the value 'n' is known. This value 'n' may be different in the two in–plane directions, ie warp and fill, and should be related to the empirical correction factor required. As an alternative the fibre thread angle which is stated by Ishikawa et al (208,209) to lead to a reduction in moduli could be used. The value of this angle θ can be determined from the number of threads per inch and the 'n' value, or the undulation ratio h/a – see Figure 22. The value of 'a' must be related to 'n' and 'h/a' to the value of 'θ'. Thus the value of the fibre Young's modulus could be multiplied by cosθ, see Figure 25, to model the reduction in modulus due to the weave.

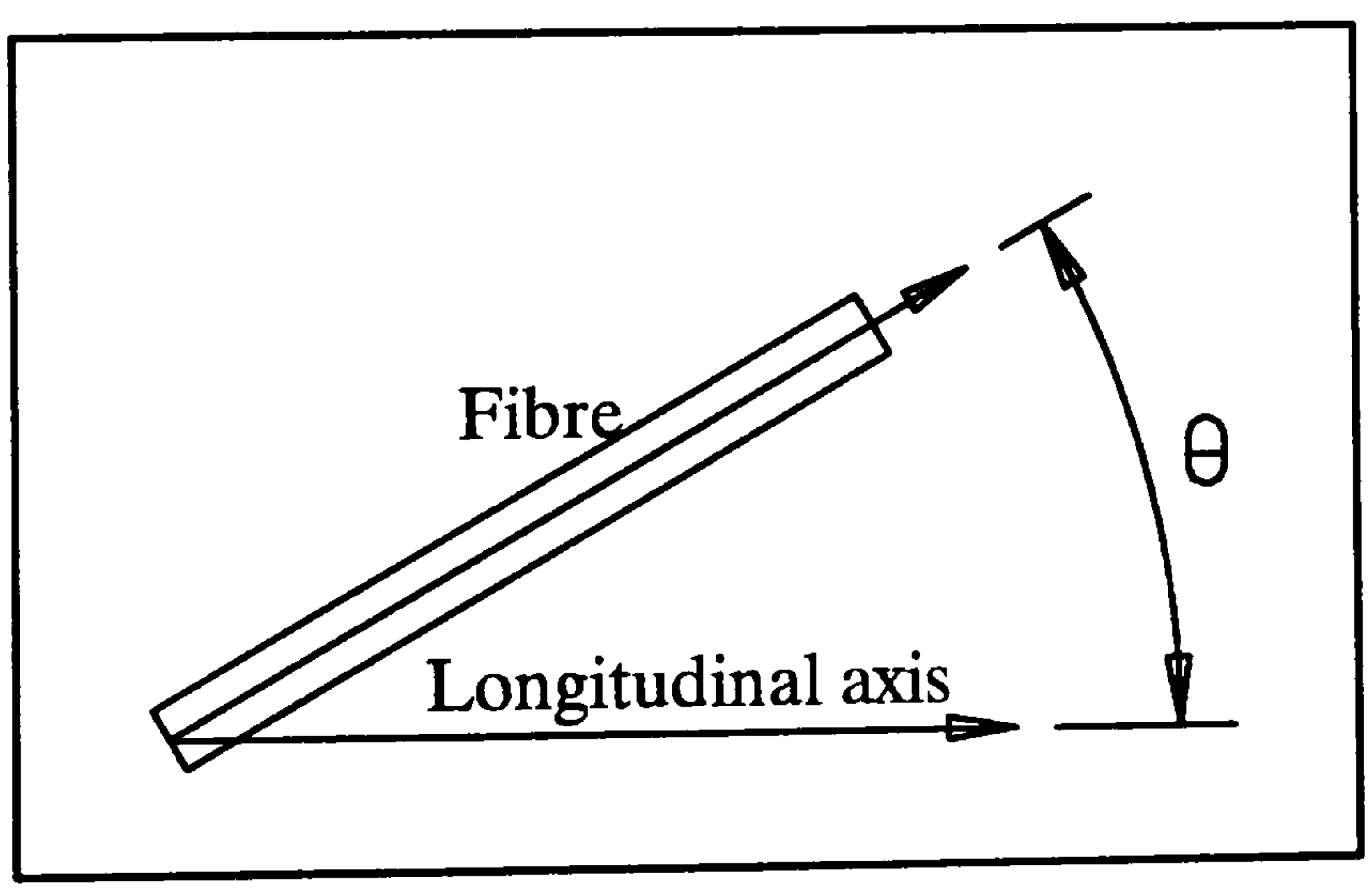


Figure 25 : Thread Angle

The information that is available from the material suppliers concerning the thread geometry does not, however, include the thread angle. It would appear that the only information that can reliably be obtained are the different fibre volume fractions and information regarding the weave pattern, ie plain weave, eighth harness weave etc. Thus the reduction factor to consider the weave must be related to the value of 'n'. The pitch of the fibres, how many per inch, may also have an effect upon the material properties as the smaller the pitch then the greater the angle between the fibres will be, but this will be related to and may be taken into account by the fibre volume fraction.

The sort of simple equation for E_x which is based on the uni–directional law of mixtures relationship would agree with the findings of Whitcomb (211) who uses finite element analysis to study the effect of fibre waviness upon the properties of a plain weave composite. Whitcomb states that the interlacing of the fibre bundles increases the out of plane strength, but at the expense of some of the in plane stiffness and strength. Whitcomb's analysis assumes that the composite is a plain weave and hence $E_x = E_y$. The analysis showed that E_x, E_z, G_{xy} and ν_{xy} decrease almost linearly with increased tow waviness in the weave pattern. Some of this decrease can be attributed to the increase in resin content as the waviness increases. However, the transverse shear modulus G_{yz} and also ν_{yz} increase essentially linearly with increased waviness. The "waviness ratio" given by Whitcomb is given as $l_1 / (l_1 + l_2)$ where l_1 is half the length of the wavy part of the thread, and l_2 is half the length of the straight part. However, the length of different parts of the thread is not the sort of information that is readily available from material suppliers. Therefore the use of an empirical factor coupled with the uni–directional relationships determined in Chapter 7 will be investigated.

Note that there is so little experimental data available that it is possible that many different relationships could fit this data, especially as the fibre volume fractions for two sets of data are so close together that the values are almost representative of the scatter for one set of data. Hence, the validity of the empirical relationships given in the following sections has not been fully tested, and therefore these equations should be treated with caution. Note that one of the sets of data used for comparison is the result of a validated FE analysis model by Paumelle et al

(217) and not experimental data, however, as so little experimental data is available it is felt that this data can be used as a check on the calculated values. Note also that whilst the empirical equations do not give the correct values at the extreme values of V_f they are acceptable for practical values of V_f.

The references by Paumelle et al (82,217) give more than one experimental value for most of the mechanical properties. For the plain weave material then the first value given has been used for comparison purposes, (except for ν_{xy} as previously stated), as the FE results calculated by Paumelle et al more closely match this data. This difference in experimental data given by Paumelle et al does highlight the variation, or scatter, possible with test data, and makes the use of empirical equations very difficult.

9.6.2.1 In Plane Young's Modulus

In a plain weave composite the in plane moduli $E_x = E_y$ (82). The value of E_x should be similar to the uni–directional E_x but reduced because of the threads weaving out of plane. Thus, if the result for E_x from the law of mixtures equation is compared with the available experimental data in Appendix C an empirical correction factor can be determined such that:–

$$E_x = \alpha\,(E_f V_f + E_m V_m).$$

The correction factor $\alpha = 0.66$. The above equation gives an average error of 7.8%.

9.6.2.2 Through Thickness Young's Modulus

It would initially appear that value of the through thickness Young's modulus E_z should be the same as the transverse Young's modulus in a uni–directional composite as they are both taken across the fibres. However, the fibres in the woven mat are not entirely two dimensional and contribute somewhat to the stiffness in the through thickness dimension, as the fibres will extend slightly out of plane when they cross one another. Thus, the result for the transverse modulus for a uni–directional composite can be increased by a correction factor when compared with the available experimental data in Appendix C such that:–

$$E_z = (E_m/\alpha)(1+\eta V_f)/(1-\eta V_f),$$

where, $\eta = \{(E_f/E_m)-1\}/\{(E_f/E_m)+1\}$.

The correction factor $\alpha = 0.66$. The above equation gives an average error of 9.2%.

9.6.2.3 In Plane Shear Modulus

It would appear from the experimental data available that the in plane shear modulus G_{xy} is increased from the value of the uni–directional modulus by the presence of the weave pattern. Thus, the equation for the uni–directional shear modulus can be increased by a correction factor when compared with the available experimental data in Appendix C such that:–

$$G_{xy} = (G_m/\alpha)(1+\xi\eta V_f)/(1-\eta V_f)$$

where, $\eta = (G_f - G_m)/(G_f + \xi G_m\}$

and, $\xi = 1 + 40V_f^{10}$

The correction factor $\alpha = 0.6$. The above equation gives an average error of 3%.

9.6.2.4 Transverse Shear Modulus

The transverse shear moduli G_{xz} and G_{yz} will be the same in a balanced composite (82). According to Smith (61) G_{xz} will be the average of the two transverse moduli G_{13} and G_{23} of the uni–directional plies. In a uni–directional material $G_{13} = G_{12}$ and thus $G_{xz} = 0.5(G_{12} + G_{23})$. The value of G_{23} for a uni–directional material was concluded to be, $G_{23} = 1.15G_{12}$ and thus $G_{xz} = 1.075G_{12}$. Thus the equation for G_{xz} is as follows and gives an average error of 4%:–

$$G_{xz} = G_{yz} = 1.075G_m(1+\xi\eta V_f)/(1-\eta V_f)$$

where, $\eta = (G_f - G_m)/(G_f + \xi G_m\}$

and, $\xi = 1 + 40V_f^{10}$

9.6.2.5 In Plane Poisson's Ratio

The in plane Poisson's ratio ν_{xy} for a uni–directional material was calculated most accurately by the law of mixtures relationship. Thus if this relationship is used here with a correction factor applied when compared with the available experimental data in Appendix C then:–

$$\nu_{xy} = \alpha(\nu_f V_f + \nu_m V_m).$$

Where the correction factor $\alpha = 0.53$. The above equation predicts the available experimental values exactly.

9.6.2.6 Transverse Poisson's Ratio

The transverse Poisson's ratio ν_{xz} and ν_{yz} will be the same in a balanced composite (82). According to Smith (61) ν_{xz} will be the average of the two transverse Poisson's ratios ν_{13} and ν_{23} of the uni–directional plies. In a uni–directional material $\nu_{13} = \nu_{12}$ and thus $\nu_{xz} = 0.5(\nu_{12} + \nu_{23})$. The value of ν_{23} for a uni–directional material was concluded to be the transverse law of mixtures relationship, and thus ν_{xz} would be the average of the longitudinal and transverse law of mixtures relationships. Therefore ν_{xz} is given by:–

$$\nu_{xz} = \nu_{yz} = 0.5\ \{(\nu_f V_f + \nu_m V_m) + \nu_f \nu_m/(\nu_f V_m + \nu_m V_f)\}.$$

The above equation gives an average error of 18%. The above equation can be used to give an average error of 7.7% if it is modified slightly when compared with the available experimental data in Appendix C such that:–

$$\nu_{xz} = \nu_{yz} = 0.66\ \{(\nu_f V_f + \nu_m V_m) + \nu_f \nu_m/(\nu_f V_m + \nu_m V_f)\}.$$

Chapter 10 : Suggested Micromechanical Equations For Continuous Fibre Composites

The following is a summary of the equations suggested in the previous three Chapters for the determination of the mechanical properties of continuous fibre reinforced composite material.

10.1 Uni–Directional Composites

As discussed in Chapter 7, apart from the longitudinal Young's modulus and Poisson's ratio which are accurately predicted by the mechanics of materials law of mixtures equations, the equations recommended for the other mechanical properties are semi–empirical ones. The results from some of these semi–empirical equations are confirmed by equations derived by other approaches.

$$E_{11} = E_f V_f + E_m V_m$$

$$E_{22}/E_m = (1+\eta V_f)/(1-\eta V_f)$$

where, $\eta = \{(E_f/E_m)-1\}/\{(E_f/E_m)+1\}$

$$E_{22} = E_{33}$$

$$\nu_{12} = \nu_f V_f + \nu_m V_m$$

$$\nu_{13} = \nu_{12}$$

$$\nu_{23} = \nu_m \nu_f / (V_f \nu_m + V_m \nu_f)$$

$$G_{12}/G_m = (1+\xi\eta V_f)/(1-\eta V_f)$$

where, $\eta = (G_f - G_m)/(G_f + \xi G_m\}$

and, $\xi = 1 + 40V_f^{10}$

$$G_{13} = G_{12}$$

$$G_{23} = 1.15\, G_{12}$$

10.2 Random Fibre Composites

The following are the mainly semi–empirical equations recommended for continuous random fibre composites.

$$E_{2D} = 3E_{11}/8 + 5E_{22}/8$$

$$E_{2T}/E_m = (1+\eta V_f)/(1-\eta V_f)$$

where, $\eta = \{(E_f/E_m)-1\}/\{(E_f/E_m)+1\}$

$$\nu_{2D} = (U_4 / U_1)$$

Where, $U_1 = (3Q_{11}+3Q_{22}+2Q_{12}+4Q_{66})/8$

$U_4 = (Q_{11}+Q_{22}+6Q_{12}-4Q_{66})/8$

and, $Q_{11} = E_{11}/(1-\nu_{12}\nu_{21})$

$Q_{22} = E_{22}/(1-\nu_{12}\nu_{21})$

$Q_{12} = \nu_{12}E_{22}/(1-\nu_{12}\nu_{21})$

$Q_{66} = G_{12}$

$\nu_{21} = \nu_{12}\, E_{22} / E_{11}$

$$\nu_{2T} = \nu_f\, V_f / (\nu_m\, V_f + \nu_f\, V_m)$$

$$G_{2D} = G_m(1+\xi\eta V_f)/(1-\eta V_f)$$

where, $\eta = (G_f - G_m)/(G_f + \xi G_m\}$

and, $\xi = 5+10^5 V_f^{10}$.

$$G_{2T} = 1.15\, G_{2D}$$

10.3 Woven Fibre Composites

The following are the semi–empirical equations recommended for woven fibre composites.

$$E_x = \alpha\, (E_f V_f + E_m V_m).$$

Where the correction factor $\alpha = 0.66$.

$$\mathbf{E_z = (E_m/\alpha)(1+\eta V_f)/(1-\eta V_f),}$$

where, $\eta = \{(E_f/E_m)-1\}/\{(E_f/E_m)+1\}.$

The correction factor $\alpha = 0.66$.

$$\mathbf{G_{xy} = (G_m/\alpha)(1+\xi\eta V_f)/(1-\eta V_f)}$$

where, $\eta = (G_f - G_m)/(G_f + \xi G_m\}$

and, $\xi = 1 + 40V_f^{10}$

The correction factor $\alpha = 0.6$.

$$\mathbf{G_{xz} = G_{yz} = 1.075G_m\,(1+\xi\eta V_f)/(1-\eta V_f)}$$

where, $\eta = (G_f - G_m)/(G_f + \xi G_m\}$

and, $\xi = 1 + 40V_f^{10}$

$$\mathbf{\nu_{xy} = \alpha(\nu_f V_f + \nu_m V_m).}$$

Where the correction factor $\alpha = 0.53$.

$$\mathbf{\nu_{xz} = \nu_{yz} = 0.66\,\{(\nu_f V_f + \nu_m V_m) + \nu_f\nu_m/(\nu_f V_m + \nu_m V_f)\}.}$$

10.4 Other Useful Equations

Some other useful equations relating the properties of the constituent materials, assuming that they are isotropic, and relating volume fraction to weight fraction are given below.

For the isotropic constituent materials:–

$$K = E\,/\,2(1-\nu-2\nu^2)$$

$$G = E\,/\,2(1+\nu)$$

To calculate volume fractions:–

$$V_f = W_f\,\varrho_c\,/\,\varrho_f$$

$$V_m = (1-W_f)\,\varrho_c\,/\,\varrho_m$$

$$\varrho_c = W_f\,\varrho_f + W_m\,\varrho_m$$

Chapter 11 : Effect Of Mechanical Property Variation

The equations given in the previous Chapter should enable the design analyst to quickly estimate the relevant mechanical properties given the fibre arrangement, the constituent properties and the expected fibre volume fraction. However, as the previous Chapters have shown, the prediction of the mechanical properties of a composite material is subject to possible error. The importance of the possible error in, or variation of, the mechanical properties thus needs investigating. Two areas need to be considered. One is the difference of the overall mechanical properties from those either expected, predicted, or advised by the material manufacturer, due for example to variations in the matrix properties. The second is the difference in discrete regions that may be caused by the manufacturing process.

To answer these questions two structural composite components in addition to plates have been analysed to study the effects of variations in the mechanical properties. The FE analysis results obtained in this Chapter have been verified by comparing them with results from experimental techniques such as stress pattern analysis by thermal emission (SPATE), photoelastic analysis and strain gauges – see references (194–196).

11.1 Automotive Subframe

The rear beam of the front subframe from a Rover 800 was re–designed to be manufactured from a composite material (219–221). The traditional steel design of this subframe consists of 27 separate fabricated parts which are welded together. The composite design of the subframe consists of two separate parts – a top half and a bottom half which are adhesively bonded together, see Figures 26 and 27.

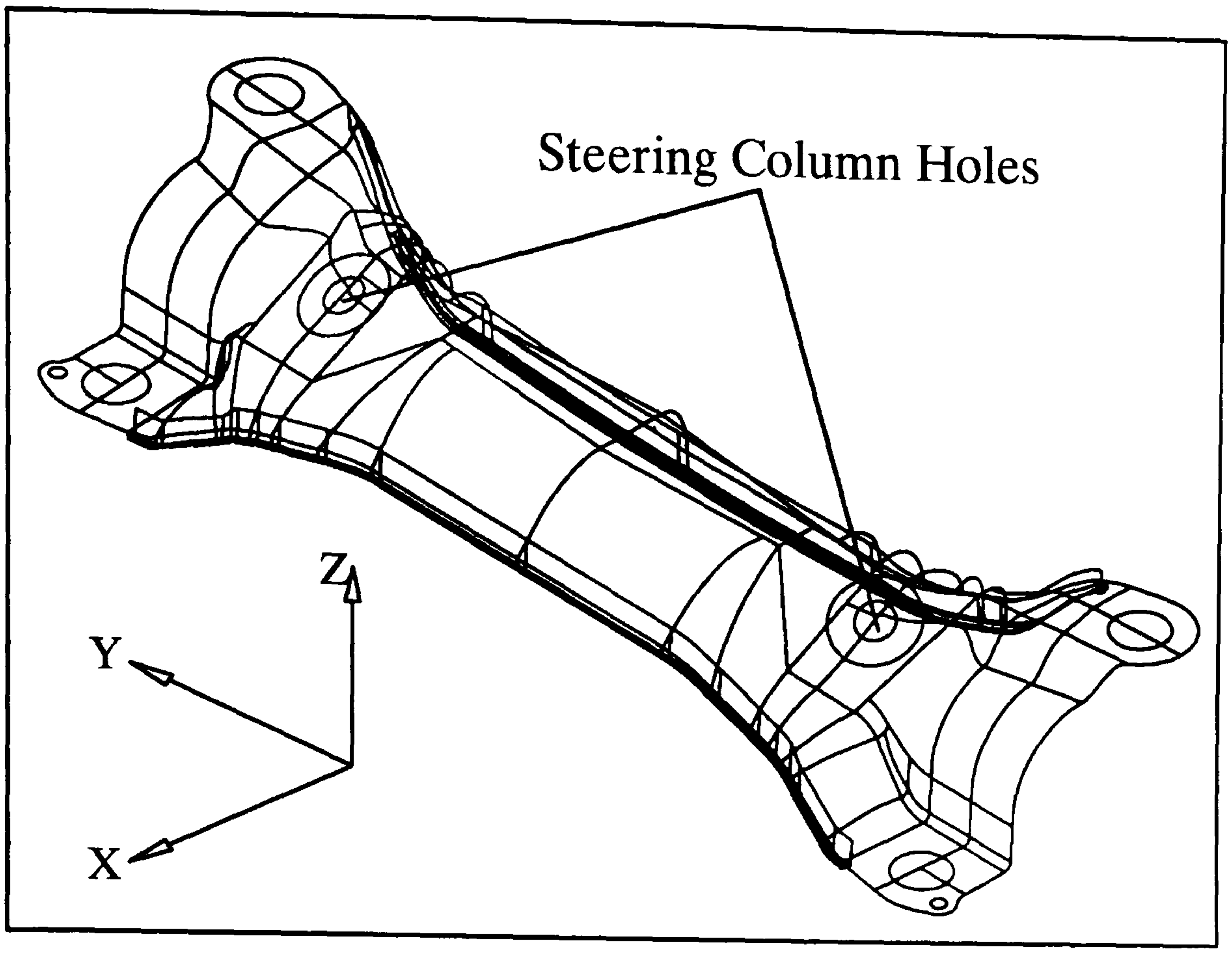


Figure 26 : Top Half Of Composite Subframe

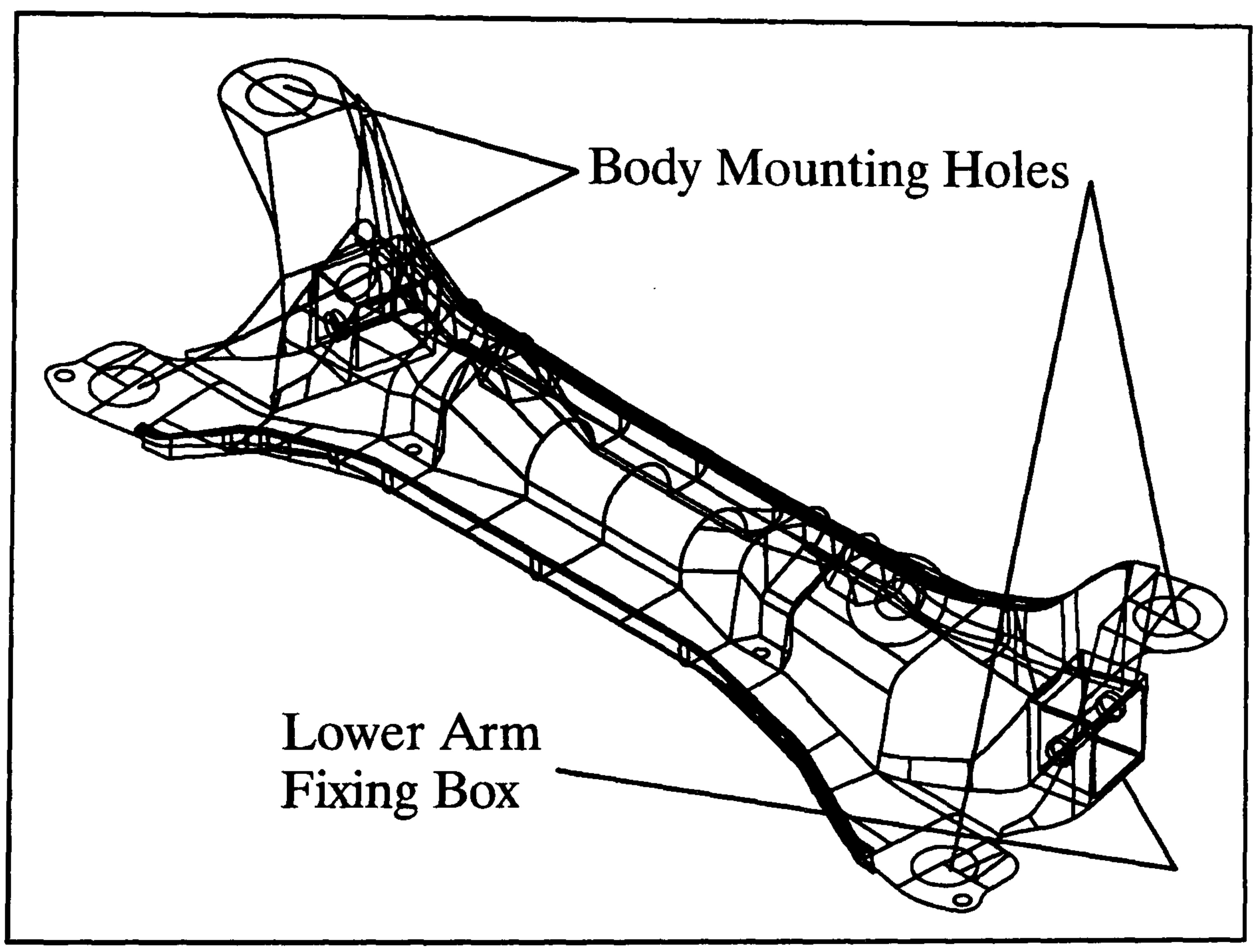


Figure 27 : Bottom Half Of Composite Subframe.

The finite element model of the subframe includes models of the steering rack, the body mounting bushes and the lower suspension arm fixings. The adhesive joint has not been modelled, and hence the subframe is assumed to consist of a continuous material over the joint area. The subframe consisted of a constant thickness of 5mm. The initial FE model of the subframe modelled in ANSYS 4.4A consisted of approximately 2500 stif91 thin shell elements plus beam elements to model a representation of the steering rack and lower suspension arm fixings, and spring elements to model the rack bushes and also the body mounting bushes.

FE analyses of the subframe were undertaken for a number of different loading and mounting conditions. The loading conditions for the subframe were obtained from the Rover Group – see references listed below for details. The material used was a continuous random fibre glass filled epoxy resin manufactured using the RTM technique for both the top and bottom halves. The component was intended to have 50% glass by weight. The mechanical properties of the composite were determined experimentally from a sample of the material to be:– $E_{11} = E_{22} = 11.05$GPa., $G = 2.87$GPa., $\nu = 0.3342$, density $= 1.38\text{x}10^{-6}$Kg/mm^3.

The details and results of the analyses undertaken on the automotive subframe described here have been published by the author in a number papers (196,219, 220) and internal reports (221–226).

11.2 Automotive Suspension Arm

A component manufactured from sheet moulding compound (SMC) has also been studied to determine the effect of processing upon the mechanical properties. This component is the suspension arm or wishbone from a Rover Metro. The existing steel lower suspension arm consists of nine pieces welded together whilst the re–designed composite component – which can be seen in Figure 28 – consists of a single moulded part. The material used to manufacture the suspension arm was a sheet moulding compound consisting of a calcium carbonate filler, a polyester resin bonding agent and a glass content of 30% by weight of randomly arranged fibres of a length of approximately 25mm. The mechanical properties used for the composite suspension arm in these analyses were obtained experimentally from a portion cut out of the centre of the suspension arm and were:– E = 10.5GPa., ν = 0.26, density = 1.8×10^{-6} Kg/mm^3.

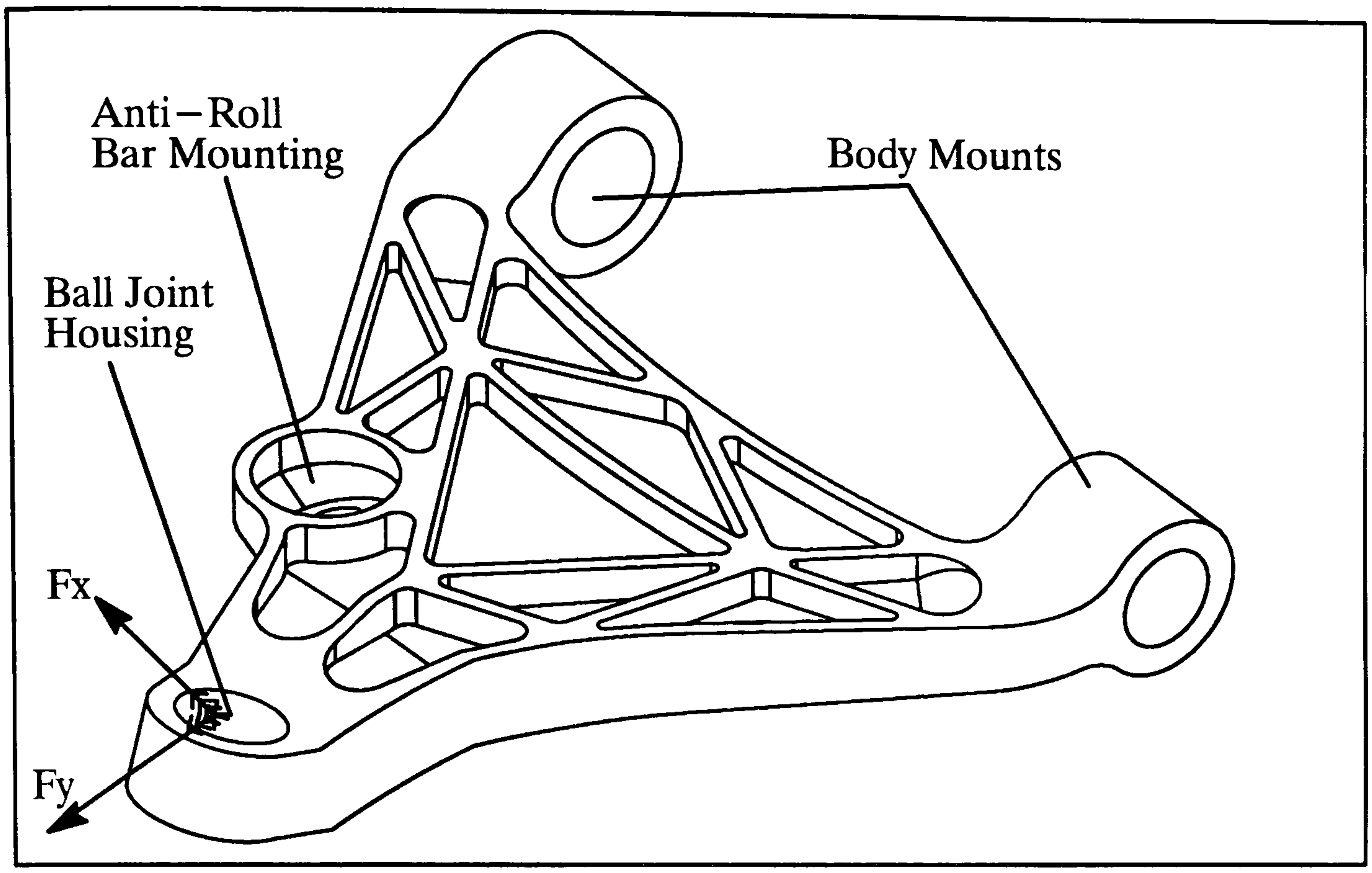


Figure 28 : The Composite Suspension Arm

The suspension arm was modelled in ANSYS 4.4A using 1267 stif45 brick elements. The body mounting bushes were modelled with spring elements. The front fixing was modelled with beam elements and the loads applied through these. The constraint and loading conditions were obtained from the Rover Group – see references listed below for details. The analyses initially assumed that the mechanical properties were constant throughout and that the material was isotropic.

The details and results of the analyses undertaken on the automotive suspension arm described here have been published by the author in a number papers (194,195) and internal reports (222,227,228).

11.3 Overall Variation In Properties

Dr. Eckold in the introduction to the Design Data Initiative programme (52) states that components manufactured "from composite materials experience much greater variability in performance than those manufactured from metals because the knowledge of material data, manufacturing effects and material behaviour is in general less than for metals and the analysis methods are more complex....The accuracy of any design method will be determined to a great extent by the quality of the data used as input, and there is little merit in fine tuning the rigour of an analysis if properties of the basic materials of construction are subject to large variability".

In Appendix A where the constituent properties are listed it can be seen that the variation in the properties of E–glass is very small. However, the properties of what is nominally called epoxy resin do vary between manufacturers. If the extreme values for E_m, ν_m and G_m are put into the suggested equations for uni–directional, random and woven mechanical properties then the graphs referred to in the following sections and shown in Appendix D can be drawn. These graphs effectively form the bounds within which the properties of an E–glass epoxy composite will lie. The variation found in the literature for E_m is 3.0–4.17GPa. (ie an increase of 39%), for ν_m is 0.3–0.447 (ie, an increase of 49%), and this equates to a variation in G_m from 1.15–1.44GPa. (ie, an increase of 25%), by using equation *[4]*. Note that this variation in resin properties also approximately covers the variation seen in the polyester resin properties. Thus, the graphs produced can also be used to determine E–glass / polyester composite properties.

11.3.1 Uni–Directional Material

The variation in the uni–directional mechanical properties with variation in resin properties can be seen in Figures 49 to 54 in Appendix D. The result for E_{11} shows that there is no significant variation as E_m changes between the two extremes highlighting that this property is fibre dominated. The variation in E_{22} for the two matrix extremes is seen to be an increase of 32% at a representative V_f of 0.6 as this property is clearly more matrix dominated. At the same value of V_f the difference in the matrix properties gives an increase in ν_{12} of 23% and ν_{23} of 12%. The figures for G_{12} and G_{23} are obviously similar as the recommended equation for G_{23} is G_{23}

= $1.05G_{12}$. The difference in G_{12} is seen to be 21% at V_f = 0.6 for the given increase in G_m..

As can be seen some properties are clearly more matrix dominated than others, and the Figures produced in Appendix D can be used to determine the uni–directional composite mechanical properties.

11.3.2 Random Fibre Material

The variation in the mechanical properties for a continuous random fibre reinforced composite with a variation in the matrix properties can be seen in Figures 55 to 57 in Appendix D. Note that only the in plane properties have been plotted as the transverse properties have previously been concluded to be the same as those transverse in a uni–directional composite. At a representative V_f of 0.3 it can be seen that E_{2D} shows an increase of 12%, G_{2D} an increase of 20% and ν_{2D} an increase of 14% as the matrix properties are increased.

11.3.3 Woven Fibre Material

The variation in the woven mechanical properties with a variation in the matrix properties can be seen in Figures 58 to 63 in Appendix D. At a representative V_f of 0.5, the variation in E_x is minimal, however E_z varies by 33% showing that this property is more matrix dominated. The two shear moduli G_{xy} and G_{xz} vary by 22% as the matrix properties vary. The two Poisson's ratios ν_{xy} and ν_{xz} vary by 27% and 21% respectively as the matrix properties vary.

11.3.4 Effect Of Matrix Variation In Analysis

In order to understand whether the difference in the mechanical properties between the two extreme values of the matrix properties is significant, the extreme values have been used in the analysis of the composite subframe for a typical load case of pot hole braking. In this load case, the loads are applied via beam elements to the lower arm fixings and are the 'pot hole brake loads' (196,219–226). This results in a quite complex load path in the subframe. The two extreme sets of mechanical property values for uni–directional, random and woven material have been taken from the previously presented Figures at representative values of V_f. The mechanical properties used can be seen in Table 1 and the results of the analyses in terms of the maximum Von Mises stress and maximum deflection can be seen in Table 2.

	Uni-Directional Material At Vf = 0.6				
	Low Matrix Props.		High Matrix Props.		Difference
E11 (Gpa)	45		45		0%
E22 (GPa)	10.4		13.7		+32%
Poissons Ratio	0.252		0.31		+23%
G12 (GPa)	4.3		5.2		+21%
	Random Material At Vf= 0.3				
	Low Matrix Props.		High Matrix Props.		Difference
E2D (Gpa)	12.3		13.8		+12%
Poissons Ratio	0.304		0.348		+14%
G2D (GPa)	3.5		4.2		+20%
	Woven Material At Vf = 0.5				
	Low Matrix Props.		High Matrix Props.		Difference
Exy (Gpa)	25.1		25.5		+1%
Poissons Ratio	0.138		0.176		+27%
Gxy (GPa)	5.28		6.45		+22%

Table 1 : Mechanical Properties Used In The Analysis.

		Low Matrix Props.		High Matrix Props.		Difference
Uni-Directional	Von Mises Stress (MPa.)	297.2		268.2		-9.80%
	Max. Deflection (mm)	-5.66		-5.54		-2.1
Random	Von Mises Stress (MPa.)	198.1		197.6		-0.20%
	Max. Deflection (mm)	-6.31		-6.12		-3%
Woven	Von Mises Stress (MPa.)	218.7		214.4		-2%
	Max. Deflection (mm)	-5.64		-5.56		-1.40%

Table 2 : Variation Of Stress And Deflection In The Composite Subframe With Varying Matrix Properties.

It can be seen from Table 1 that the difference in the mechanical properties between the two extremes is in some cases relatively large. The corresponding difference that this change of properties causes in the analysis results seen in Table 2 is, in all except one case, very small. The change in the two sets of mechanical properties for a subframe modelled from uni-directional material is seen to make nearly a 10% difference in the maximum Von Mises stress. However, as discussed in Chapter 5 a variation in the results of 10% is deemed to be acceptable especially as the majority of engineering designs are not engineered to such tight tolerances.

11.4 Variation In Discrete Regions

The finite element analyst will often assume that unless the composite component has specifically been designed to have different mechanical properties in different areas then the properties will be constant throughout the component. This makes the composite much easier to model as different properties do not have to be attached to different groups of elements. This assumption of constant properties may cause inaccuracies in the case of a glass preform being formed into shape as the continuous fibres in the glass mat may be stretched, aligned or moved apart in different areas according to the shape that is being formed. Also, the processing of a compression moulded material may result in the material becoming directional in any ribs for example (229,230). This means that in different areas of the component there may be different properties. In order to answer the question of the importance to the analysis results of variations in the mechanical properties caused by processing, a number of FE analyses have been undertaken on plates and components. The results of these analyses are presented here.

11.4.1 Resin Transfer Moulding (RTM)

In a resin transfer moulding (RTM) the glass mat is first heated and pressed into a mould to create a preform. Resin is then injected or transferred into the mould to create the composite component.

11.4.1.1 Automotive Subframe

The composite automotive subframe discussed previously has not been able to be cut up and examined in order to study the resultant fibre orientation and concentration after manufacturing. This is due to there being only a limited number of subframes manufactured which were required for tests. However, a visual examination suggested that the processing did not have a significant effect upon the resultant fibre orientation.

11.4.2 Compression Moulding

Composite components may be manufactured from compression moulding, such as an SMC (sheet moulding compound) or GMT (glass mat thermoplastic). In this process heated blanks or charges of the material are placed in the pre–heated mould. The mould is then closed which forces the material to flow over the surface of the mould to form the component. An SMC material usually consists of polyester resin reinforced with randomly oriented chopped glass fibres of approx. 25mm in length, together with a percentage of filler such as calcium carbonate and a small amount of thickening and curing agents (231,232). For the material of interest in this work the manufacturers of the glass mat state that the average fibre length is 25mm whilst the fibre diameter of E–glass is 7 microns (233). This gives a fibre length to diameter (aspect) ratio of 3570. Thus, due to this fibre aspect ratio, the discontinuous random fibre reinforced material used in SMC will be assumed to behave as a continuous random fibre reinforced material – see Chapter 2. GMT usually consists of continuous random fibre reinforced polypropylene. During the compression moulding process the fibres will tend to flow with the matrix when the charge material is compressed. This suggests that a variation in the mechanical properties will be found throughout a compression moulded component.

The following sections will investigate whether detailed knowledge of the variation of the mechanical properties is necessary.

11.4.2.1 Automotive Suspension Arm

To investigate the variation caused by the compression moulding process the automotive suspension arm considered earlier has been studied. X–rays taken of the component appear to show significant fibre orientation in some areas, and a variation in fibre distribution. Examples of the X–rays can be seen in the following Figures.

The X–rays appear to show significant fibre alignment as the initially random fibre SMC material flows from the centrally placed charge position into the body mount areas and the ball joint housing area. Fibre alignment can also be observed in the majority of ribs – alignment in the direction of the ribs. In the other areas the SMC has a random fibre orientation. However, even in these areas there does not appear to be a consistent concentration of fibres. Whilst the X–rays are useful for studying the orientation of the fibres in the finished component they cannot give quantitative information concerning fibre concentration, ie fibre volume fraction.

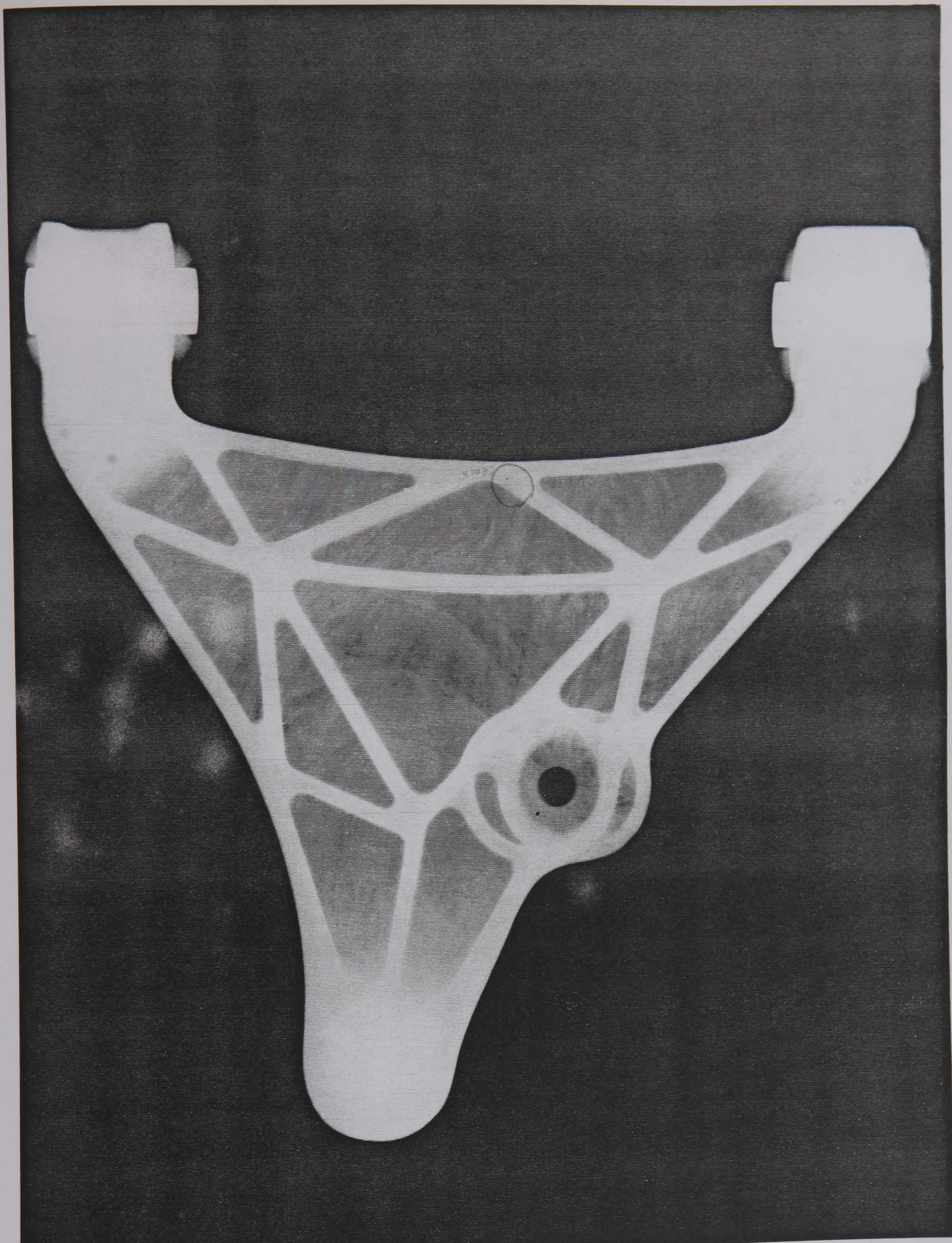


Figure 29 : X–Ray Of Composite Suspension Arm.

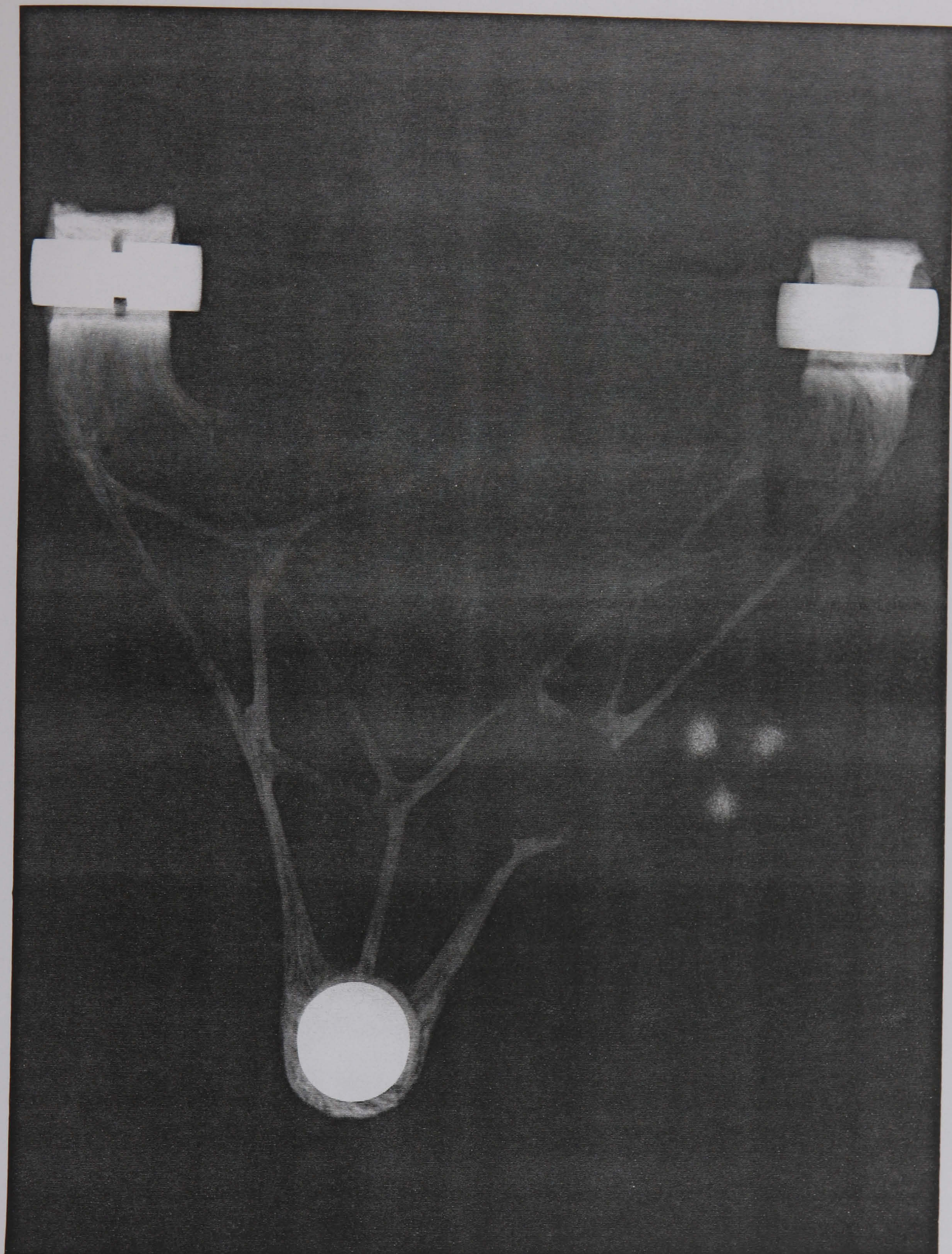
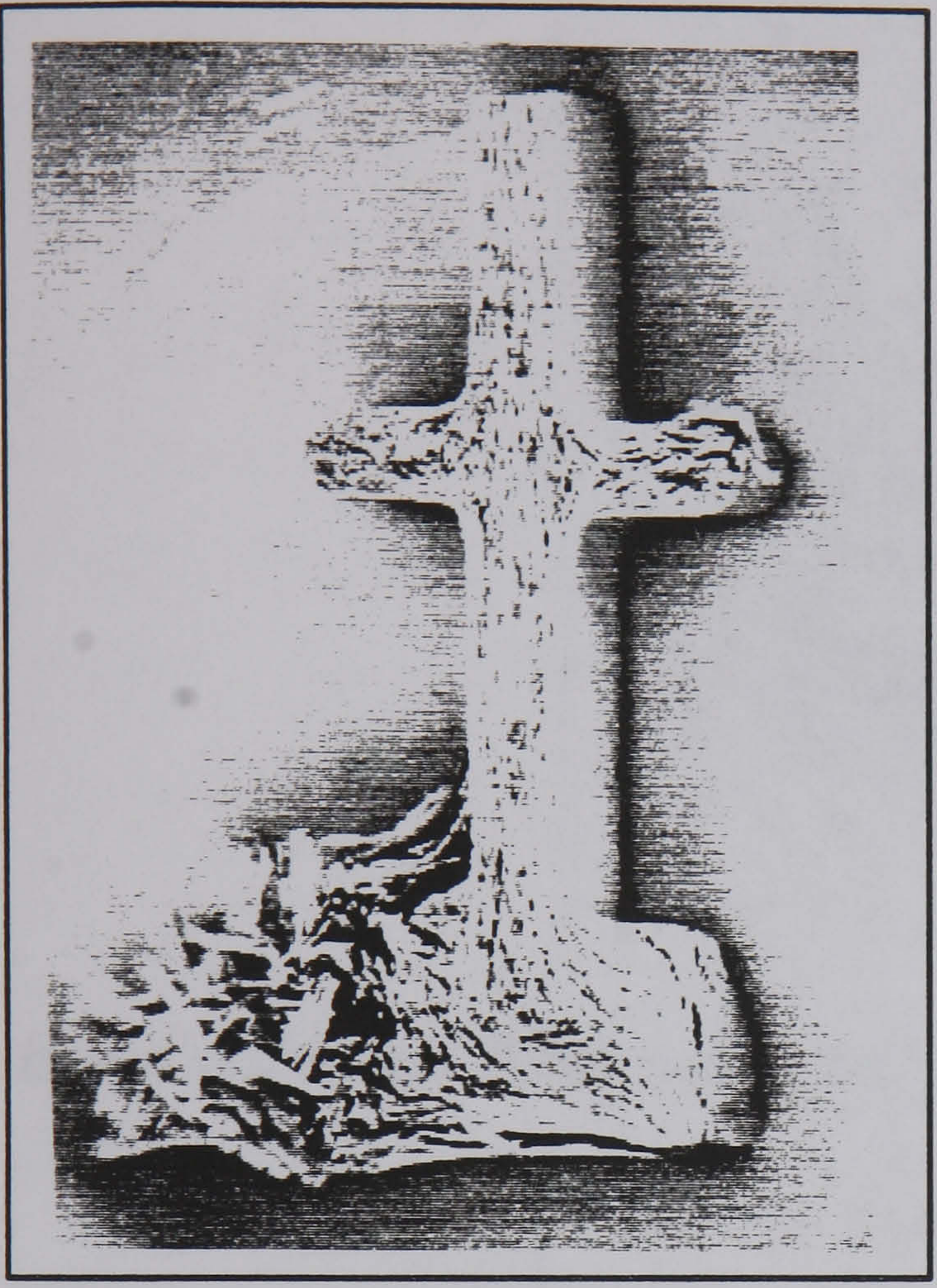


Figure 30 : X–Ray Of Composite Suspension Arm.

The X-ray results appear to be confirmed from sections cut from the suspension arm that have been subjected to resin burnoff – see Figures 31 to 33 (230).



Figures 31 and 32 : Sections Of Suspension Arm (230).

Resin burnoff also showed that the volume percentage of glass is reasonably constant at 19% to 24.6% with an average value of 21% (230). Sections taken through the thickness of the suspension arm show that the fibres are in the 1–2 plane in the pocket areas and hence the material is not isotropic in these areas. In the ribs the material is folded up to form the ribs and the ribs appear to be well filled with glass. Thus the areas around the ribs tend to be more uni-directional.

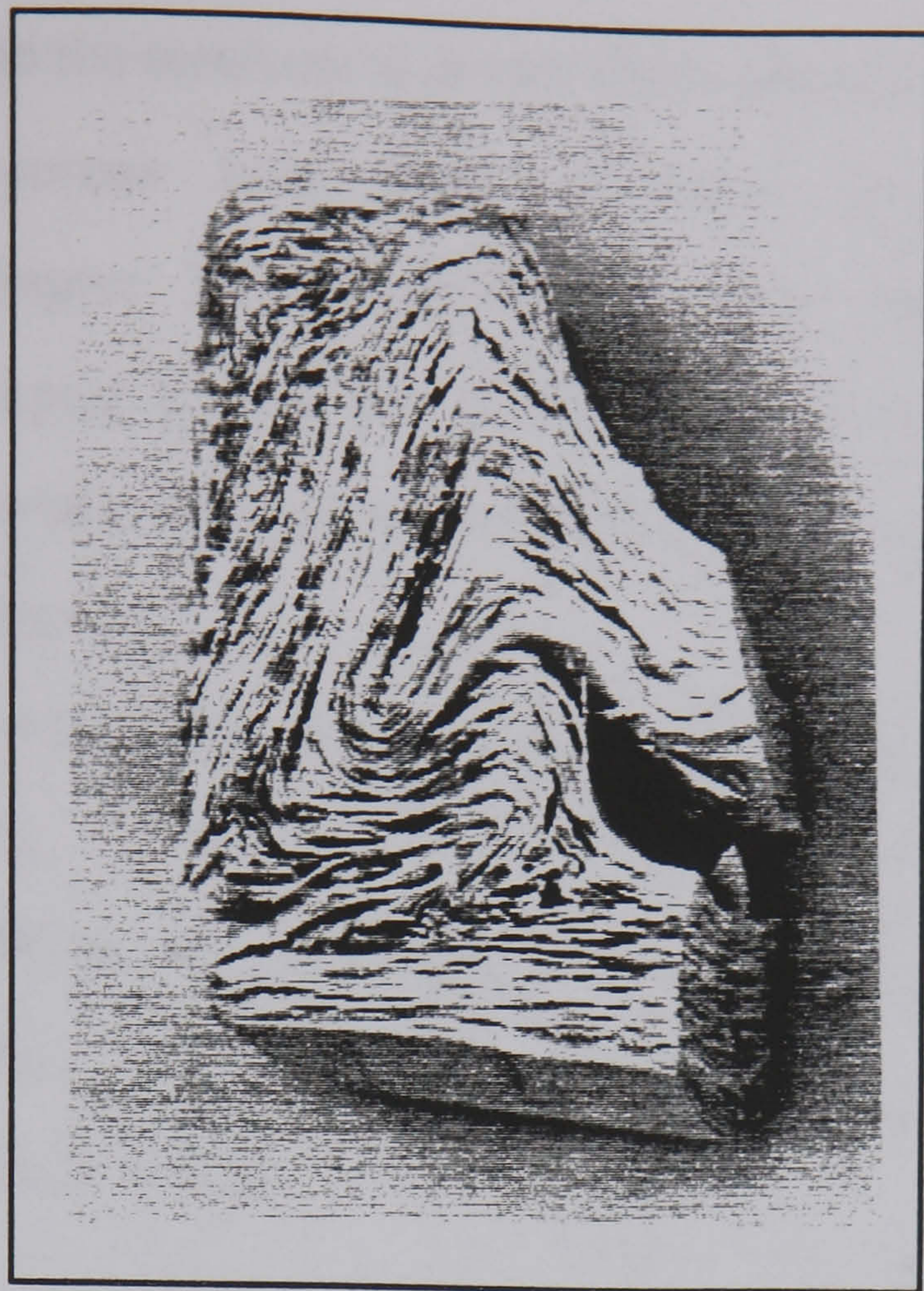


Figure 33 : Section Of Suspension Arm (230).

The matrix material for SMC consists not just of a resin but also various fillers such as calcium carbonate in various quantities. Therefore to determine the mechanical properties of the matrix by using micromechanics is not easy as the V_f and properties of each of the filler materials needs to be known, and these are not known in enough detail for the SMC used for the suspension arm. If these properties and quantities are known then various micromechanical equations would appear to be available to determine the matrix properties, ie the properties of a statistically isotropic composite (7,8,13,42,74,108,110,234,235). Once these matrix properties are known then the equations recommended in Chapter 10 can be used to determine the overall properties of the composite component.

The mechanical properties of the composite used in the suspension arm can be determined by using a resin / calcium carbonate modulus of 6GPa. (236,237) and using this as the resin modulus in the micromechanical equations. It has been assumed that the mechanical properties within the ribs and pockets would be constant and consist of the average V_f ie $V_f = 0.21$. This average V_f has therefore been used to determine the mechanical properties required for the analyses. The uni–directional properties have been calculated from the equations recommended in Chapter 7, the random properties from the equations recommended in Chapter 8, and the isotropic properties have been those determined experimentally and given previously. The properties used in the analyses were as follows:–

Uni–directional: E_{11} = 20.1GPa., E_{22} = 8.6GPa., ν_{12} = 0.282, ν_{23} = 0.279, G_{12} = 3.3GPa., G_{23} = 3.8GPa.

Random : E_{2D} = 12.9GPa., E_{2T} = 8.6GPa., ν_{2D} = 0.28, ν_{2T} = 0.279, G_{2D} = 4.5GPa., G_{2T} = 5.2GPa.

Isotropic : E = 10.5GPa., ν = 0.26.

Finite element analyses have been undertaken for three cases all of which assumed that V_f was constant throughout, however, the fibre direction in each region have been taken as those shown in the X–rays. The analyses have been undertaken for the typical load case of pot hole braking – see (194,195,222,227,228). The first case assumed that the material was uni–directional in the ribs, with the fibres aligned in the direction of the ribs. In the rest of the suspension arm the fibres are assumed to be randomly arranged. The second analysis assumed that the ribs and the pockets all had fibres randomly arranged in the relative 1–2 plane. The third analysis assumed all areas to have isotropic properties. The results for the maximum Von Mises stress have been published in references (238,239) and can be seen in Figures 34 to 36.

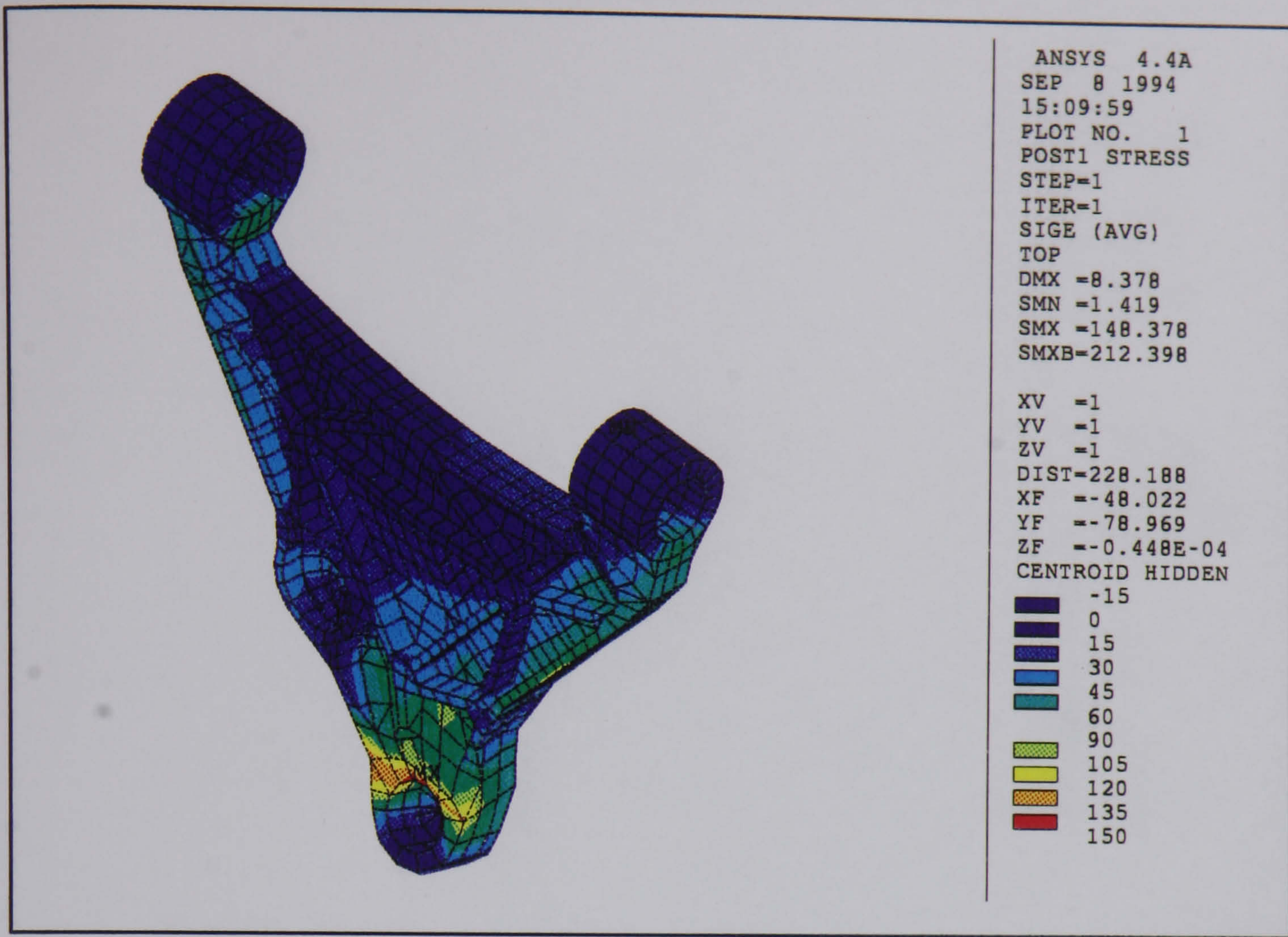


Figure 34 : Maximum Stress Assuming UD Ribs.

Figure 35 : Maximum Stress Assuming In Plane Random Properties.

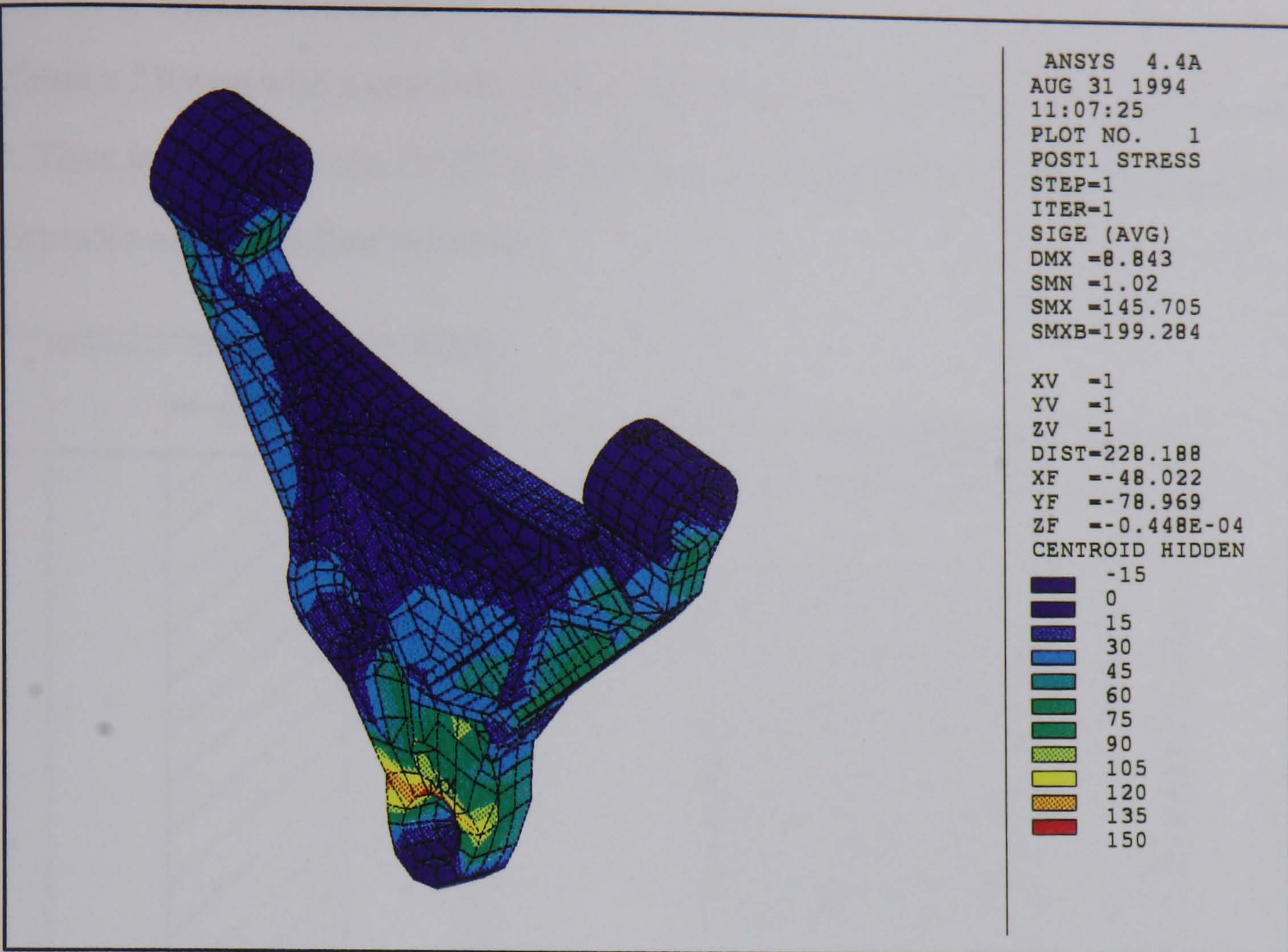
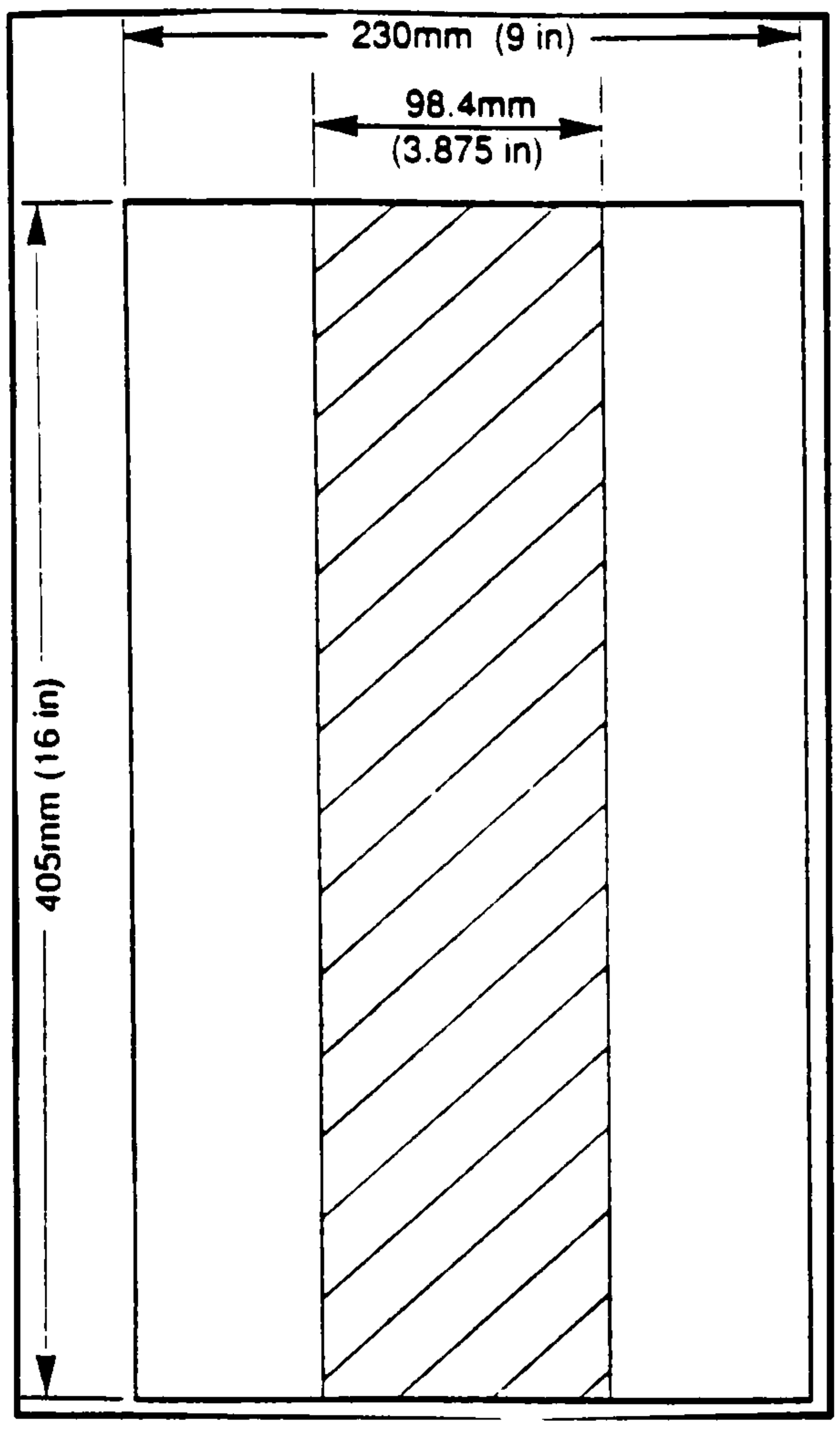


Figure 36 : Maximum Stress Assuming Isotropic Properties.

As can be seen from the above results there is no appreciable difference in either the magnitude or distribution of the maximum stress for the different cases. The maximum value of stress is given by the analysis assuming random properties as 150.2MPa., with the lowest value given by the analysis assuming isotropic properties as 145.7MPa. The difference is only 3%. Similarly, the difference between the maximum deflections observed in the analyses is 8.84mm with isotropic properties and 8.37mm with uni–directional ribs, giving a difference of only 5%. This confirms that the initial analysis assumption of an isotropic material is a valid one in this instance. Upon investigation of the manufacturing process it was found that the initial charge of material covered the majority of the mould area.

11.4.2.2 GMT Plate

An example of the variation experienced in a compression moulded part can be seen in the experimental work undertaken by Stokes (240,241). The GMT plate studied by Stokes consisted of 40% glass by weight. The GMT plate measured 405mm x 230mm with a centrally placed charge of 405mm x 98.4mm – see Figure 37. Thus in this plate the GMT undergoes a one dimensional flow, and the glass orientates along the flow direction.



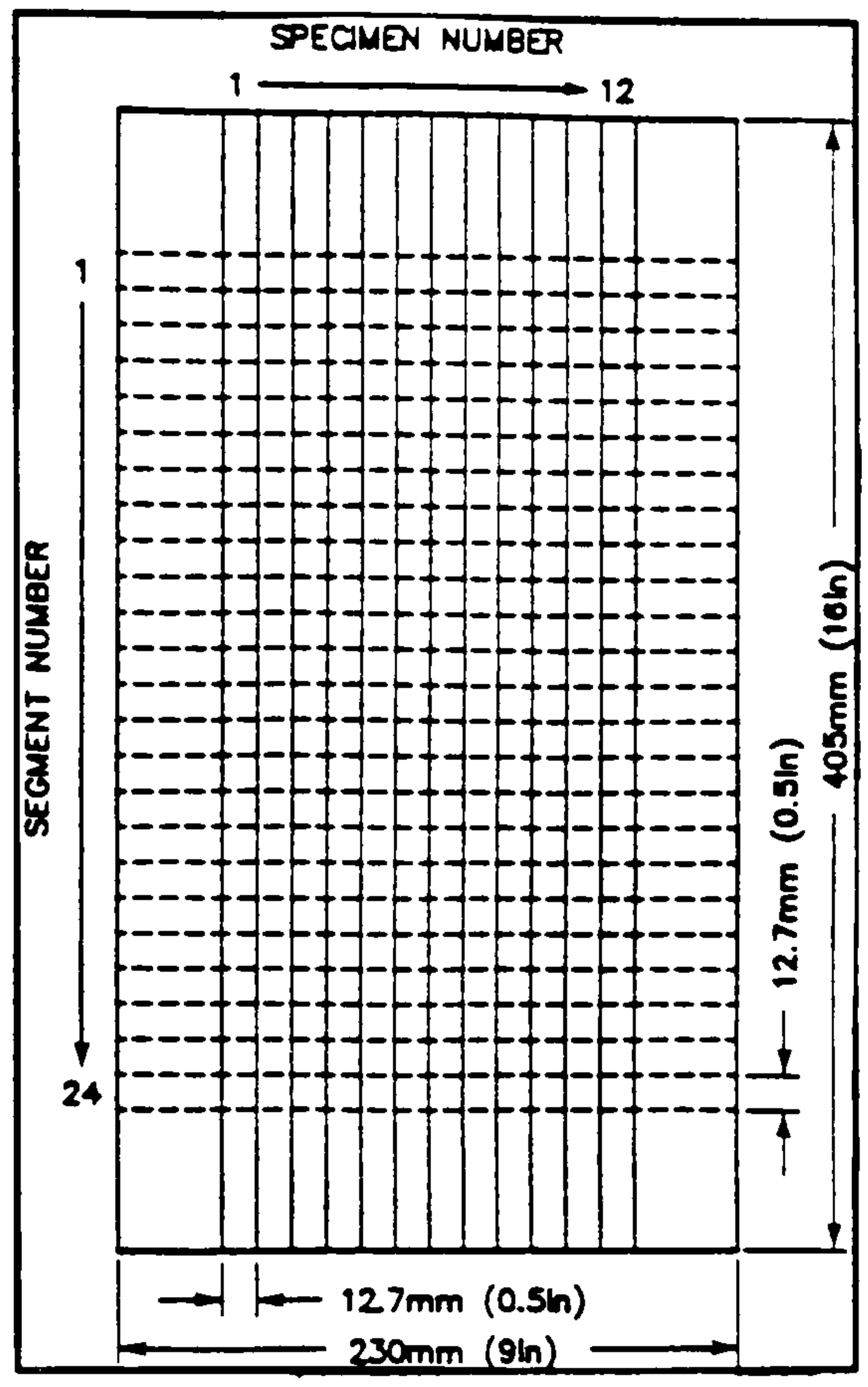


Figure 37 : Dimensions And Charge Position Of GMT Plate (240).

Figure 38 : Measurement Positions (240).

In the centre portion (150mm x 305mm) of this GMT plate Stokes made measurements of the Young's modulus in 288 12.7mm squares – see Figure 38. The measurements have been made along the left hand edge, right hand edge and on the face of each of these 288 regions. The average of the left and right moduli was found to be a good approximation to the measured face modulus. Stokes presents tables showing the modulus in each of the 288 regions. Stokes has found that over the specimen length of 12.7mm the average Young's modulus of the left hand and right hand edges, E_A, varied by a factor of over 2:1. The density was also found to vary. Stokes found that the average modulus E_A correlated well with the density and thus was an appropriate measure of the local tensile modulus.

As a result of the above findings Stokes asks some interesting questions such as : If the modulus varies by a factor of over 2:1 over a length of 12.7mm what value is measured in tests ? How repeatable is this data ? How can standard properties be defined for this material ? How can these properties be used in design ? These questions do not appear to be answered in the papers. Stokes does not discuss at all the effect or importance of having different moduli in different regions, but only discusses how to determine an overall average modulus value. Thus the major points raised by these papers have not been answered, ie what effect upon the components performance do such variations in the Young's modulus have? The study by Stokes whilst informative and valuable does leave questions unanswered. For example, how do the other mechanical properties vary over the plate? Are the properties in the regions anisotropic rather than isotropic? How does the strength vary? Stokes has thus studied only part of the problem. From the work of Stokes it can be seen that test pieces machined from different areas of a plate to experimentally determine modulus values may give different values depending where in the plate they were machined from. Stokes states that GMT can be looked upon as a nonhomogeneous material that is made up of a number of small discrete regions each having its own material properties.

The GMT plate moulded and tested by Stokes has been analysed with 288 four noded stif63 thin shell elements in ANSYS 5.0, and the results published in reference (242). The analysis modelled the average tensile modulus E_A determined by Stokes for each of the elements – see Table 3 of Stokes paper (240) reproduced here as Table 3. The variation in the modulus is shown in Figure 39 as a contour plot. The Figure shows that there is a potentially significant variation in the modulus values throughout the plate and suggests that the plate cannot be assumed to have a constant value of modulus.

Segment Number	Tensile Modulus GPa. Specimen Number											
	1	2	3	4	5	6	7	8	9	10	11	12
1	3.98	4.13	4.79	5.33	4.06	5.16	4.34	4.52	3.86	4.07	3.46	4.64
2	4.08	4.22	5.28	4.58	4.71	4.68	4.08	3.67	4.01	4.11	4.06	4.37
3	3.43	3.98	5.4	4.31	5.25	4.2	4.07	4.5	4.37	3.71	3.56	4.32
4	3.11	3.51	5.07	4.78	4.64	4.55	4.22	5.09	4.84	4	3.63	4.44
5	3.37	3.84	5.24	5.52	4.45	4.67	4.67	4.95	4.53	4.34	4.02	3.76
6	3.1	3.98	5.64	5.87	4.01	4.83	4.61	4.16	4.36	4.44	4.22	4.07
7	2.78	4.48	5.12	5.86	5.46	5.08	4.89	4.51	4.62	3.92	3.87	4.25
8	2.81	3.49	5.39	5.07	4.8	4.89	4.42	4.43	4.58	3.87	3.79	4.21
9	3.29	4.2	5.68	5.44	4.74	4.46	3.48	4.47	4.41	4.27	3.87	4.19
10	2.45	4.5	5.68	5.24	4.44	4.57	4.76	4.18	4.23	3.95	3.81	4.32
11	3.56	5.38	4.45	4.79	5.04	4.68	4.91	4.1	3.97	4.07	3.52	4.41
12	3.58	4.59	4.54	4.7	4.98	4.96	4.47	4.3	4.05	3.9	3.29	4.25
13	3.32	4.84	4.72	4.39	4.57	4.52	4.79	4.49	4.35	3.96	3.54	4.03
14	3.02	5.08	4.86	3.91	4.07	4.52	4.2	3.95	4.4	3.98	3.87	3.84
15	3.57	5.45	4.47	3.91	4.65	4.79	4.32	4.1	4.45	3.84	3.78	3.51
16	3.46	5.82	4.61	3.8	4.95	4.49	4.66	4.36	4.09	4.03	3.75	3.93
17	3.6	5.26	4.98	4.33	4.23	4.54	4.6	4.27	3.81	4.31	3.5	3.32
18	3.51	4.17	4.81	4.54	4.17	4.71	4.48	4.35	4.06	4.14	3.49	3.47
19	3.56	4	4.59	4.24	5.29	5.21	5.12	5.14	3.9	4.36	3.21	3.32
20	4.17	3.81	4.3	4.81	4.29	4.71	3.93	5	3.88	4.04	3.16	3.08
21	3.55	3.88	4.35	4.47	5.39	4.74	5.05	5.28	4.49	3.59	3.1	2.99
22	3.51	4.63	4.53	4.71	4.86	4.78	4.86	4.4	4.82	3.98	3.12	3.47
23	4.39	4.22	5.18	4.88	5.35	4.95	4.78	4.21	3.97	5.05	3.29	4.48
24	4.22	4.58	4.99	5.28	5.3	4.74	5.17	5.34	4.37	5.03	3.8	4.49

Table 3 Average Measured Modulus Values (240).

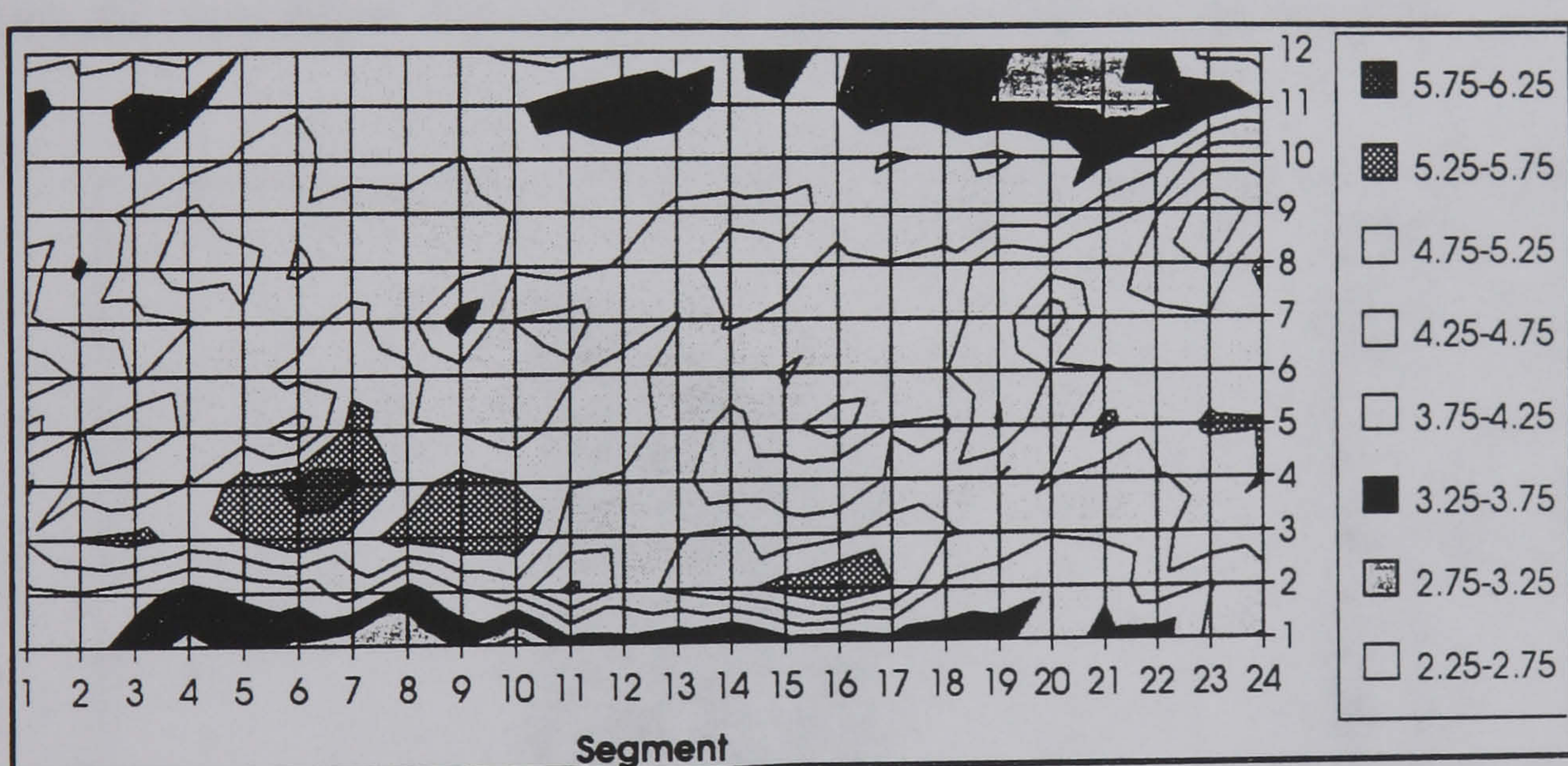


Figure 39 : Contour Plot Of The Variation In Modulus.

For the analysis the material thickness was 3.18mm and each element was assumed to have isotropic properties (a Poisson's ratio of 0.3 was assumed). The analysis assumed that the plate was rigidly fixed at one end and a nominal tensile load of 36KN was applied to the other end. The resultant plot of the Von Mises stress can be seen in Figure 40.

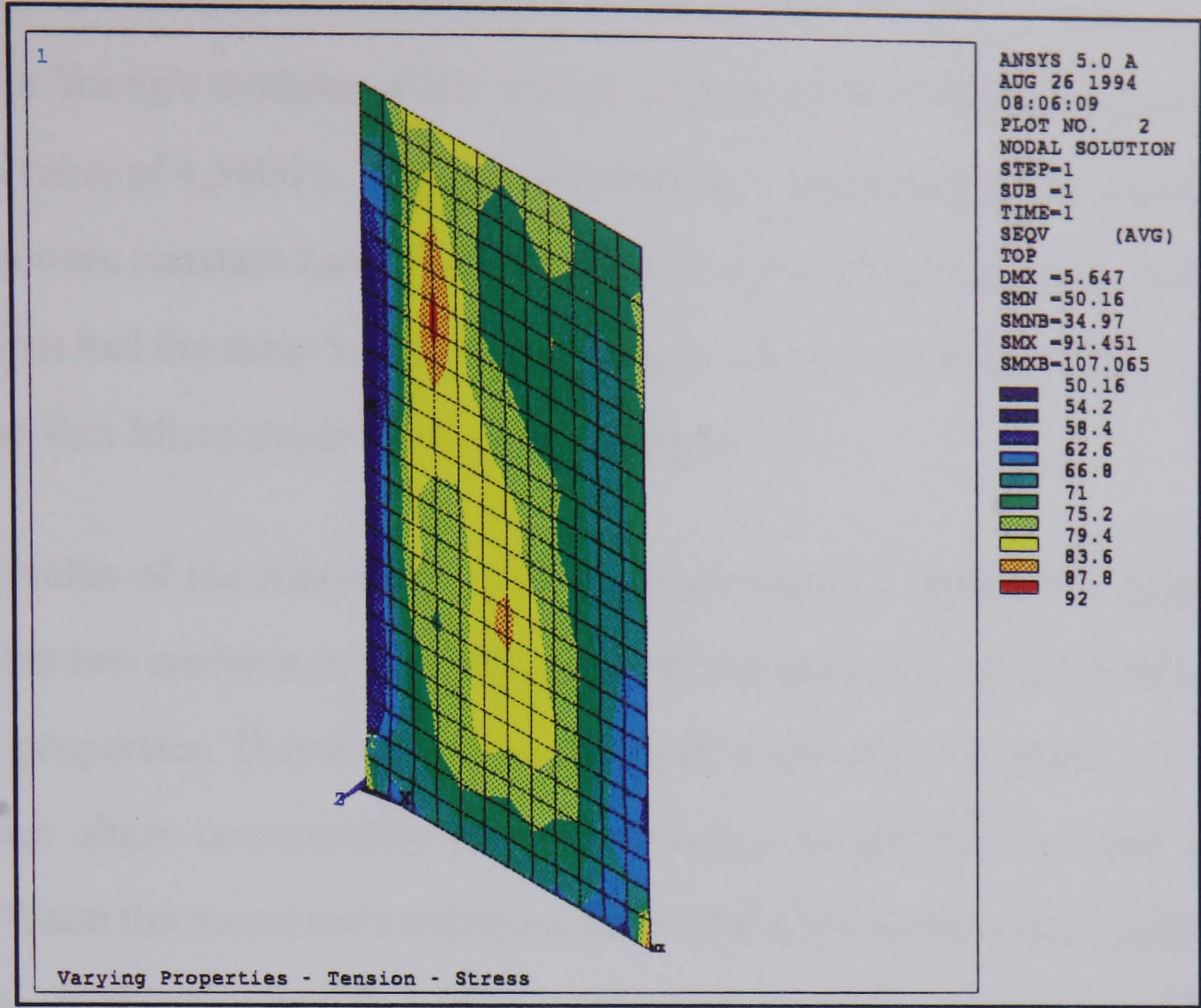


Figure 40 : Von Mises Stress In Plate – Variable Properties – Tension.

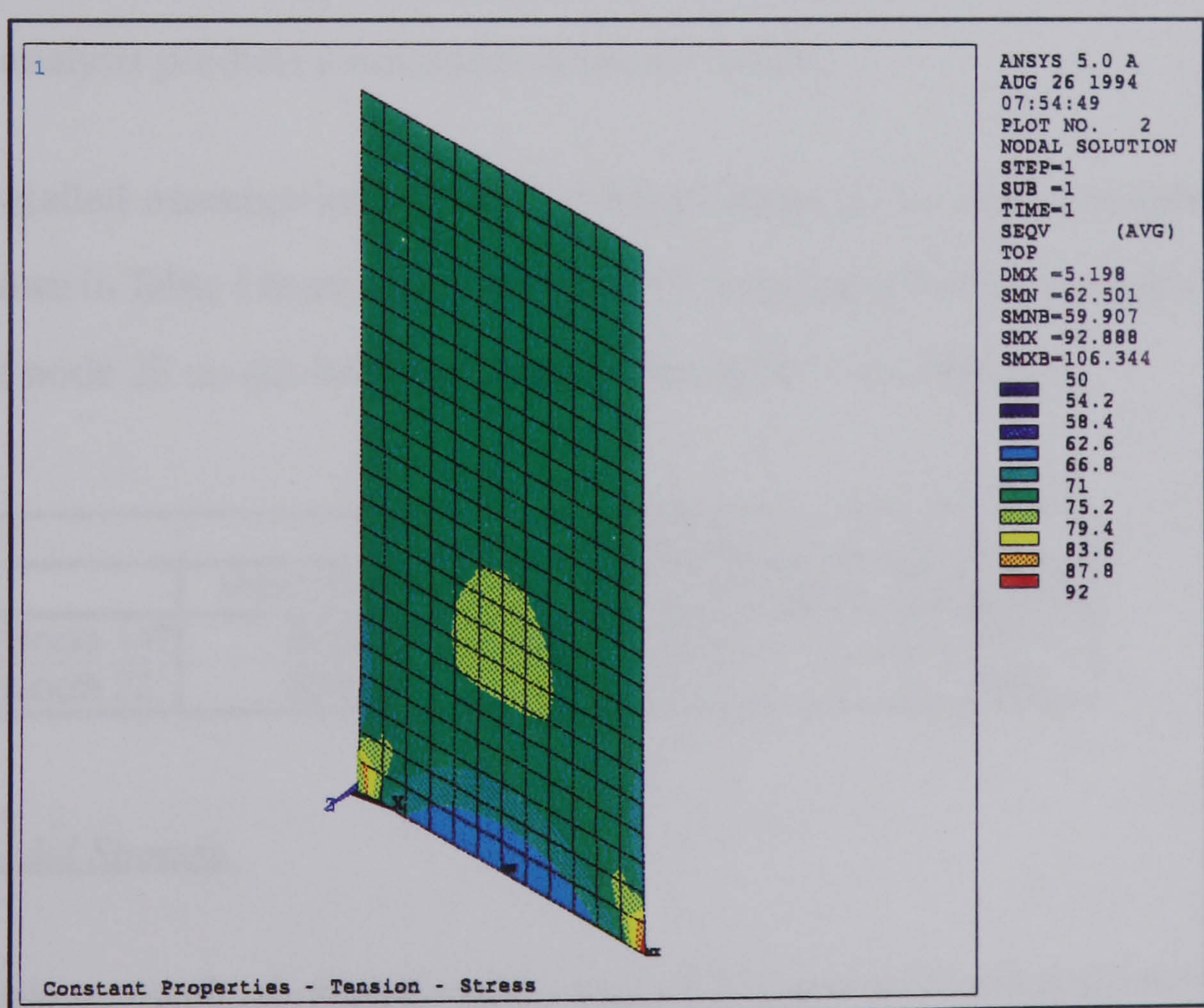


Figure 41 : Von Mises Stress In Plate – Constant Properties – Tension.

A further analysis was undertaken assuming that all of the elements in the plate had a Young's modulus which was an average of all of the individual values used, ie a value of 4.34GPa. This analysis therefore assumed that the mechanical properties were constant throughout the plate and that the material was isotropic. This analysis had the same boundary conditions as the previous one. The resultant plot of the Von Mises stress can be seen in Figure 41.

The value of the maximum stress observed does not show much difference between the two analyses, ie 91.4 MPa. with varying properties and 92.8MPa. with constant properties. However, as can be seen from the two plots, the stress distribution alters considerably when the varying properties are used in the analysis. When the measured varying properties have been used in the analysis the Von Mises stress in the upper left hand region of the plate is in the range of 92MPa. – see Figure 40. The results of the constant property analysis show the stress to be in the range of 75.2MPa. Along the left hand edge of the plate using the varying properties in the analysis gives a minimum stress of 50.1MPa., whilst the constant property analysis predicts a minimum stress of 71MPa..

A detailed examination of the Von Mises stress in the two areas gives the results shown in Table 4 from node numbers 147 in the upper left hand region of the plate, and node 23 on the left hand edge of the plate – see Figure 42.

	Von Mises Stress MPa.		
	Variable Props	Constant Props	Difference
Node 147	89.3	74.3	20%
Node 23	50.5	74.2	32%

Table 4 Nodal Stresses.

The above results show that a difference of 20% may be found in the results in the upper left hand region of the plate, whilst along the left hand edge a difference in the results of 32% may be observed. These differences could be significant as the strength would also vary over the plate as well as the modulus, and hence the plate may contain regions of lower strength.

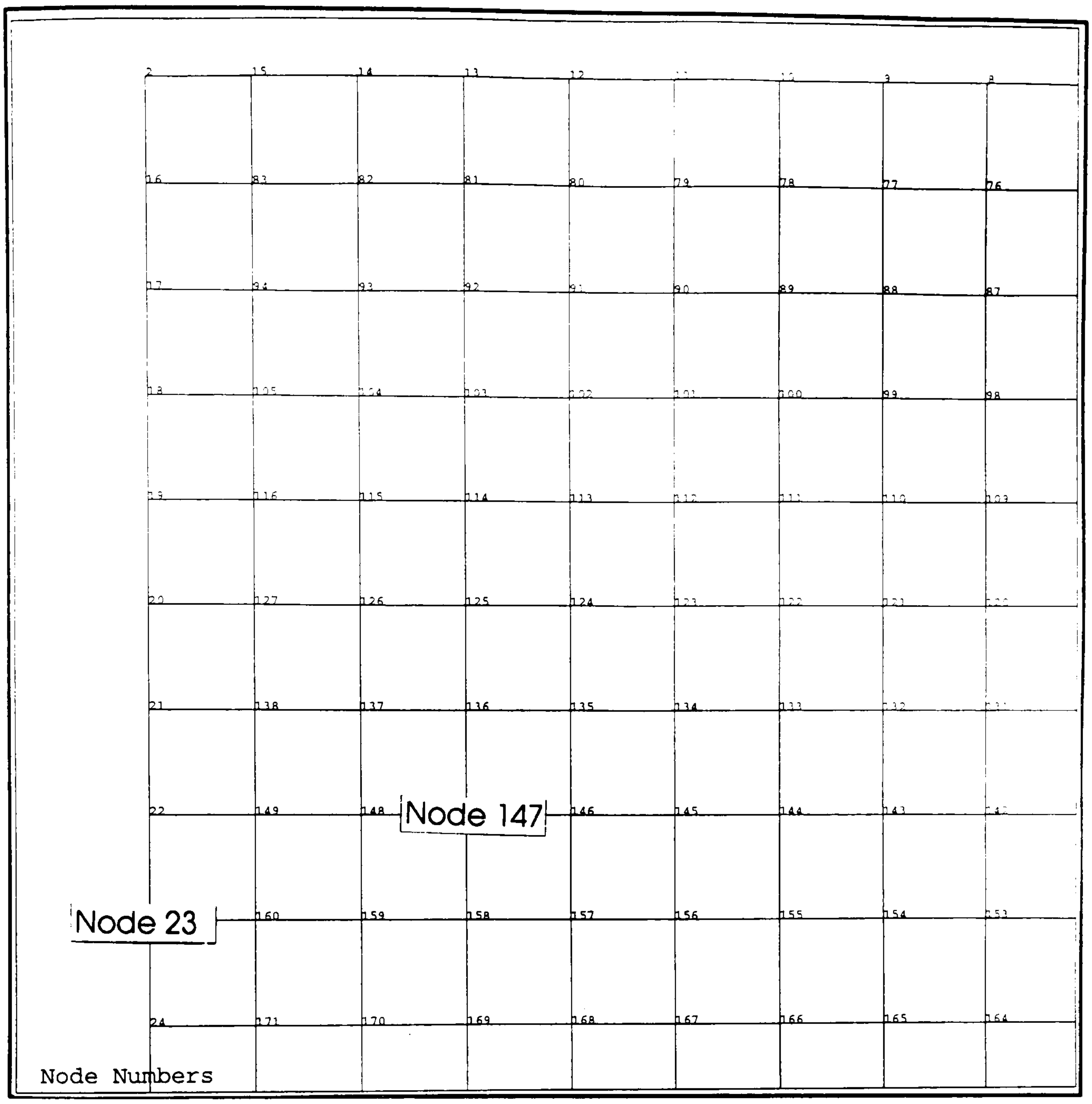


Figure 42 : Nodal Positions – Top Left Hand Corner Of Plate.

Similar observations can be made if the same analyses are repeated for different loading conditions, ie, with a nominal cantilever bending load of 2.4N. The results for the Von Mises stress distribution for the plate with varying properties can be seen in Figure 43, whilst the results for the plate assuming constant properties can be seen in Figure 44. From these two Figures it can be seen that the higher stressed area in the analysis with varying properties is extended into regions that have a lower stress in the analysis with constant properties. However, some areas, ie, towards the bottom corners, have lower stress when the accurate varying properties are used.

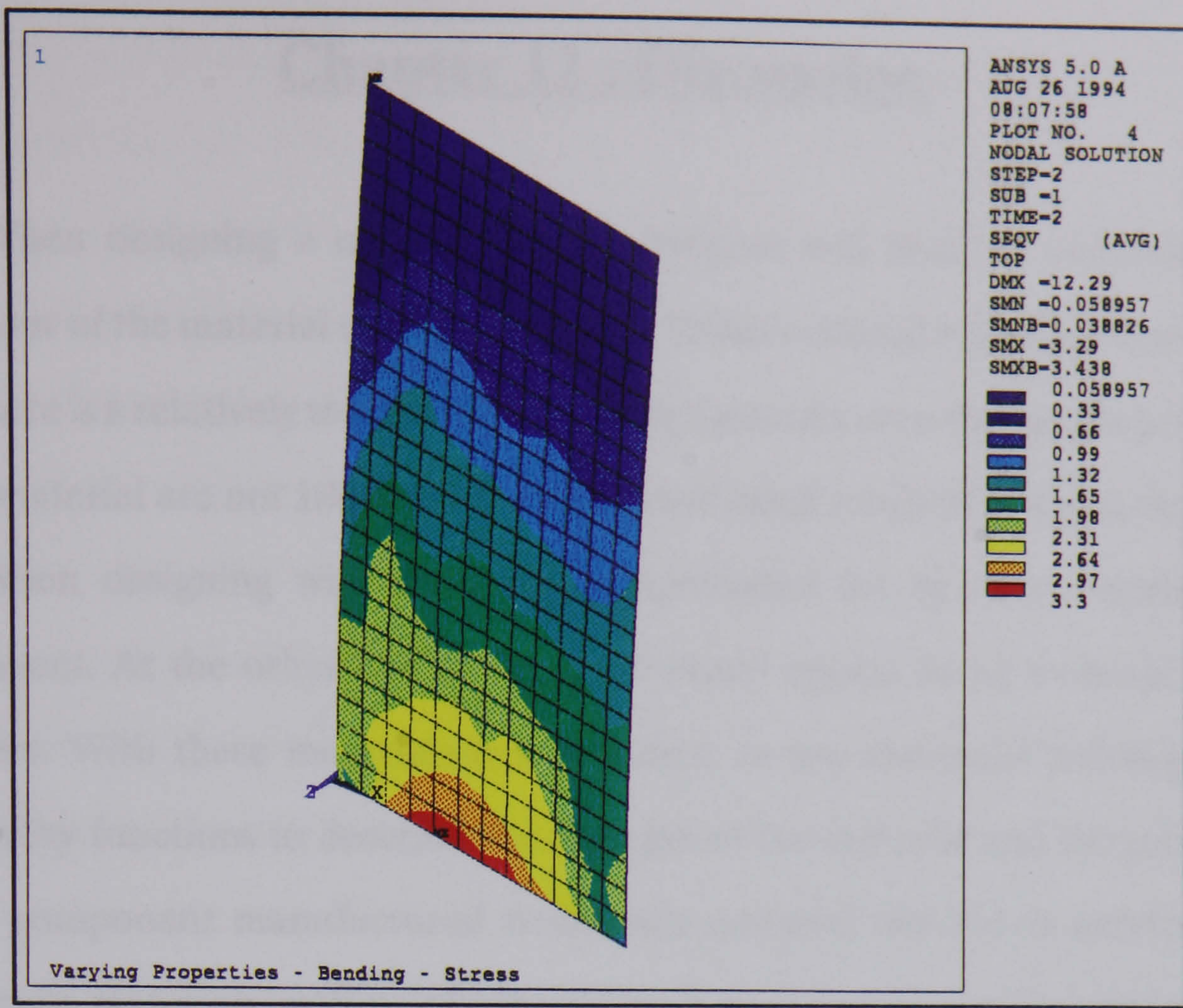


Figure 43 : Von Mises Stress In Plate – Variable Properties – Bending.

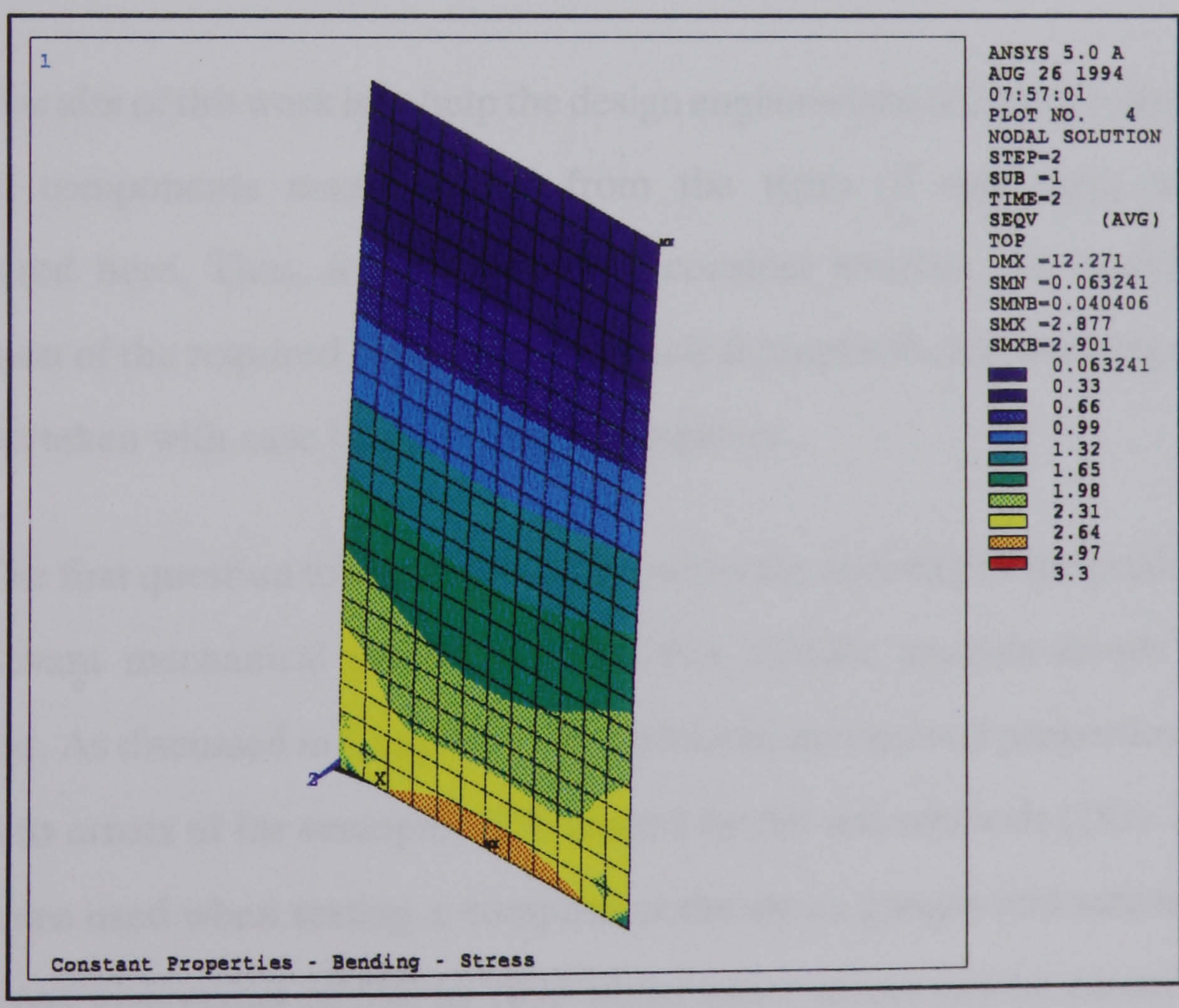


Figure 44 : Von Mises Stress In Plate – Constant Properties – Bending.

Chapter 12 : Discussion

When designing a component the designer will need to understand the behaviour of the material that is being used. If that material is a metal such as steel then there is a relatively well established design process even though the properties of the material are not 100% predictable. Such small levels of uncertainty that do exist when designing with steel are compensated for by over designing the component. At the other end of the scale would appear to lie material such as ceramics. With these materials it is common to use statistical techniques and probability functions to describe the strength of the material and the probability that a component manufactured from such material will fail in service (243). Composite materials such as considered here, would appear to lie somewhere in between these two materials. If it is assumed that both the fibre and matrix constituent properties are known then the composite properties can be predicted mathematically with the use of micromechanical equations.

The aim of this work is to help the design engineer who is having to design and analyse components manufactured from the types of composite materials considered here. Thus, it is necessary to consider whether the mathematical prediction of the required composite mechanical properties is something that can be undertaken with ease by such a design engineer.

The first question to be addressed concerns the accuracy of the prediction of the relevant mechanical properties such that reliable analysis results can be obtained. As discussed in Chapter 5 any measured mechanical properties may be subject to errors of for example, 12% caused by the test methods (193). If strain gauges are used when testing a component the strain gauges and measurement system can give errors of 5% to 10% whilst other errors can be caused by the orientation, location and bonding of the gauges (53). In addition, most composite components will have a thin resin rich skin at their surface which can lead to errors in experimental moduli which would underestimate the actual composite moduli by as much as 10–20% (20, 198). As can be seen from Appendix A the experimental data available is subject to some scatter. It is concluded in a report published by the Royal Aircraft Establishment (35) that experimental values of the mechanical properties of composites are subject to greater scatter than in isotropic materials (+/− 15% must be expected). This is stated to be probably mainly due to the types and frequencies of faults occurring in the composite (such as pores and air bubbles), the different fibre and matrix properties of the individual specimens, and inadequate testing techniques. The report concludes that the differencies between theoretical predictions and experimental results can, in many cases, be ascribed to the assumptions made in the theoretical analyses which are not always valid. Thus, an error of the predicted mechanical properties when compared to experimental data of 10% was concluded to be acceptable for the composites considered here.

12.1 Determination Of Properties By Micromechanical Equations

The derivation of the various micromechanical equations for predicting the mechanical properties of composite materials is based on a number of simplifying assumptions, such as assuming a perfect bond between the fibre and the matrix. Different researchers also use different mathematical models to derive their micromechanical relationships. This results in many different equations being available in the literature to predict a single property. These equations often give considerably different results. Thus, there exists no definitive micromechanics equations for all of the mechanical properties of interest although the so called law of mixtures equation is used by the majority of researchers to determine the longitudinal properties of a uni–directional composite material.

The accuracy of the micromechanical theories is often checked by comparing the results obtained from these equations with results obtained from experimental tests. Often empirical factors are included in the micromechanical equations so that their predictions match more closely the experimental results. However, these empirical factors need to be determined for the material being considered, and may vary from material to material. It is rare to manufacture a perfect composite component, and thus these empirical factors inherently account for the simplifying assumptions made during the derivation of the equations, such as assuming a perfect bond between the fibres and the matrix. The empirical factors also account for the imperfections caused by the manufacturing process, such as voids, that are present in the composites themselves as these equations are thus 'tuned' to match experimental values more closely. Thus, the equations containing empirical factors attempt to consider the effects that the processing of the material may have upon these properties.

Glass reinforced composite structures are often relatively thin and are therefore usually assumed to be subject to plane stress conditions. This thin sheet of composite material would usually be modelled with thin shell finite elements where only the four elastic constants, E_{11}, E_{22}, G_{12}, ν_{12}, are required to specify the stiffness properties for analysis purposes. However, if plane stress conditions cannot be assumed in the composite component such that solid finite elements are used to model the material then the other mechanical properties will need to be determined. Many composite materials are transversely isotropic which thus reduces the number of required mechanical properties from nine to six as, in a uni–directional material for example, $E_{22} = E_{33}$, $G_{12} = G_{13}$, and $\nu_{12} = \nu_{13}$. Hence only ν_{23} and G_{23} may need to be additionally determined.

12.1.1 Determination Of Uni–Directional Composite Properties

Many equations have been found in the literature for the prediction of the mechanical properties of uni–directional composites. An initial comparison of the equations against one another has shown that many equations often predict the same result for a particular property. It has been shown that some equations – see Chapter 6 for details – give results that are not what would be expected, including negative values, at some values of the fibre volume fraction. This has lead to the number of equations considered being reduced and the results from this reduced number of equations being compared to the available experimental data. The comparison with the experimental data has shown that the accuracy of the equations varies from giving a 1% error up to giving a 177% error when compared to the available test data. However, a number of equations do give results that would be considered acceptable according to the criterion previously discussed, ie to within 10% of test data. It should be noted that the equations that give the less acceptable results are generally those which attempt to predict the bounds within which the properties will always lie.

The well known law of mixtures equations have been shown to give the closest results to the available test data for E_{11} and ν_{12}. The other often used mechanical properties E_{22} and G_{12} have been shown to be best predicted by using the semi-empirical Halpin Tsai equations where the geometrical factor can be fine tuned to match the experimental data. However, it is to be noted that such fine tuning of the geometrical factor may not be necessary for an initial design study. The other mechanical properties that may sometimes be required, G_{23} and ν_{23} have been shown to be able to be determined from relatively simple equations. This work has shown that the required mechanical properties of a uni-directional composite are able to be predicted from relatively simple micromechanical equations with an error of less than 10% when compared to experimental data.

12.1.2 Determination Of Random Fibre Composite Properties

Much less information has been found in the literature concerning the micromechanical prediction of the mechanical properties of continuous random fibre reinforced composites. The majority of the equations found rely upon the use of the uni-directional mechanical properties. The equations found in the literature only predict the in plane properties with no equations being found to predict the transverse properties. When the available equations have been compared to one another it has been found that some of them predict the same result, and some predict results that are not what would be expected. This again has lead to the results from a reduced number of equations being compared to the available experimental data. Whilst the amount of experimental data available for the continuous random fibre material is less than that available for the uni–directional material, there is enough data available to enable the equations to be assessed. The results show that E_{2D} and ν_{2D} can be predicted with equations found in the literature giving 5% and 11% errors respectively when compared to the available test data – see Chapter 8. These equations use the uni–directional properties predicted for the required fibre volume fraction. With the equations found in the literature the other in plane property, G_{2D} cannot be predicted with less than a 26% error when compared to the available test data. However, the Halpin Tsai equation for G_{12} in a uni–directional material has a geometrical factor which alters the result between the longitudinal and transverse law of mixtures results. As a random fibre material is between these two extremes then it has been shown that if the Halpin Tsai equation for G_{12} in a uni–directional material has the geometrical factor modified, then this equation can be used to predict the in plane shear modulus G_{2D} of a continuous random fibre composite with an error of only 5%.

If the transverse properties for a continuous random fibre composite are required then these can be estimated by using the equations recommended for the transverse properties of a uni–directional composite. For example, the through thickness Young's modulus in a continuous random fibre composite would initially appear to be the same as the transverse Young's modulus in a uni–directional composite as they are both taken across the fibres. However, the fibres in the random fibre material may not be entirely two dimensional and may contribute somewhat to the stiffness in the through thickness dimension. As there is a lack of experimental data concerning the transverse properties then, if required, a reasonable approximation of the through thickness modulus can be found by using the expression for the transverse Young's modulus in a uni–directional composite. Of the three transverse properties there is only experimental data available for ν_{2T} and the equation adopted from a uni–directional material is shown to give only a 3% error.

12.1.3 Determination Of Woven Fibre Composite Properties

The literature survey has highlighted that in order to determine the mechanical properties of woven composites a number of researchers create detailed models of the micro geometry of the weave pattern using the FE technique. However, it would appear that the only data readily available concerning woven material are the fibre volume fraction, weave pattern and constituent properties. Detailed information of the weave geometry for parameters such as 'θ', 'a' and 'h' – see Chapter 9 – whilst used by researchers to create analytical models of the woven material, are not readily available from the material suppliers. Thus the use of such analytical models is not practical in a commercial design environment. Relatively simple methods such as the laminate analogy, which only use the uni–directional properties determined from the constituent properties, are therefore required to calculate the mechanical properties of a woven material. The laminate analogy however, only allows the calculation of the three in plane mechanical properties, ie, E_x, G_{xy} and ν_{xy}, and when the results obtained from this method are compared with the available experimental data they have been found to give errors of between 42% to 89%.

As woven material is similar to two sheets of uni–directional material at 90° to each other, but weaving in and out of one another, it would seem reasonable to use the uni–directional equations to determine the woven mechanical properties. However, in a woven material the two uni–directional layers are not merely laid one upon the other, rather the fibres weave in and out of one another in different weave patterns. This means that the fibres are not purely uni–directional as they do bend somewhat into the third dimension.

The above reasoning has lead to the development of empirical expressions for woven material based on those used to predict the mechanical properties of a uni–directional material. These uni–directional equations have been empirically modified to take account of the weave geometry and give errors of less than 10% when compared to the available experimental data. Such empirical equations inherently take into account that the thread in a woven material is itself a composite with a V_f of less than unity. These empirical relationships for woven material are, however, based on very little experimental data. As Raju et al (246) state, there is a lack of experimental data available to verify the accuracy of the analytical models for woven composites. Most of the properties calculated only have two experimental values available. These two values are at almost the same V_f and are thus probably more representative of the scatter that can occur in experimental results rather than providing a check on the calculations. Therefore, the empirical relationships developed here for woven composites should be treated with caution as more experimental data is needed in order to verify them. They can however, be used to give an estimate of the properties expected.

The bending or undulation of the fibres whilst stated by Wenger and McIlhagger (244) to increase the Young's modulus in the fibre direction due to the interlocking of the fibres, is stated by Bowyer and Bader (245) to reduce the in plane stiffness. It has been seen here in Chapter 9 that it has been necessary to reduce the longitudinal Young's modulus and increase the through thickness modulus compared to the uni–directional moduli to match with the available experimental data, thus tending to agree with Bowyer and Bader.

Should more experimental data become available and the equations presented here for the woven properties prove not to be accurate, then the empirical factors used in the equations may need slight modification. However, the approach taken here shows that the woven properties can be predicted by simply modifying the uni–directional equations rather than creating a detailed model of the weave geometry. This approach has previously been used by two researchers – see Appendix C – to predict only E_x but has not previously been used to predict the other mechanical properties. It is worth noting that the lack of experimental data may be the reason why such an empirical approach has not been adopted by other researchers. However, the approach adopted here is felt to have merit and to predict the required mechanical properties accurately enough for initial design studies. These equations show that by using the uni–directional equations the properties of woven material are able to be predicted with micromechanical relationships based on semi–empirical uni–directional relationships. It is thus not necessary to resort to the creation of detailed models of the weave geometry where the individual threads, and thread angles, are modelled using the FE technique to determine the overall composite properties.

12.1.4 The Accuracy Of Micromechanical Predictions

The micromechanical equations presented in Chapter 10 have been shown to be able to predict the required mechanical properties with an accuracy when compared to the available experimental data as shown in the following tables.

Property	Equation	Average Error
E11	Law of mixtures	1%
E22	Modified Halpin Tsai	2%
v12	Law of mixtures	7%
v23	Transverse law of mixtures	No data
G12	Modified Halpin Tsai	7%
G23	Empirical	10%

Table 5 : Accuracy Of Uni–Directional Composite Mechanical Property Prediction

Property	Average Error
E2D	5%
E3D	No data
v2D	11%
v3D	3%
G2D	5%
G3D	No data

Table 6 : Accuracy Of Random Fibre Composite Mechanical Property Prediction

Property	Average Error
Ex	8%
Ez	9%
vxy	0%
vxz	8%
Gxy	3%
Gxz	4%

Table 7 : Accuracy Of Woven Fibre Composite Mechanical Property Prediction

It is apparent that the micromechanical prediction of the mechanical properties of various types of continuous fibre reinforced composites is not an exact science. If micromechanical equations are used to predict the composite mechanical properties, then these equations only predict the properties that may be obtained. Many researchers, such as Smith (61), state that for final design purposes the theoretical estimates of moduli should not be regarded as a substitute for reliable test data, and that the best that may be achieved from such a micromechanical prediction is an estimate of the actual composite properties. The safest way of determining the properties is to measure them experimentally, and thus theoretical estimates are only useful for evaluating initial designs and studying the influence of the various parameters upon the overall performance of the composite. This is because of the possible variation caused by the manufacturing process and the limiting assumptions upon which the micromechanical analysis is based. As Shenoi and Voilette (189) state, the defects caused by processing and the local variation in mechanical properties are usually neglected in an analysis as they cannot be identified and quantified before manufacture. However, Chamis (27,56) notes that even with the simplifying assumptions the micromechanics theories can predict the properties of a uni–directional composite within "acceptable engineering accuracy", even though most of the assumptions upon which the micromechanics theory is based are violated by the real material.

In this work it has been seen that whilst the variation in the properties of E–glass is very small, the properties of the matrix materials, for example what is nominally called epoxy resin, do vary between manufacturers. In addition, the in situ matrix properties may not be the same as the properties of the bulk matrix material due to, for example, the geometry of the in–situ matrix – see Chapter 4 and references (29,47,48). Thus the matrix properties used in any micromechanical calculations may not reflect the matrix properties in the actual composite. If the extreme matrix values found in the literature are used in the recommended micromechanical equations, then this results in relatively high variations, of up to 32% in one case, in the matrix dominated composite properties see Table 1 in Chapter 11. However, when these properties are used in an analysis of a complex component, ie the subframe, with a typical load case and complex load paths, it has been shown in Chapter 11 that in all cases the variation in the maximum Von Mises stress and maximum deflection is less than 10%, and in most cases much smaller than this. These figures are confirmed by other analyses (226,228). As design engineers are not usually able to design to such tight tolerances then such a variation can be considered insignificant. Thus, the accuracy, when compared to test data, of the equations recommended here for the prediction of the composite properties is considered acceptable, and the graphs presented in Appendix D of the variation of the mechanical properties against fibre volume fraction can be used for initial design purposes.

The above results suggest that the variation seen in the matrix properties is not very significant. The results also suggest that up to a 12% variation in the in plane Young's modulus (longitudinal Young's modulus in the case of a uni–directional material), a 27% variation in the Poisson's ratio, and a 22% variation in the shear modulus do not significantly effect the resulting stress and deflection within a complex component. Note that in Section 11.3 the transverse Young's modulus in a uni–directional composite was altered by 32% without causing a significant difference in the analysis results. From these results it can be concluded that the most important property to predict accurately is the in plane (or longitudinal) Young's modulus when the component has complex load paths. These results also suggest that the assumption that an acceptable error of 10% in the accuracy of the predicted mechanical properties, as discussed in Chapter 5, is a valid one. These observations are supported by other work concerning other real composite components also with complex loading conditions that the author has been involved with (224,226,228,247).

To summarise, the micromechanical equations for the uni–directional material have been shown to be the basis of the equations used to predict both the continuous random fibre results and the woven composite results. Thus, the uni–directional material is shown to be the basic building block in the analysis of the other fibre arrangements. The recommended equations for all three types of composite considered here have been summarised in Chapter 10. The possible variation in the properties of different matrix materials has been shown to be relatively insignificant. In addition, it has been shown that variations in the composite properties of the order shown above do not significantly effect the analysis results. This suggests that if the required composite mechanical properties can be predicted to within 10% of the experimental data, which was the criterion adopted here, then these properties will be good enough for initial design purposes. However, all of the above results assume that the properties remain constant throughout the component. If the processing of the material is such that the mechanical properties do not vary throughout the component, then errors in the predicted properties of the sort of values seen above, due to variation in the matrix properties for example, can be considered acceptable. Further, these results have shown that micromechanical equations can predict the required mechanical properties with sufficient accuracy for the ordinary designer. It is thus undesirable to create detailed analytical models that require a large amount of microscopic geometric information to be known about the materials which may not be readily available. Another problem with using these analytical models is that every time any of the input parameters is altered such as the fibre volume fraction, the matrix material, or the fibre layup geometry then a new analysis needs to be undertaken. These detailed analytical models are however, useful if more detailed information is required concerning the interaction of the constituent materials and the importance and behaviour of the interface region.

12.1.5 The Use Of Micromechanics By Designers

As has been discussed, this work has shown that it is possible to mathematically predict the mechanical properties of composite materials by using knowledge of the constituent materials, the relative amount of each, and the fibre arrangement. As the composite mechanical properties is not something that is readily available from most material suppliers, is this the sort of information that is readily available to the design engineer? A knowledge of the constituent properties should be available through the material suppliers data sheets. The fibre arrangement and fibre volume fraction are parameters that should be determined by the designer to suit the particular component requirements. The design engineer, however, is unlikely to have knowledge of the micromechanical equations necessary to determine the required mechanical properties. It is only by undertaking a large literature survey and reading a document such as this thesis, or various books and papers, that the designer would be able to find this information. As has been seen, many of the micromechanical equations require the use of an empirical factor. The knowledge of a value for such a factor is again something that would not be readily available to the designer. Such information would again only be found from undertaking a search of relevant literature, or by performing tests upon samples of the material. The design engineer would therefore have to input a large amount of effort in order to be in a position to perform the sort of micromechanical calculations seen here. There does however, appear to be some commercial software available that attempts to aid the designer in this task.

Commercial computer software for predicting composite properties, such as that developed by NPL (12,65), often considers a much wider variety of materials than have been considered here, ie glass, kevlar and carbon reinforcement. This software allows the user to input the constituent material properties and fibre arrangement. The software will then use micromechanics theories to calculate the lamina or layer properties. The particular micromechanical equations used by the software are hidden from the user, thus making the software attractive to the designer who has no knowledge of such equations. However, it is important that the designer using such software should realise the limitations, such as those highlighted in this work, of such predictions of the required mechanical properties. For example, as the NPL project only had a very limited number of specimens of each material available for testing, the properties predicted by NPL required a different correction factor for each material and each property. These correction factors were then averaged to give an overall correction factor for each property. Despite such limitations, the software is a very useful tool for designers to determine the required mechanical properties for an initial design study, although more information concerning the correction factors used would be helpful.

Other software, such as the SDRC package (66), gives the user a choice of micromechanics theories to use in order to determine the mechanical properties. This is something that is not helpful to the ordinary designer who will not have enough knowledge to make a reasoned choice of which theory to use to calculate the properties. Hence, in order to use this software effectively the user needs to be familiar with the micromechanical theories. This sort of information is something that most design engineers would not possess. However, if the designer did not possess this sort of knowledge use of the software would at least allow the designer to use a micromechanical theory to calculate the required properties and hence to undertake an initial design study. In order to use the NPL or SDRC packages, or any other commercial software, the user should really have some knowledge of the micromechanics approaches used in order to appreciate the assumptions made and thus the possible inaccuracy of the predicted values of the moduli. There is obviously a commercial opportunity for software to be developed that gives the novice user more information concerning the equations and empirical factors to be used to determine the required mechanical properties. Such software should also highlight any possible errors to be obtained from such predictions and the relative importance of such errors.

12.2 Further Development Of Micromechanical Equations

Whilst many researchers still concentrate on the choice and refinement of micromechanical equations, this work has shown that the predictions made from the equations already available from the literature are acceptable. This work has shown that further development of the micromechanical theories is therefore not the most important problem concerning the prediction of the mechanical properties. These properties can currently be predicted with acceptable engineering accuracy when compared to the available test data. This is something that does not appear from the literature to have been shown or discussed previously. Rather than concentrating on improving micromechanical predictions, this work shows that effort should be made to understand the influence of other factors upon the mechanical properties of composite materials. In particular, the behaviour and flow of the material during the manufacturing process has been highlighted as being of importance. The work undertaken here suggests that this is now the most serious variable to be considered as it will cause a significant variation in the resultant mechanical properties of the composite. This flow of the material has been shown to have a greater effect upon the resultant predicted stresses than either the possible variation in matrix properties or the effect of the percentage error in the predicted properties adopted here as a criterion for evaluation. The flow of material during the manufacturing process is considered in detail in Section 12.8. Some of the other effects of the flow of the material during the manufacturing process, together with other important factors, should also be concentrated upon for further research. These will now be considered briefly in the following sections.

12.3 Weld Lines

Weld lines are formed when two flow fronts meet. They give areas of local weakness since no fibres are found across a weld line, instead fibres tend to align along the weld line, and therefore the area around the weld line contains directional properties. Jackson et al (248) state that in a compression moulding such a weld line represents a local flaw which is an order of magnitude weaker than the parent material. The position of weld lines can be predicted by using flow simulation tools, however, the properties of such flaws would appear to need quantifying such that they are considered in a structural analysis. If such flaws can be considered to be pure polymer, then the results from a flow analysis can give the position of the weld lines to the structural analysis such that they can be considered.

12.4 Fibre / Fibre Interaction

At high volume fractions of fibres the fibres will overlap or touch. This leads to a reduction of the interfacial surfaces between the fibres and matrix caused by the fibres touching and a lack of matrix between closely spaced fibres. The effect of this fibre interaction upon the resultant mechanical properties would appear to require quantifying as it is stated by Karam (249) that this can reduce strength at high values of the fibre volume fraction.

12.5 Voids

Most composites with a high fibre volume fraction contain voids due to packing problems and incomplete penetration of the matrix (249). Many researchers (12,40,215) state that the presence of voids adversely effects the composite and can reduce the mechanical properties. Voids should therefore should be avoided or minimised by improving the manufacturing process such that, for example, the temperature and pressure during moulding are better controlled. Ideally the void content should be kept below 1% (12). The relative importance and amount of voids to be found in components produced commercially therefore requires study.

12.6 Temperature Effects

The strength of E–glass relative to its strength at 20°C is stated to be reduced to approximately 75% at 375°C and 50% at 500°C (105). Additionally, thermosetting resins are stated to suffer gradual thermal degradation in their mechanical properties until exposure to temperatures of around 350 – 500°C (105). A uni–directional composite loaded in the fibre direction will have a temperature dependence similar to the fibres (105). However, under other types of loading, ie compression, shear and transverse tensile loads, the temperature dependence of the mechanical properties can be dominated by the matrix behaviour (105). At very low temperatures, ie below zero, glass fibre reinforced composites will retain their stiffness and strength and in some cases increase them (61). The effect and importance of temperature upon the mechanical properties of composites would thus appear to require more study.

12.7 Strain Rate Effects

Under impact loads the mechanical properties of composite materials may differ from the static properties of the composite. Smith (61) states that data describing the influence of strain rate upon the stiffness and strength of glass reinforced composites are sparse and contradictory. However, the data that is available indicates that both the modulus and the strength will increase with increasing strain rate (61). Thus, the strain rate effects upon the composite mechanical properties require quantifying in order to understand their importance.

12.8 Variation In Mechanical Properties Due To Processing

What has not been discussed so far is the possibility that a composite component, due to the way in which it was manufactured, may not have constant properties throughout its structure. A lack of adequate data concerning the mechanical properties of a composite may mean that an analysis assumes that the component analysed is manufactured from an isotropic material if the material used consists of random fibre orientation. Such an assumption when analysing compression moulded or resin transfer moulded composites may be incorrect. The fibre lengths of these materials may mean that at best the orientation of the fibres is only random in two directions because of the through thickness dimensions of the component. Although, if the material is formed into a die then the fibres may be folded up into the third dimension making the material tend towards isotropy. If the component is modelled using thin shell finite elements then no account can be taken of the mechanical properties through the thickness which may mean that the analysis is not accurate. An accurate analysis may be performed using solid or brick finite elements where the full three dimensional mechanical properties may be modelled if these properties are known.

It is suggested in the literature that the processing of the composite material may have an effect upon the fibre orientation and concentration and cause a variation in the mechanical properties throughout the component. This variation in mechanical properties is important as this would affect the overall behaviour of the component as well as the accuracy of the analysis. Also, all micromechanical prediction methods are based on a number of assumptions such as the value of the fibre volume fraction, fibre orientation etc. An example of the variation in glass content caused by the manufacturing process can be seen in a commercial compression moulded GMT heat shield that was designed to be 40% glass by weight. Upon testing this component was found to have the glass weight fraction varying between 35.9% and 52.4% in different areas (236). Thus, the micromechanical predictions of composite mechanical properties will be inaccurate if the manufacturing process causes these factors to vary in the finished component. Any analysis based on the designed properties is only using predicted properties and it must be recognised that these may be inaccurate because of the effects of the manufacturing process.

12.8.1 Resin Transfer Moulding (RTM)

In the resin transfer moulding (RTM) technique some authors (127, 250−256) state that the preforming of the fibres into the dies may cause the fibre arrangement to be altered from the original layup due to fibre movement and re−orientation during preforming. Rudd et al (256) and Karbhari et al (257) state that a frequently encountered problem in RTM is the movement of the fibre reinforcement during either preforming of the fibre mat or injection of the resin. If areas of the reinforcement move this would cause a different fibre concentration (ie, V_f) to be found in different areas as well as altering the fibre alignment. This would then give rise to a variation in the mechanical properties throughout the component. Owen et al (251) state that if the fibres in the preform are not completely wetted by the resin during RTM then this can also have an adverse effect upon the mechanical properties.

If data from test samples is used for analysis purposes it does not take into account any modification of the fibre arrangement that may be caused by the preforming process when manufacturing the component. Thus, the analysis results for the composite subframe described and analysed earlier that use the mechanical properties which have been obtained from a test sample should be treated with caution, ie results included in references (196,219–226). These observations have been made previously (226,258) concerning the analysis of this component and are supported in general by Rudd et al (256). Unfortunately, whilst considered important, the subframe has yet to be cut up and examined as all of the components manufactured are currently required for test purposes. Rudd et al suggest that a 40% reduction in the modulus perpendicular to the direction of stretch can result from a 30% stretch when preforming for a random fibre material such as that used for the subframe. This work by Rudd et al suggests that the manufacturing process of preforming should be modelled to determine the mechanical properties after preforming, and that these properties, not those obtained from test samples, should be used in any subsequent analysis. Rudd et al describe the use of analysis software to perform a simulation of the preforming process with the objective of providing mechanical property information.

12.8.2 Compression Moulding

During the compression moulding process the fibres will tend to flow with the matrix when the charge material is compressed. This suggests that a variation in the mechanical properties will be found throughout a compression moulded component. The glass will therefore be redistributed during the flow and this means that the fibre orientation and concentration will tend to vary throughout the component (236,240,241,252,259–265). The literature suggests that this process can result in "a significant change in the mechanical properties of the material" (231,240). Thus, as a result the elastic moduli show a significant amount of scatter and the material may become highly anisotropic (240,241,266). The materials data therefore found in manufacturers data sheets may be incorrect if the influence of the flow during moulding is neglected (267). The fibre length of these materials may mean that at best the orientation of the fibres is random only in two dimensions because of the through thickness dimensions of the component. Advani and Tucker (268) state that where parts manufactured from SMC are typically large thin shell like structures, there is no evidence for through thickness variations of fibre orientation. Thus, SMC parts can be modelled as having a two dimensional variation of fibre orientation. The initial position of the charge will obviously effect the resultant fibre orientation and distribution.

Considerable variation in local fibre content in both the charge and a moulded plaque have been observed by Kline (231) when microscopically examining moulded SMC plaques. Kline found that in a nominal 50% W_f SMC plaque the fibre content within a plaque varied between 38% W_f and 54% W_f. During a project by the National Physics Laboratory on the "Predictive Modelling Of Fibre Reinforced Composites" (12,65) the material properties found in different areas of a glass mat thermoplastic (GMT) box were commented upon (41,48). It was found that the stiffness, fibre volume fraction and composite density at the edges of the box were relatively low due to these being resin rich areas. The stiffness, fibre volume fraction and composite density on the sides of the box were also found to be much lower than those corresponding properties on the base of the box.

It is stated by Akey and Reifschneider (269) that the design of compression moulded components is difficult because of the anisotropic material behaviour of SMC. They add that the mechanical properties can vary greatly with the orientation of the fibres and thus designing with these materials requires more analytical tools. Akay and Reifschneider describe the use of a computer simulation package PLASTEC (270) to model the compression moulding process. Kau (232) states that the determination of the orientation of the fibres in SMC is an important step in correlating the effects of the moulding process upon the properties of the finished component. However, Kau also states that the flow of SMC is complex and has not been fully analysed even for a rectangular plate moulded with a central charge. The use of simulation software would add another step, and extra cost, into the design cycle, but may be the quickest and easiest way to obtain the mechanical properties of the component required for a structural analysis. This does of course require factors such as the material data, mould geometry, charge position to be known and controlled during manufacture.

One source of possible error when analysing compression moulded composites is thus suggested by the available literature to be caused by a variation in the mechanical properties in the component due to the flow of the material during the compression moulding process. Whilst the literature can be seen to highlight the effect of processing upon compression moulded composite properties, and states that it may be significant, there appears little or no attempt to quantify how significant the processing effects are upon the resultant behaviour of the material. No attempt has been made to study whether a larger variation in properties is found in one type of compression moulded material as compared to another, the cause of such a larger variation, and whether it is significant.

12.8.2.1 Sheet Moulding Compound (SMC)

To investigate the variation caused by processing in a real component the automotive suspension arm considered earlier and manufactured from SMC was investigated. Two papers published by the author (194,195) have detailed the correlation of the experimental and analysis work undertaken on the suspension arm. The papers compare the results from FE, photoelastic and SPATE analyses and strain gauges. However, the FE, photoelastic and SPATE analyses all assumed that the material was isotropic, and hence the results may not be correct. The analysis results were based upon the mechanical properties obtained from material cut from one area of the actual component, whilst the SPATE system was calibrated from a disc of the material. All three analysis techniques also assumed that the suspension arm consisted of constant mechanical properties throughout its structure. Thus, all three analysis techniques were based on assumptions that were not necessarily correct.

The fact that the results from the three analysis techniques correlate fairly well does not necessarily mean that they are predicting the correct levels of stress throughout the component. This is because of the limiting assumptions made in the analyses such as the effect of the fibre distribution and orientation which has been ignored in all three techniques. The only significant comparisons that indicate how the analysis results compare with test results are those that are compared to the strain gauges. However, even these may not indicate the true stress distribution within the component as the possible effect of the fibre distribution and orientation was not realised at the time, and thus the strain gauges were not necessarily positioned in the areas where the anisotropic effects of the material flow were highlighted.

X–rays taken of the component since the papers were published appear to show significant fibre alignment as the initially random fibre SMC material flows from the centrally placed charge position into the body mount areas and the ball joint housing area – see Chapter 11. Fibre alignment can also be observed in the majority of ribs – alignment in the direction of the ribs. In the other areas the SMC appears to have retained its random fibre orientation. However, even in these areas there does not appear to be a consistent concentration of fibres. Whilst the X–rays are useful for studying the orientation of the fibres in the finished component they cannot give quantitative information concerning fibre concentration, ie fibre volume fraction, or even fibre alignment. The X–ray results appear to be confirmed from sections cut from the suspension arm that have been subjected to resin burnoff.

When the information found from the sections and X–rays concerning the fibre directions was used in finite element analyses of the component for a typical load case, it was found that the position and value of the maximum equivalent stress only varied by at most 3% when compared to the value obtained when assuming that the material was isotropic. The distribution of the stress was also seen to vary very little. The value of the maximum deflection was found to vary by only 5%. These relatively small variations are due to the initial charge covering the majority of the mould area thus allowing little flow of the material. This therefore suggests that as the flow of the material in the compression moulding process for this component is not significant, the breaking up and modelling of this component into separate regions where different mechanical properties can be applied is not important as an assumption of isotropy, whilst much easier to model, does not mean a great loss in accuracy.

Other related work that the author has recently been involved in, and is still ongoing, shows that processing can have a significant effect upon the analysis results for SMC if the flow path is greater than that for the suspension arm. Two ribbed SMC plates – see Appendix D – but with different charge positions were analysed. One analysis of each plate assumed that the mechanical properties were constant throughout, whilst another analysis used the varying mechanical properties predicted by a simulation programme and confirmed by tests – see Appendix D. A bigger difference between the two sets of results was observed when the charge position was such that the material had a longer flow path. In the plate with the lateral charge position, ie the longest flow path, the stress distribution in the plate was seen to vary such that a difference in the predicted value of the stress between the analysis using constant properties and that using varying properties was as great as 25% in some areas. In other areas of this plate a difference of 15% was found. A detailed examination of the results shows that in some areas a reduction in the stress was observed, whilst in others an increase in the stress was observed. In the plate with the centrally placed charge, where the flow path was not as long, the difference between the results was seen to be at most 12%. Such quantitative information showing the effects of the manufacturing process have not been seen previously in the literature.

The above results highlight that the position of the charge in the mould and the resultant flow of the material is an important factor in an analysis of a component manufactured from such material. Thus, the design engineer is required have some knowledge about, and be concerned with, the manufacture of the component. The sort of values seen here for the differencies in stress levels are much greater than those seen earlier caused by either variation in the matrix properties, or by the percentage error in predicted properties. This therefore highlights the relative importance of the variation of the properties caused by the manufacturing process. This is something that has not been highlighted in the literature survey undertaken here. More work is obviously required to study the length of the flow path and its effect upon the resultant mechanical properties.

12.8.2.2 Glass Mat Thermoplastic (GMT)

An example of the variation in the mechanical properties experienced in a GMT plate manufactured by compression moulding has been found in the work of Stokes (240,241). The work of Stokes has shown that a variation in the Young's modulus of over 2:1 can be expected in a flat plate. Stokes also showed that the density, and hence fibre concentration varied. The study by Stokes does leave some important questions unanswered. For example, throughout the plate, how do the other mechanical properties including the strength vary?

The plate studied by Stokes has been analysed in Chapter 11 using the data provided for the variation of the Young's modulus, and compared to results obtained if the plate was assumed to have a constant Young's modulus throughout. The analyses undertaken show very little difference in the predicted value of the maximum equivalent stress. However, the stress distribution does change significantly when the accurate and varying properties are used instead of constant average properties. The difference in the predicted value of stress between these two cases was as great as 32% in some areas, see Chapter 11. In other areas of the plate a difference between the results of 20% was found. Similar observations could be made if the same analyses were repeated for different loading conditions. From the results it can be seen that the higher stressed area in the analysis with varying properties is extended into regions that have a lower stress in the analysis with constant properties. However, some areas of the plate, have lower stress when the accurate varying properties are used. These are generally bigger differences between the two sets of results than was observed with the SMC component and plates. Such differences could be significant as the strength would also vary over the plate as well as the modulus, and hence the plate may contain regions of lower strength.

12.8.2.3 GMT vs SMC

A bigger difference between the analysis results from using the accurate varying properties compared to constant properties has been found for GMT than for SMC. This bigger difference seen in GMT is observed because the ratio of the composite modulus E_c to the matrix modulus E_m is much greater in GMT than it is in SMC. The matrix (ie, the mixture of limestone filler and resin) modulus of SMC is approximately 6GPa. (236,237) whilst a typical composite modulus is 10GPa. giving an E_c/E_m ratio of 1.6. The matrix modulus of GMT is that of polypropylene, ie approximately 1GPa. (229) whilst the composite modulus is a minimum of around 4GPa., therefore giving an E_c/E_m ratio of at least 4. As the composite material flows during the compression moulding process the fibres will become aligned to some extent rather than retaining their random nature, and therefore the composite modulus E_c will tend towards the uni-directional moduli E_{11} and E_{22}. As the transverse modulus E_{22} is more of a matrix dominated property than the longitudinal modulus E_{11} which is fibre dominated, the transverse modulus will be greater in SMC than in GMT. Thus the ratio between E_{11} and E_{22} will be greater in GMT than in SMC and the effects of the fibre orientation will be greater in GMT. The result of the flow of the material in the compression moulding process therefore has more effect on GMT than it does on SMC. This is something that appears from the literature to not have been known previously.

It has been seen that a variation in the mechanical properties in discrete regions can have a profound effect upon the pattern of the stress distribution. The variation in stress distribution seen for the GMT plate is much greater that that observed for the SMC plates even though the amount of flow of the material is less. The SMC plates analysed in Appendix D showed that the variation in the stress distribution was greater with a longer flow path. As the GMT plate analysed here had a relatively small flow path, and differences in the stress of up to 32% were observed, then this suggests that much larger differences would be observed if the GMT was subject to a longer flow path similar to the SMC plates.

The amount of variation of the mechanical properties will depend upon the length of the flow and the geometry of the component. This means that higher stresses may be found in areas of predicted and expected low stress if variations in discrete regions are not modelled, and vice versa. These results suggest that the effect of the processing of the material upon the fibre orientation, and its resultant effect causing a variation of the mechanical properties, does need to be considered before undertaking a structural analysis. Thus, the modelling of the processing of compression moulded material, whether it is SMC or GMT, is considered to be an important step in the analysis of components manufactured from these materials.

In order to determine the different properties in the different areas of a component caused by the processing method prototype components may first need to be manufactured and hence more test work is required. Thus, until the component is manufactured and the actual mechanical properties can be checked, constant properties are usually assumed. If constant properties are used in an FE analysis then these are the predicted properties and the accuracy of this prediction needs to be verified once the component has been manufactured. This means that even if the mechanical properties could be accurately predicted using micromechanical equations then these predicted properties could in areas be quite different to the actual properties if constant properties are assumed. Thus such micromechanical predictions can at best be used to obtain an initial indication of the integrity of the designed component. As the composite material is formed during the manufacture of the component there will not be any stock material available that can be tested to provide information on the variation of the mechanical properties. The mechanical properties of the finished component itself need testing to determine the fibre concentrations and orientations due to the manufacturing process and to check the accuracy of the predicted mechanical properties by cutting a number of test samples out of the finished component. The consistency of the manufacturing process also needs to be checked by examining a number of components.

12.9 Flow Modelling

Computer programmes are currently available that predict the flow of plastic into the mould in the injection moulding process. These programmes can also predict the flow and orientation of the short glass fibres which may be present, and hence can predict the mechanical properties of the composite by using micromechanical equations. What appears to be required for a more structural composite that is manufactured by the compression moulding technique is similar predictive programmes that will predict the flow and final orientation and distribution of the long fibres in such material. From such a prediction the variation of the mechanical properties over the component can be predicted by using micromechanics methods and transferred into an FE analysis of the component. However, as has been seen different micromechanical equations can give quite different results such that the user needs to have confidence in the equations used in such software. Thus, such flow analysis programmes would appear to be useful for determining the fibre orientation due to processing, but the prediction of the mechanical properties depends upon the micromechanical equations that the programmes subsequently use. Whichever micromechanical equations are used to predict the composite properties by the flow analysis programmes, they are only predicting the properties expected in the finished component, and the accuracy of this prediction will require validation at some stage. Such programmes that are able to model the conditions expected in a finished component caused by the processing of the material thus make the model used in the subsequent structural analysis more representative of the actual component.

The flow modelling of the compression moulding process has been studied by authors such as Folgar and Tucker (271). They state that the orientation pattern of the fibres as caused by the flow during processing is important as it controls the mechanical properties of the component. The study of this process has lead to the development of software, such as PLASTEC (270) to model the compression moulding process and to provide the varying mechanical properties for a subsequent structural analysis. However, there would appear to be limitations in the flow analysis software that is currently available – a discussion of these has been published in references (238,239,242). For example, use of such software means that each element will have a separate material property data table. This increases the amount of data stored for the analysis. If the structural analysis showed that the finite element mesh required refining in certain areas, then because the material property data is attached to each element it may mean going back to the original mesh in the flow analysis program. If the mesh is refined in the flow analysis program then a further flow analysis may be necessary before transferring the data once more to the structural analysis software.

The flow analysis software typically only uses thin shell elements and therefore the through thickness properties cannot be predicted. This is not important if the component consists of a relatively thin shell of material. However, if the component consists of relatively thick sections such as in the automotive suspension arm considered here then the accuracy of any flow analysis data has to be questioned. If the suspension arm was analysed by flow analysis software then two different analysis models would be required, one for flow analysis and one for structural analysis.

The development of the flow analysis software to model the compression moulding process is relatively new. The software has therefore not yet been adequately validated and thus the importance of the limitations in it are not currently understood (264). The flow analysis software currently does not consider variations in the fibre concentration which have been seen in the GMT tested by Albrecht et al (267) and Stokes (240,241). The importance of considering this variation in fibre concentration is not currently understood as although the fibre volume fraction will vary, the predictions of the mechanical properties made by the software have yet to be fully validated. In addition, the flow analysis software currently only determines the variation in the moduli and not the strength variation throughout. Whilst some designs are stiffness limited and thus strength is not so important, many others are strength limited and thus the prediction of the strength is important. The flow simulation software currently available would appear only to analyse SMC and not GMT (271,272). The software also uses triangular elements which means that some translation of the results needs to be performed if they are to be used in a structural analysis where quadrilateral elements are being used.

12.10 Prediction Of Stress

The stresses predicted by various techniques such as FE analysis do not necessarily tell the analyst whether the composite component can survive the loads applied to it. In many cases, to understand how the component behaves under the service loading conditions the analyst will compare the predicted stresses with the strength of the material. However, in a composite the strength will depend upon the fibre orientation and distribution and therefore in material such as SMC or GMT may vary throughout the component due to the manufacturing process.

Thus, as discussed in reference (242), the concept of an acceptable level of stress as predicted by FE has little meaning unless the strength in that discrete region is also known.

As there is doubt over the accuracy of the prediction of the mechanical properties of continuous fibre composites, there is the question of how important the degree of accuracy needs to be and how the analysis results are effected. The work undertaken here shows that the FE predictions of stress distribution can be greatly affected by local changes in the modulus caused by the manufacturing process. The strength will also vary due to the manufacturing process.

What is required is a flow modelling programme that predicts the strength values in each discrete region as well as the modulus. The modulus and strength values can be read into the FE programme for the structural analysis. The stress results for each discrete region can then be combined with the strength values into a relevant failure criterion. The result could then be shown as a percentage of failure or a failure index in each discrete region. Because of the uncertainties in the predictions of modulus and strength, a recommendation that a failure index above a certain value would be too close to failure would be needed.

Chapter 13 : Conclusions

1. An accuracy of the predicted mechanical properties of composite materials to within +/− 10% of experimental data is considered acceptable as designers do not usually require greater accuracy. This level of tolerance does not affect the resultant stress distribution significantly as long as the mechanical properties are constant throughout the component.

2. The micromechanical prediction of the mechanical properties of uni−directional composites can be made to within +/− 10% of experimental data using the rule of mixtures relationships for E_{11} and ν_{12} and the semi−empirical Halpin Tsai relationships for E_{22} and G_{12} providing the empirical factors are known. However, the determination of the required empirical factors is not easy and a designer may need to either undertake a large literature survey, or manufacture and test samples to determine this factor. Thus, the usefulness of the semi−empirical equations relies upon the knowledge of the empirical factors.

3. The mechanical properties of continuous random fibre composites can be predicted to within +/− 10% of experimental data with semi−empirical relationships based on those found for uni−directional material.

4. Creating a detailed model of a unit cell of the weave pattern is undesirable when analysing a component manufactured from woven material as many of the required parameters are not readily available from manufacturers data sheets. The mechanical properties of plain weave composites are able to be predicted with semi−empirical relationships based on those of a uni−directional material to within +/− 10% of experimental data. However, the lack of experimental data means that these relationships are not fully validated and may need fine tuning when more experimental data is available.

5. The uni−directional micromechanical relationships have been shown to provide the basis of the equations for the prediction of the mechanical properties of

composites with continuous random fibre or woven fibre arrangements. Thus, the uni–directional material is shown to be the basic building block in the analysis of the other materials.

6. This work has shown that further development of the micromechanical theories is not the most important problem concerning the prediction of the mechanical properties. These properties can currently be predicted with acceptable accuracy when compared to the available test data from the micromechanical equations already available in the literature. Rather than concentrating upon improving the micromechanical predictions, this work shows that effort should be made to understand the influence of other factors upon the mechanical properties of composite materials. In particular, the behaviour and flow of the material during the manufacturing process has been highlighted as being of importance. This is something that does not appear from the literature to have been shown or discussed previously.

7. The variations observed in the resin properties can cause a large difference in the matrix dominated composite properties. However, when these are used in analyses of real components less than a 10% difference in the maximum stress and deflection was observed. As design engineers do not usually require greater accuracy such a variation can be concluded to be insignificant. Thus the graphs presented in Appendix D of the variation of the mechanical properties against fibre volume fraction can be used for initial design purposes.

8. The micromechanical predictions of composite properties can at best be used for an initial design study. Due to the variations in properties caused by the manufacturing process more accurate predictions cannot be made until the component has been manufactured and then investigated to determine the fibre concentrations and orientations.

9. Composites manufactured by resin transfer moulding may show a variation in their mechanical properties from those expected from tests on samples of the

material. The test samples do not take into account any fibre movement or re–orientation that may occur during processing. Properties obtained from such samples should only be used for initial design studies. To obtain accurate mechanical property information, either samples should be taken from the manufactured component, or the preforming process should be simulated using numerical modelling techniques.

10. Compression moulded material such as sheet moulding compound and glass mat thermoplastic can show large variations in the mechanical properties in some areas due to the manufacturing process. The variation in the mechanical properties throughout a compression moulded composite means that any analysis cannot assume that the component consists of constant properties throughout. The variation in the stress distribution between analyses assuming constant properties and those assuming varying properties in simple plates subject to simple loading conditions has been shown to be as much as 32% in some areas of a GMT plate and 25% in some areas of an SMC plate. Such quantitative information showing the effects of the manufacturing process have not been seen previously in the literature.

11. Larger variations in the stress distribution between analyses assuming constant properties and those assuming varying properties are observed in GMT compression moulded components than are observed in SMC components. This is because the ratio of composite to matrix modulus, and hence longitudinal to transverse modulus is greater in GMT. This is something that previously appears not to have been known or discussed in the literature.

12. Compression moulding simulation software should be used to provide the varying mechanical properties for a structural analysis of compression moulded components. However, the variation of mechanical properties throughout a compression moulded composite means that the prediction of a maximum stress level has little meaning unless the variation of the strengths are also known, and

thus such simulation software requires further development to consider strength prediction. The predictions of the fibre orientation and hence mechanical properties from such software need to be validated and the limitations in the software understood. This may lead to a further requirement for the software to be developed such that variations in the fibre concentration are also modelled. This is something that has not been discussed previously in the literature.

Chapter 14 : Recommendations For Further Work

The following areas have been highlighted as requiring further research.

1. The effect of factors such as weld lines, fibre / fibre interaction, voids, temperature and strain rate upon the composite mechanical properties.

2. Long term effect of environmental factors such as temperature, humidity, and moisture absorption on the composite mechanical properties should be examined.

3. Compression moulding simulation programmes require work to validate their predictions of mechanical properties. Once the current limitations of the software are understood, then the software may need developing to consider strength prediction and the effect of a variation in the fibre concentration caused by processing.

4. The percentage coverage of the mould area of the charge material and the initial charge position to cause significant variation in the mechanical properties requires quantifying.

5. The use of a definitive failure criteria for composites needs investigating.

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Appendices

Appendix A : Micromechanical Determination Of Uni–Directional Mechanical Properties

A.1 Mechanics Of Materials

The longitudinal law of mixtures equations are recommended by many authors and are given below (6,7,12,17,20,23,24,27–30,32–34,42,54,56,61,75,77, 80,105–108,110,125,134,245,273–277). Note that the subscript 'm' denotes matrix and 'f' denotes the fibre properties.

$$E_{11} = E_m V_m + E_f V_f \qquad [16]$$

$$\nu_{12} = \nu_f V_f + \nu_m V_m \qquad [17]$$

Where E_{11} is the longitudinal Young's modulus, V_m the matrix volume fraction, V_f the fibre volume fraction and ν_{12} is often termed the 'major' Poisson's ratio, ie in the fibre direction.

Also, from Hill (42):–

$$K = V_f K_f + V_m K_m \qquad [18]$$

The density of the composite can also be expressed as a law of mixtures relationship as follows (12,54,105):–

$$\rho_c = \rho_f V_f + \rho_m V_m \qquad [19]$$

or,

$$\rho_c = 1 / \{(W_f/\rho_f) + (W_m/\rho_m)\} \qquad [20]$$

The transverse law of mixtures equations are given below in the form used by Jones (28) although Clyne (30) states that for E_{22} the relationship is very inaccurate and that it is common to use the Halpin Tsai relationship instead:–

$$E_{22} = E_f E_m / (V_f E_m + V_m E_f) \qquad [21]$$

$$G_{23} = G_f G_m / (V_f G_m + V_m G_f) \qquad [22]$$

$$\nu_{23} = \nu_f \nu_m / (V_f \nu_m + V_m \nu_f) \qquad [23]$$

The in plane shear modulus is also given by this transverse relationship using the assumption that the shear stresses are the same in the matrix and the fibre

(28,29,54,105,107,276):–

$$G_{12} = G_m G_f / (V_m G_f + V_f G_m) \qquad [24]$$

However, this equation is stated to give lower values than those obtained by experiment (54). A number of researchers (11,24,26,29,34,56,61,70,80,86,108) give the isotropic relationship for the transverse shear modulus:–

$$G_{23} = E_{22}/2(1+\nu_{23}) \qquad [25]$$

However, for the reasons already given in Chapter 4, this equation will not be considered further.

A modification to the longitudinal and transverse law of mixtures equations is proposed by Ekvall (278) which takes into account the "triaxial stress state in the matrix due to fibre restraint." Ekvall's modified equations are given below:–

$$E_{11} = V_f E_f + V_m E_m' \qquad [26]$$

and, $$E_{22} = E_f E_m' / \{V_f E_m' + V_m E_f (1-\nu_m^2)\} \qquad [27]$$

Where,

$$E_m' = E_m / (1-2\nu_m^2) \qquad [28]$$

A similar modification to equation *[27]* is given in the IACFA report (75) for a composite consisting of a rectangular array of fibres embedded in a matrix containing an idealised distribution of voids. However, the equation requires the determination of the void volume fraction and will not be included here.

Unlike many authors Chamis (56,77,274) does not distinguish between isotropic and transversely isotropic fibres. The following equations are presented by Chamis and are derived from the mechanics of materials approach:–

$$G_{12} = G_m / \{1 - V_f^{0.5}(1 - G_m/G_f)\} \qquad [29]$$

$$E_{22} = E_m / \{1 - V_f^{0.5}(1 - E_m/E_f)\} \qquad [30]$$

and, $$G_{23} = G_m / \{1 - V_f^{0.5}(1 - G_m/G_f)\} \qquad [31]$$

Note that the equation presented by Chamis for G_{23} is exactly the same as that for G_{12}. No experimental validation of these equations is presented. However, Chamis does state that if the results from these equations are compared to experimental data then the limitations of the equations may be assessed and modifications made to them, and that any apparent discrepancies may be due to the fact that the shear modulus is difficult to measure accurately.

An alternative definition for ν_{23} is reported by Whitney and McCullough (29) and Foye (279) by considering "the constraints that the stiffer fibres place on the tendency of the softer matrix material to contract in the fibre direction when stretched normal to the fibres" (279). The equation is given as:–

$$\nu_{23} = \nu_f V_f + \phi \nu_m V_m, \qquad [32]$$

with, $\phi = \{1+\nu_m-\nu_{12}(E_m/E_{11})\}/\{1-\nu_m^2+\nu_m\nu_{12}(E_m/E_{11})\}$.

Where, E_{11} and ν_{12} are determined from the law of mixtures relationships. Foye (279) reports that the results obtained from this equation appear to be reasonable when compared to those obtained from FE analysis. A limitation on the above formula noted by Foye is that it can only be applied to composites containing stiff, circular, isotropic fibres in a soft matrix material.

Equations for E_{22} are presented by Shaffer (275) when the cross sectional geometry of the fibres and matrix are considered. No comparison of the results obtained from these equations is made with experimental data. For $V_f < 0.68$:–

$$E_{22} = E_m \frac{\{1 - (1-E_m/E_f)(0.8247V_f^{0.5} - V_f)\}}{\{1 - 0.8247\, V_f^{0.5}\,(1 - E_m/E_f)\}} \qquad [33]$$

For $V_f > 0.68$ then:–

$$E_{22} = E_m / \{1-V_f(1-E_m/E_f)\} \qquad [34]$$

A.2 Bounding Techniques

The equations presented in the following sections are for the general case of uni–directional hollow fibres in a matrix. However, these equations can be reduced to the form shown here when considering solid fibres.

A.2.1 Plane Strain Bulk Modulus

For a hexagonal array of hollow uni-directional fibres the lower and upper bounds of the plain strain bulk modulus K_{23}^* are given by (10):-

$$K_{23-}^* = K_m/(v_1/m_k + v_2) \qquad [35$$

$$K_{23+}^* = K_m(v_1 m_k + v_2) \qquad [36]$$

From the geometry of the hexagonal array, the fractional volumes:-

$$v_1 = 0.907, \text{ and } v_2 = 1-v_1.$$

The constant m_k is given by (10):-

$$m_k = \frac{\phi(1+2\nu_m\beta^2) + (1-\beta^2)2\nu_m}{\phi(1-\beta^2) + (\beta^2+2\nu_m)} \qquad [37]$$

and, $\phi = K_f/K_m$ *[38]*

Also, 'β^2' can be given in terms of the fractional volumes such that:-

$$\beta^2 = V_f/v_1 \qquad [39]$$

For a random array of fibres the bounds coincide and hence, (10):-

$$K_{23}^* = K_m m_k \qquad [40]$$

and, $\beta^2 = V_f$ *[41]*

Hence, equation *[35]* can be rewritten as (10):-

$$K_{23}^* = K_m \left\{\frac{\phi(1+2\nu_m V_f) + 2\nu_m V_m}{\phi V_m + (V_f+2\nu_m)}\right\} \qquad [42]$$

Hashin (7,109,110), and also Christensen (13), presents equations for the bounds of the bulk modulus and shear modulus of a composite that is statistically transversely isotropic and the constituent materials are assumed to be linear and isotropic. The fibres are assumed to be solid and parallel and so long that end effects can be neglected. The bounds for K_{23} for this case are given below (13,109):-

$$K_{23-}^* = K_m + V_f /\{1/(K_f-K_m)+V_m(K_m+G_m)\} \qquad [43]$$

$$K_{23+}^* = K_f + V_m /\{1/(K_m-K_f)+V_f(K_f+G_f)\} \qquad [44]$$

However, there is a discrepancy between the papers presented by Hashin. In reference (7) the last term in equation *[43]* is $V_f(K_m+G_m)$, and in reference (110) the last term in equation *[44]* is $V_m(K_f+G_f)$.

A.2.2 Shear Modulus

The equations given by Hashin and Rosen (10) for the bounds on G_{23} are very unwieldy and will not be repeated here. The bounds of G_{12} are as follows for a hexagonal array of fibres (10):–

The lower bound is given as:–

$$G_{12-}^{*} = G_m/(v_1/m_G + v_2) \qquad [45]$$

The upper bound is given by:–

$$G_{12+}^{*} = G_m(v_1 m_G + v_2) \qquad [46]$$

From the geometry of the hexagonal array, the fractional volumes:–

$$v_1 = 0.907, \text{ and } v_2 = 1-v_1.$$

The constant m_G is given by (10):–

$$m_G = \frac{\eta(1+\beta^2) + (1-\beta^2)}{\eta(1-\beta^2) + (1+\beta^2)} \qquad [47]$$

Where, β^2 is given by equation *[39]*, also,

$$\eta = G_f/G_m \qquad [48]$$

For a random array of fibres Hashin and Rosen state that the bounds coincide such that:–

$$G_{12}^{*} = G_m \left\{\frac{\eta(1+V_f) + V_m}{\eta V_m + (1+V_f)}\right\} \qquad [49]$$

Comparison of the above random array results with experimental data shows that there is poor agreement for G_{12} (34).

Equations are presented by Hashin (7,45,109,110), and also Christensen (13), for the bounds of the shear modulus of a composite that is transversely isotropic and the constituent materials are assumed to be linear and isotropic:–

$$G_{23-}^{*} = G_m + V_f/\{1/(G_f-G_m)+V_m(K_m+2G_m)/2G_m(K_m+G_m)\} \quad [50]$$

$$G_{23+}^{*} = G_f + V_m/\{1/(G_m-G_f)+V_f(K_f+2G_f)/2G_f(K_f+G_f)\} \quad [51]$$

Also,

$$G_{12-}^{*} = G_m + V_f/\{1/(G_f-G_m)+V_m/2G_m\} \quad [52]$$

$$G_{12+}^{*} = G_f + V_m/\{1/(G_m-G_f)+V_f/2G_f\} \quad [53]$$

A.2.3 Young's Modulus

The bounds of E_{11} coincide, and are given by Hashin and Rosen (10) to be as follows for a random array of fibres:–

$$E_{11}^{*} = m_E\, E_m \quad [54]$$

Where,

$$m_E = \{(V_fE_f/E_m)+V_m\} \frac{\{E_m(D_1-D_3F_1)+E_f(D_2-D_4F_2)}{E_m(D_1-D_3) + E_f(D_2-D_4)\}} \quad [55]$$

and,

$$D_1 = 1 - \nu_f \quad [56]$$

$$D_2 = \{(1+V_f)/V_m\} + \nu_m \quad [57]$$

$$D_3 = 2\nu_f^2 \quad [58]$$

$$D_4 = 2\nu_m^2V_f/V_m \quad [59]$$

$$F_1 = (\nu_mV_fE_f + \nu_fV_mE_m)/(\nu_fV_fE_f + V_mE_m) \quad [60]$$

$$F_2 = V_fF_1/V_m \quad [61]$$

Hashin and Rosen state that equation *[54]* is unwieldy, and a good approximation for E_{11}^{*} is given by the law of mixtures as in equation *[16]*.

For a hexagonal array of fibres, Hashin and Rosen (10), give the following bounds on E_{11}:–

$$E_{11+}^{*} = E_m(m_E v_1 + p v_2) \quad [62]$$

$$E_{11-}^{*} = E_m/\{(v_1/m_E)+v_2) \quad [63]$$

Where,

$$p = (1-\nu_m-4\nu_m\nu_{12}^{*} + 2\nu_{12}^{*2})/(1-\nu_m-2\nu_m^2) \quad [64]$$

and, ν_{12}^{*} is given by equation *[65]*

Also, V_f in equations *[55]* and *[65]* is replaced by V_f/v_1,

V_m in equations *[55]* and *[65]* is replaced by $\{1-(V_f/v_1)\}$

A.2.4 Poisson's Ratio

The bounds for a random array of fibres for ν_{12}^* coincide and are given by (10):-

$$\nu_{12}^* = (V_f E_f L_1 + V_m E_m L_2 V_m)/(V_f E_f L_3 + V_m E_m L_2) \qquad [65]$$

Where,

$$L_1 = 2\nu_f(1-\nu_m^2)V_f + V_m(1+\nu_m)\nu_m \qquad [66]$$

$$L_2 = V_f\{1 - \nu_f - 2\nu_f^2\} \qquad [67]$$

$$L_3 = 2(1-\nu_m^2)V_f + (1-\nu_m)V_m \qquad [68]$$

Hashin and Rosen (10) state that the bounds of ν_{12} for a hexagonal array of fibres cannot be obtained directly and that those obtained from the method presented in their paper "may be of little practical value" and hence they are not quoted here.

A.2.5 The CCA Model

Various references (6,7,26,42,45,57,59,110,112,117) give equations relating the constituent material properties to the effective material properties of the composite based on the analysis of the CCA model. These equations are given below.

G_{12} is given by equation *[52]*.

$$\nu_{12} = \nu_m V_m + \nu_f V_f + \frac{(\nu_f - \nu_m)(1/K_m - 1/K_f)V_m V_f}{(V_m/K_f) + (V_f/K_m) + (1/G_m)} \qquad [69]$$

$$E_{11} = E_m V_m + E_f V_f + \frac{4(\nu_f - \nu_m)^2 V_m V_f}{(V_m/K_f) + (V_f/K_m) + (1/G_m)} \qquad [70]$$

$$K_{23} = \frac{K_m(K_f+G_m)V_m + K_f(K_m+G_m)V_f}{(K_f+G_m)V_m + (K_m+G_m)V_f} \qquad [71]$$

It is noted by Hashin (6,4,110) and other authors (29,42,45) that when the fibres are considerably stiffer than the matrix material, as is usually the case, then the third term in equations *[69]* and *[70]* becomes negligible leaving the equations as the law of mixtures results. Various authors (13,29,42,110,112) state that equation *[70]* can be used to determine the bounds of E_{11}. The lower bound for E_{11} can be determined from the equation as it appears here, and the upper bound determined by simply exchanging the constituent terms in this equation. Similarly a number of authors (13,29,110) also state that equation *[69]* can be used to determine the bounds of ν_{12}. The lower bound for ν_{12} can be determined from the equation as it appears here, and the upper bound determined by simply exchanging the constituent terms in this equation.

The transverse shear modulus is also given for the CCA model by a number of authors (6,7,26,59,117) for when the fibre shear modulus is greater than the matrix shear modulus:–

$$G_{23} = G_m \left\{1 + \frac{(1 + \beta_m)V_f}{A - V_f[1+(3\beta_m^2 V_m^2)/(BV_f^3 +1)]}\right\} \qquad [72]$$

Where :–

$$B = (\beta_m - \omega\beta_f) / (1 + \omega\beta_f) \qquad [73]$$

$$A = (\omega + \beta_m)/(\omega - 1) \qquad [74]$$

$$\omega = G_f/G_m \qquad [75]$$

$$\beta_m = 1 / (3 - 4\nu_m) \qquad [76]$$

$$\beta_f = 1 / (3 - 4\nu_f) \qquad [77]$$

Rosen and Hashin (6) give a further equation for the case when the fibre shear modulus and bulk modulus are less than the values of the matrix moduli, but as this is not the case for any of the materials considered here the equation has not been quoted.

In one paper Whitney (33) gives the following expression for E_{22} using the CCA model:–

$$E_{22} = 2K(1-\nu_{12})E_{11} / (E_{11} + 4K\nu_{12}^2) \qquad [78]$$

Where,

$$K = \{K_m(K_f+G_m)+(K_f-K_m)G_mV_f\}/\{(K_f+G_m)+(K_f-K_m)V_f\} \qquad [79]$$

Alternative transverse definitions are presented by Hashin (45) for a long fibre reinforced cylinder. These equations assume that both of the constituent materials are transversely isotropic where the fibres are carbon or graphite. However, these Hashin equations for a random distribution of uni−directional fibres are also given for when the fibres are assumed to be isotropic (6,7,10,26,54,59,76,117):−

$$E_{22} = 4KG_{23}/(K+MG_{23}) \qquad [80]$$

$$\nu_{23} = (K-MG_{23})/(K+MG_{23}) \qquad [81]$$

Where,

$$M = 1 + 4K\nu_{12}^2/E_{11} \qquad [82]$$

and,

$$K = \frac{\{K_f(1+2\nu_m V_f)+K_m(1-V_f)2\nu_m\}K_m}{K_f(1-V_f)+K_m(V_f+2\nu_m)} \qquad [83]$$

Hashin (7,45) and Whitney and McCullough (29) state that equation *[80]* allows the bounds of E_{22} to be determined from the bounds of the other properties. Hashin (45) also reports that equation *[81]* can be used to determine the bounds of ν_{23} from the bounds of the other properties.

According to Tsai and Hahn (54) and Hahn (76) all of the relevant composite moduli can be obtained from just one equation which is a simplification of the equations derived by them from the CCA model. Hahn's equation is as follows:−

$$P = (V_f P_f + \eta V_m P_m) / (V_f + \eta V_m) \qquad [84],$$

and the value of E_{22} can be obtained from the CCA model, ie equation *[80]*. The terms used in the above equation can be found from Table 8 below:−

Modulus	P	P_f	P_m	η
E_{11}	E_{11}	E_f	E_m	1
ν_{12}	ν_{12}	ν_f	ν_m	1
G_{12}	$1/G_{12}$	$1/G_f$	$1/G_m$	η_6
G_{23}	$1/G_{23}$	$1/G_f$	$1/G_m$	η_4
K_{23}	$1/K_{23}$	$1/K_f$	$1/K_m$	η_k

Table 8: Constants For Hahn's Equation (76).

The other terms given in Table 5 are given by the following:–

$$\eta_6 = (1 + G_m/G_f)/2 \qquad [85]$$

$$\eta_4 = \{3 - 4\nu_m + (G_m/G_f)\}/4(1-\nu_m) \qquad [86]$$

and,
$$\eta_k = \{1 + (1 - 2\nu_f)(G_m/G_f)\}/2(1 - \nu_m) \qquad [87]$$

It should be noted that for E_{11} and for ν_{12} the equation reduces to the law of mixtures.

A.3 Self Consistent Method

The self consistent model is also used by Whitney and McCullough (29) to derive the following expressions which are similar in form to those derived above from the CCA model:–

$$K_{23} = \{K_m(K_f+G_m)+(K_f-K_m)G_mV_f\}/\{(K_f+G_m)-(K_f-K_m)V_f\} \qquad [88]$$

$$G_{12} = G_m\{(G_f+G_m)+(G_f-G_m)V_f\}/\{(G_f+G_m)-(G_f-G_m)V_f\} \qquad [89]$$

$$G_{23} = G_m \frac{\{K_m(G_m+G_f)+2G_fG_m+K_m(G_f-G_m)V_f\}}{\{K_m(G_m+G_f)+2G_fG_m-(K_m+2G_m)(G_f-G_m)V_f\}} \qquad [90]$$

$$\nu_{23} = (2E_{11}K_{23}-E_{11}E_{22}-4\nu_{12}^2K_{23}E_{22})/2E_{11}K_{23} \qquad [91]$$

and, E_{22} is given by equations *[80]* and *[82]*.

Whitney and McCullough also give equation *[70]* for the longitudinal Young's modulus E_{11}, and equation *[69]* for the Poisson's ratio, although in slightly different forms. Note that the above expression for K_{23} is very similar to that given by equation *[79]*.

Whitney (33) also gives equation *[89]* for G_{12}, but states that this equation gives results that "seem low" when compared with experimental values. Whitney thus multiplies the results from equation *[89]* by a factor of 1.4 to "give closer agreement with experimental data". Equation *[91]* is used by Whitney and McCullough (29) to determine the bounds of ν_{23} from the bounds of the other properties.

For fibres packed in a random array Whitney and McCullough (29) also use equation *[88]* to determine the lower bound of the bulk modulus K_{23}. The upper bound can then be found by exchanging the constituent terms in this equation. Similarly, the lower bounds of G_{12} and G_{23} can be found from equations *[89]* and *[90]* as they are presented here, and the upper bounds by exchanging the constituent terms in these equations.

A.4 Semi–Empirical Relationships

Semi–empirical equations are given by some researchers (17,280) for E_{22} and G_{12} such that:–

$$E_{22} = \{E_m/(1-\nu_m^2)\}(1+0.85V_f^2)/\{(1-V_f)^{1.25}+\{E_mV_f/E_f(1-\nu_m^2)\}\} \quad [92]$$

$$G_{12} = \{E_m/2(1+\nu_m)\}(1+0.6V_f^{0.5})/\{(1-V_f)^{1.25}+\{V_fE_f(1+\nu_f)/E_m(1+\nu_m)\}\} \quad [93]$$

Note that equation *[93]* for G_{12} gives a result that gets progressively smaller as V_f increases. As this is not what is expected from a uni–directional glass reinforced composite material then this equation will not be considered further. It should be noted that the terms $(1+0.85V_f^2)$ and $(1+0.6V_f^{0.5})$ are 'correction factors' obtained by comparing the equations with experimental results (280).

The following equations are presented by Noga and Woodhams (281) for the determination of the moduli of a uni–directional composite material. The longitudinal Young's modulus is obtained by comparing the law of mixtures equation with the experimental data of Lees (282) where for glass fibres:–

$$E_{11} = E_fV_f(1.31+0.31\log_{10}(E_m/E_f))^2 + E_mV_m \quad [94]$$

The other mechanical properties are found from the following relationships:–

$$E_{22} = \frac{2\{1-\nu_f+V_m(\nu_f-\nu_m)\}\{M_f(2M_m+G_m)-G_mV_m(M_f-M_m)\}}{\{2(M_m+G_m)+2V_m(M_f-M_m)\}} \quad [95]$$

$$\nu_{12} = \frac{M_fV_f\nu_f(2M_m+G_m)+M_mV_m\nu_m(2M_f+G_m)}{M_f(2M_m+G_m)-G_mV_m(M_f-M_m)} \quad [96]$$

Where, 'M' is the 'areal modulus' and given by:–

$$M_m = E_m / 2(1-\nu_m) \quad [97]$$

$$M_f = E_f / 2(1-\nu_f) \quad [98]$$

Noga and Woodhams also give equation *[52]* for the in plane shear modulus.

A.4.1 Elasticity Solutions With Contiguity

Equations for moduli are presented by Jones (28) and Smith (61) which attempt to take account of the random packing of the fibres in the matrix. The equation for E_{11} is a modified version of the law of mixtures to take account of imperfections in fibre alignment and is given below:–

$$E_{11} = Z(V_fE_f + V_mE_m) \quad [99]$$

Where 'Z' is a 'fibre misalignment factor' which varies between 0.9 and 1.0, is experimentally determined, and highly dependent upon the manufacturing process.

The transverse modulus is given by (28,61):–

$$E_{22} = 2[1-\nu_f+(\nu_f-\nu_m)V_m]\{(1-C)\frac{[K_f(2K_m+G_m)-G_m(K_f-K_m)V_m]}{(2K_m+G_m)+2(K_f-K_m)V_m} + C\frac{[K_f(2K_m+G_f)+G_f(K_m-K_f)V_m]\}}{(2K_m+G_f)-2(K_m-K_f)V_m} \quad [100]$$

The term 'C' denotes the degree of contiguity, and C=0 corresponds to no contiguity and C=1 corresponds to perfect contiguity, ie when C = 0, then the fibres do not touch, when C = 1, then all of the fibres are in contact (28). With a high fibre volume fraction C would be expected to approach C=1. Jones (28) states that this approach could be classified as either a semi–empirical technique or as a bounding technique, as the contiguity factor is a so called 'fudge factor' determined from the comparison of experimental data with theoretical predictions. The value of 'C' is stated to usually be low.

Similar equations are given for the Poisson's ratio and shear modulus as follows (28,61):–

$$\nu_{12} = (1-C)\frac{[K_f\nu_f(2K_m+G_m)V_f + K_m\nu_m(2K_f+G_m)V_m]}{K_f(2K_m+G_m) - G_m(K_f - K_m)V_m} + C\frac{[K_m\nu_m(2K_f+G_f)V_m + K_f\nu_f(2K_m+G_f)V_f]}{K_f(2K_m+G_m) + G_f(K_m-K_f)V_m} \qquad [101]$$

$$G_{12} = (1-C)G_m\frac{\{2G_f - (G_f-G_m)V_m\}}{2G_m + (G_f-G_m)V_m} + CG_f\frac{\{G_f+G_m)-(G_f-G_m)V_m\}}{(G_f+G_m)+(G_f-G_m)V_m} \qquad [102]$$

Equations *[100]*, *[101]* and *[102]* are also presented by Nielsen and Chen (200). They assume that the fibres do not touch and that there exists perfect bonding between the fibres and the matrix, ie that the value of 'C' in the equations is set to zero.

A.4.2 The Halpin Tsai Semi–Empirical Relationship

The Halpin Tsai relationship for G_{12} of a continuous uni–directional fibre reinforced composite is (12,23,28,29,34,40,61,75,94,105,134,135):–

$$G_{12} = G_m(1+\xi\eta V_f)/(1-\eta V_f) \qquad [103]$$

$$\eta = (G_f - G_m)/(G_f + \xi G_m\} \qquad [104]$$

Where, the value of the geometrical factor, $\xi = 1.0$. A modification to the geometrical factor is given by Hewitt and Malherbe (135) to be:–

$$\xi = 1 + 40V_f^{10} \qquad [105]$$

Figure 45 shows the performance of the Halpin Tsai relationship using $\xi=1.0$ and equation *[105]* against experimental data (135).

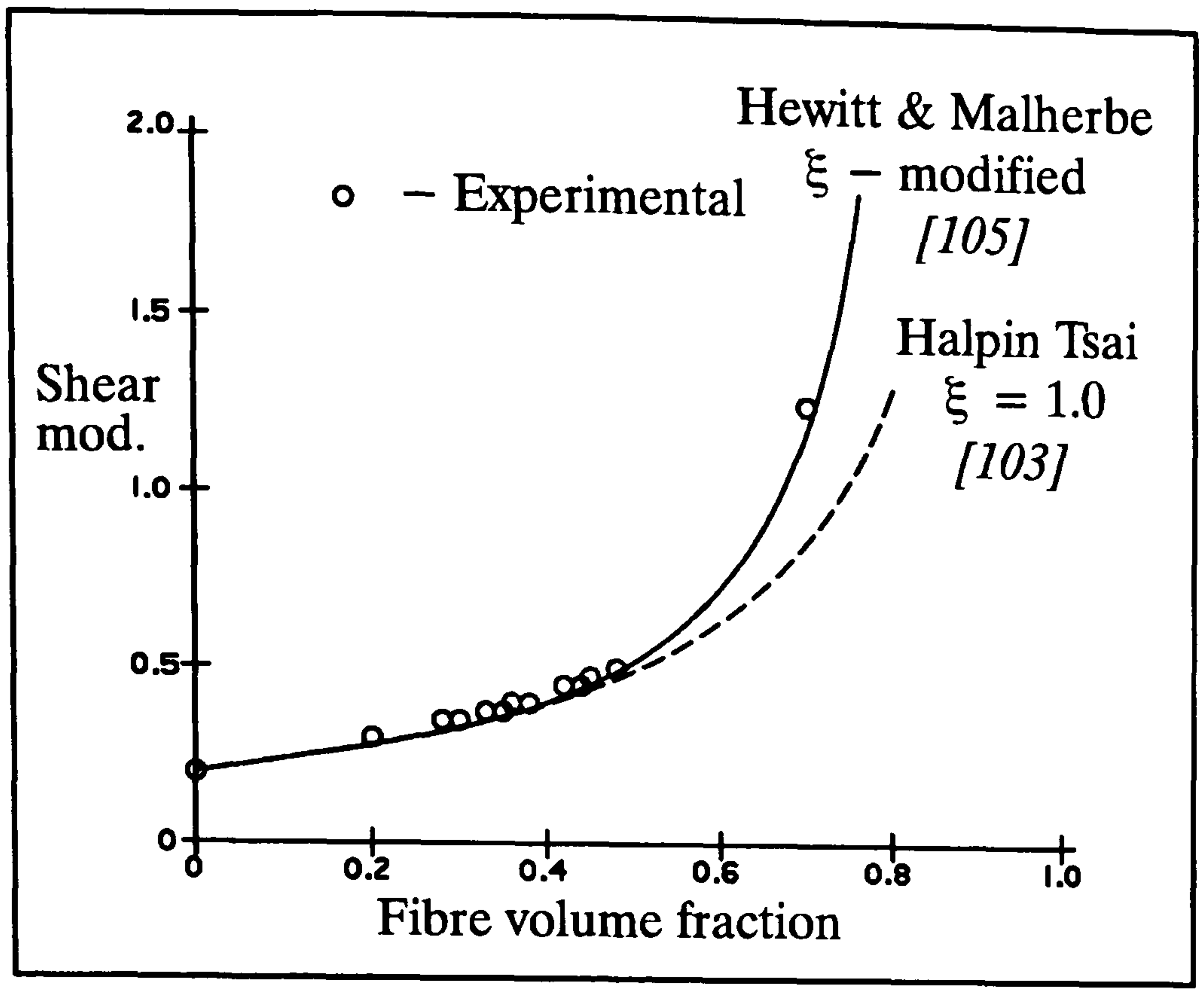

Figure 45 : G_{12} Calculated From Equations [103] And [105] For An E–Glass Polyester Resin. (135)

The Spencer (136) semi–empirical equations based on the Halpin Tsai relationship introduce a "fibre separation parameter" δ which is related to V_f by :–

$$\delta = 1 / \{(1.1V_f^2 - 2.1V_f + 2.2)V_f\}^{0.5} \qquad [106]$$

Note that the terminology used by Spencer has been altered in some instances to make it consistent with the other equations presented in this work.

The shear modulus as given by Spencer is (136):–

$$\frac{G_{12}}{G_m} = \left\{\frac{(\delta-1)}{\delta}\right\} + \left\{\frac{1}{(1-G_m/G_f)}\right\}\left\{(-\pi/2) + \frac{2\delta \tan^{-1}[(\delta+[1-G_m/G_f])/(\delta-[1-G_m/G_f])]^{0.5}}{[\delta^2-[1-G_m/G_f]^2]^{0.5}}\right\} \qquad [107]$$

Spencer reports that the above formula finds "good agreement with experimental values" however no numerical evidence of this is presented in the paper. A comparison of the above formulae with equation *[103]* where the constant ξ is equal to 1.0 is shown in Figure 46.

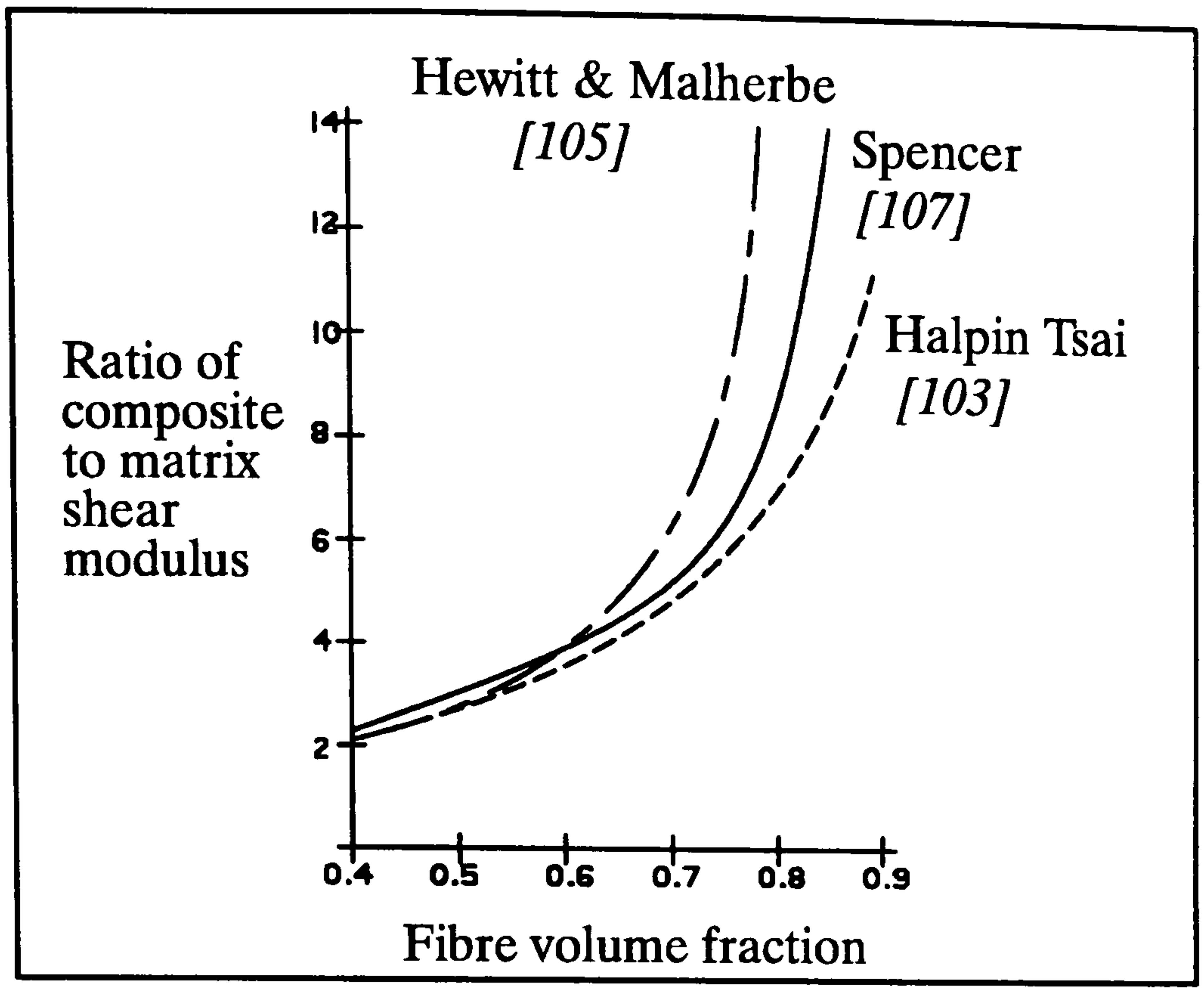

Figure 46 : A Comparison Of Equations [103], [105] And [107] (136).

There appears to be good correlation between experimental data and Hewitt and Malherbe's modified equation for G_{12} from Figure 45. In Figure 46 it can be seen that the Spencer equation gives lower values than the Hewitt and Malherbe one. Thus, it would appear that the former will not determine G_{12} as accurately as the latter. However, when the Spencer equation as presented is used it gives larger than expected values for G_{12} which suggests that there is an error in the paper that the equation was obtained from – see Chapter 4.

The Halpin Tsai equation can also be used to predict E_{22} where the geometrical factor is taken to be 2.0. The equation is given as (12,23,28,29,34, 40,75,94,105,134,135):–

$$E_{22} = E_m(1+\xi\eta V_f)/(1-\eta V_f), \qquad [108]$$

with, $$\eta = (E_f - E_m)/(E_f + \xi E_m). \qquad [109]$$

A comparison of results obtained from the above equation with experimental data is shown in Figure 47.

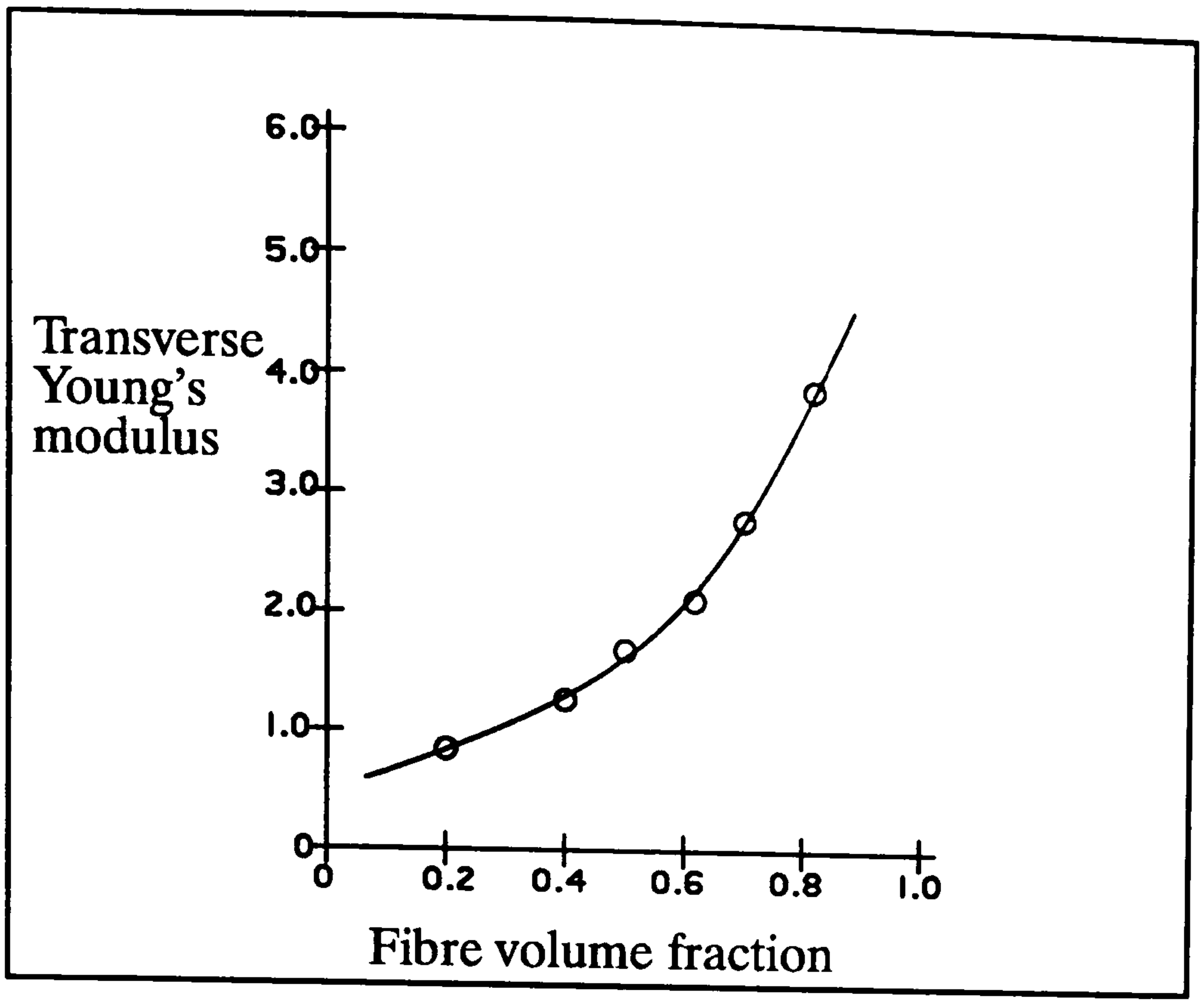

Figure 47 : E_{22} Calculated From The Halpin Tsai Equation (28)

A modification to the geometrical factor for E_{22} is said by Halpin (34) to give better results above $V_f = 0.65$. This geometrical factor is:–

$$\xi = 2 + 40V_f^{10} \qquad [110]$$

A similar modification to the Halpin Tsai equation for E_{22} as that given for G_{12} is presented by Spencer (136) and is reported to give good agreement with experimental values. The Spencer equation is:–

$$\frac{E_{22}}{E_m} = \left\{\frac{(\delta-1)}{\delta}\right\} + \left\{\frac{1}{(1-E_m/E_f)}\right\}\left\{(-\pi/2) + \frac{2\delta \tan^{-1}[(\delta+[1-E_m/E_f])/(\delta-[1-E_m/E_f])]^{0.5}}{[\delta^2-[1-E_m/E_f]^2]^{0.5}}\right\} \qquad [111]$$

The value of the fibre separation parameter δ is given by equation *[106]*. A comparison between the modified Spencer equation and the original equation is shown in Figure 48. Once again the Spencer equation gives different results to the Halpin Tsai equation which appears to fit the available experimental data (see Figure 47). This suggests that the Spencer equation is less accurate than equation *[108]*. Also, when the Spencer equation is used it gives larger than expected answers suggesting that there may be an error in the paper that the equation was obtained from. Thus the Spencer equation will not be used in any further work.

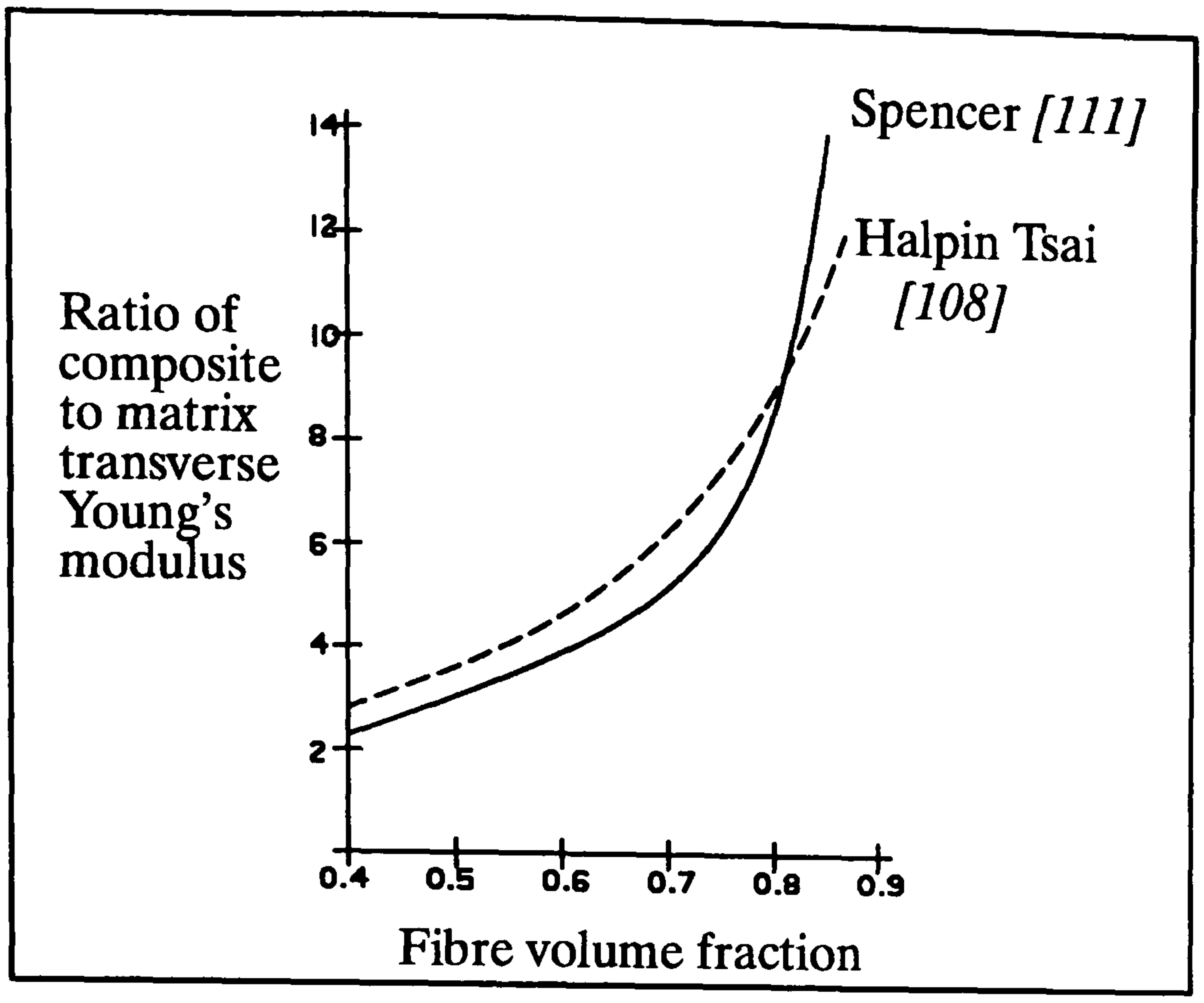

Figure 48 : A Comparison Of Equations [108] And [111] (136).

The Halpin Tsai relationship for G_{23} is given by Halpin (34) as:–

$$G_{23} = G_m(1+\xi\eta V_f)/(1-\eta V_f) \qquad [112]$$

$$\eta = (G_f - G_m)/(G_f + \xi G_m\} \qquad [113]$$

Where, $\xi = 1/(4-3\nu_m)$ *[114]*

This version of the Halpin Tsai equation is also given by Smith (61). However, Smith gives the geometrical factor in a fuller form such that:–

$$\xi = (K_m/G_m)/\{(K_m/G_m)+2\}, \qquad [115]$$

where, K_m is defined by equation *[7]*. The equation for the geometrical factor thus reduces to :–

$$\xi = 1/(3-4\nu_m) \qquad [116]$$

This is the same definition as given by other authors (23,29). Thus, it would appear that there is an error in the reference by Halpin, and therefore the form of the geometrical factor given by equation *[114]* will not be used further.

Halpin (34) also gives ν_{23} as:–

$$\nu_{23} = 1 - E_{22}/G_{23} \qquad [117]$$

Whitney and McCullough (29) also use the Halpin Tsai relationship determine values of the plane strain bulk modulus K_{23} from:–

$$K_{23} = K_m(1+\xi\eta V_f)/(1-\eta V_f) \qquad [118]$$

$$\eta = (K_f - K_m)/(K_f + \xi K_m\} \qquad [119]$$

Where, $\xi = (1-2\nu_m)$ *[120]*

The Halpin Tsai equations are also modified by Nielsen (20,201) to take into consideration the packing geometry of the fibres. The Nielsen equations introduce a term 'ϕ_m' representing the 'maximum packing fraction'. Nielsen modifies equations *[103]*, *[108]* and *[112]* to those given below:–

Transverse Young's modulus,

$$E_{22}/E_m = (1+ABV_f)/(1-B\Psi V_f), \qquad [121]$$

$$A = 0.5,$$

$$B = \{(E_f/E_m)-1\}/\{(E_f/E_m)+A\}, \qquad [122]$$

and, $\Psi = 1 + V_f\{(1-\phi_m)/\phi_m^2\}$. *[123]*

In plane shear modulus,

$$G_{12}/G_m = (1+ABV_f)/(1-B\Psi V_f), \qquad [124]$$

$$A = 1.0,$$

B = as above with 'G' replacing 'E',

and, Ψ = as above.

Transverse shear modulus,

$$G_{23}/G_m = (1+ABV_f)/(1-B\Psi V_f), \qquad [125]$$

$$A = 0.5,$$

B = as above with 'G' replacing 'E',

and, Ψ = as above.

The value of 'ϕ_m' in the above equations is given by Nielsen as 0.785 for a cubic array of fibres, 0.907 for a hexagonal array, and 0.82 for a random array. Thus, knowing which value of ϕ_m to use means that an assumption needs to be made by the design engineer about the packing geometry of the fibres. This will probably be beyond the designers area of knowledge. A random array of fibres is probably what exists in a real composite and thus a value of $\phi_m = 0.82$ has been used in all of the calculations made here.

A.5 Transformation Properties

The mechanical properties of a composite material at an angle other than the fibre direction, or transverse to the fibres may be obtained from the following equations given by a number of researchers (20,25,30,183,200,281) for a uni-directional composite:-

$$1/E_\theta = n^4/E_{11} + (1/G_{12} - 2\nu_{21}/E_{11})m^2n^2 + m^4/E_{22} \qquad [126]$$

$$1/G_\theta = 1/G_{12} + 4\{(1+2\nu_{12})/E_{11} + 1/E_{22} - 1/G_{12}\}m^2n^2 \qquad [127]$$

$$\nu_\theta = E_{11}\{\nu_{12}/E_{11} - ((1+2\nu_{12})/E_{11} + 1/E_{22} - 1/G_{12})m^2n^2\} \qquad [128]$$

Where, $m = \sin\theta$, and $n = \cos\theta$. The angle θ is the angle of the fibres measured from the longitudinal axis. Thus, when the angle in the above equations is taken to be 0° the properties calculated should be the longitudinal ones, and when the angle is taken to be 90° the properties should be those in the transverse direction.

A.6 Comparison Of Equations Against Each Other

The composite materials studied in this work consist of an E-glass reinforcement together with a matrix of either epoxy or polyester resin. The mechanical properties of these constituent materials are given in the following sections. The units used for the Young's modulus, shear modulus and the plane strain bulk modulus are GPa., whilst the units used for the density are Kg/mm^3.

A.6.1 Constituent Properties

The properties given by various references for E-glass are summarised in the table below.

Reference	Young's Modulus GPa.	Poisson's Ratio	Density Kg/mm3	Shear Modulus Gpa.
(30)	75			
(46)	73	0.22	2.60E-06	
(51,61,88,105)	72	0.2		
(80)	71	0.22		
(82)	72	0.22		
(90)	76	0.21		
(117)	72.4			
(205)	72	0.3		27.7
(283)	72			

Table 9 : E–Glass Properties

From Table 9 it can be seen that for E–glass the average quoted Young's modulus is 73GPa., whilst the Poisson's ratio is given as 0.22. In Chapter 4 many authors were quoted as stating that the composite constituents could be considered as isotropic. Thus, if E–glass is considered to be isotropic then equation *[4]* can be used to calculate the shear modulus. The plane strain bulk modulus K of the fibres can be calculated from equation *[7]*.

The properties quoted below in Table 10 by the various references for epoxy resin show quite a variation. The shear modulus has been calculated from equation *[4]* for both the epoxy and polyester resins.

Reference	Young's Modulus GPa.	Poisson's Ratio	Density Kg/mm3	Shear Modulus GPa.
(46,88)	3.45	0.35		1.3
(51,61,105)	3	0.37		
(80)	3.5	0.38		
(82)	3.13-3.6	0.3-0.34		
(90)	3.45	0.334		
(117)	4.1	0.35		
(205)	3.5	0.35		1.3
(217)	4	0.34		
(284)	3.53-4.17	0.373-0.447		
Experimental	3	0.415	1.14E-06	1.06

Table 10 : Epoxy Resin Properties

The properties quoted below in Table 11 by the various references for polyester resin also show quite a variation.

Reference	Young's Modulus Gpa	Poisson's Ratio	Density Kg/mm3	Shear Modulus Gpa.
(51,61,105)	3.2-3.6	0.36		
(90)	3.5	0.35		
(92)	4.1	0.36		1.5
Experimental	3.39	0.36	1.10E-06	1.177

Table 11 : Polyester Resin Properties

A.6.2 Mechanical Property Calculations

In all of the following calculations it is assumed that the composite is an E–glass / epoxy resin composite. The properties of the constituent materials are assumed to be as follows:–

E–Glass – $E = 73$GPa., $\nu = 0.22$, $G = 29.92$GPa., $K = 53.43$GPa.

Epoxy – $E = 3.0$GPa., $\nu = 0.415$, $G = 1.06$GPa., $K = 6.23$GPa.

The graphs of the properties calculated from each equation against the fibre volume fraction can be seen in Chapter 4. The following two sections identify some of the equations used in the calculation of E_{22} and ν_{23}.

It is worth noting that when the value of 'K' is inserted into equations presented here to determine the properties of continuous uni–directional fibre reinforced composites, it is observed that in most cases there is no difference in the end result whether the expression for the bulk modulus – equation *[5]* – or the plane strain bulk modulus – equation *[7]* – is used. The only equation in which the different values of 'K' are observed to make a difference is in equation *[51]* for the transverse shear modulus. The two definitions of 'K' when used in this equation make approximately a 5% difference in the end result.

A.6.2.1 Transverse Young's Modulus

In equation *[78]* E_{11} is determined by equation *[70]*, ν_{12} by equation *[69]* and K by equation *[79]*. Note also that for equation *[80]* the value of G_{23} is determined by equation *[72]*, E_{11} by equation *[70]*, and ν_{12} by equation *[69]*. Note that the

Nielsen (20) modification to the Halpin Tsai equation – equation *[121]* gives negative values at a fibre volume fraction above 0.8.

A.6.2.2 Transverse Poisson's Ratio

In equation *[32]* in the expression for ϕ the values of E_{11} and ν_{12} determined by the law of mixtures relationships *[16]* and *[17]*. Note also that in equation *[81]* the value of G_{23} is determined by equation *[72]*, E_{11} by equation *[70]*, and ν_{12} by equation *[69]*. For equation *[91]* the value of E_{11} is determined by equation *[70]*, E_{22} by equation *[80]*, ν_{12} by equation *[69]*, and K_{23} by equation *[88]*.

A.6.3 Comparison Of Equations With Experimental Data

The experimental properties obtained from the literature are for an E–glass / epoxy composite (in GPa.), and are as follows:–

Reference	E11	E22	v12	G12	G23	Vf
(284)	50	15	0.227	4		0.7
(283)	45	12	0.28	5.5		0.6
(117)	38.3	12.2	0.266	4.07	4.48	0.5
(250)	35.6					0.46
(54,138,142)	38.6	8.27	0.26	4.14		0.45
(88)	26.76	6.36	0.299	2.44		0.34
(88)	21.27	5.53	0.311	2.08		0.26

Table 12 : E–Glass/Epoxy Experimental Data

The properties of the constituent materials used in the calculations are assumed to be the average of those given in the literature and are as follows. Note that K has been calculated by equation *[7]*.

E–Glass – E = 73GPa., ν = 0.22, G = 29.92GPa., K = 53.43GPa.

Epoxy – E = 3.45GPa., ν = 0.35, G = 1.28GPa., K = 4.2GPa.

Reference	E11	E22	v12	G12	Vf
(26)	47.5	15.9	0.27	6.23	0.6
(61,105)	30				0.43

Table 13 : E–Glass/Polyester Experimental Data

The properties of the polyester resin used in the calculations are as follows: –

Polyester – E = 3.5GPa., ν = 0.36, G = 1.29GPa., K = 4.6GPa.

Note that the properties are the average of those given by different researchers.

The average error shown in the following sections is obtained by comparing the results of the calculations with the experimental values given in the above table. Note that the relevant resin property values given above have been used in the calculations.

			Fibre Volume Fraction					
Eqn. No.	0.26	0.34	0.45	0.46	0.5	0.6	0.7	Ave. Error
16	21.53	27.09	34.7	35.44	38.5	45.18	52.1	2.60%
94	17.9	22.3	28.45	29	31.2	36.8	42.3	19%
99	20.58	25.74	32.96	33.67	36.3	42.92	49.5	5.40%
Exp.	21.27	26.76	38.6	35.6	38.3	45	50	

Table 14 : Calculated Longitudinal Young's Modulus E–Glass/Epoxy

	Fibre Volume Fraction		
Eqn. No.	0.43	0.6	Ave. Error
16	33.38	45.2	8%
94	27.5	36.96	15.30%
99	34.1	45.7	8.70%
Exp.	30	47.5	

Table 15 : Calculated Longitudinal Young's Modulus E–Glass/Polyester

			Fibre Volume Fraction				
Eqn. No.	0.26	0.34	0.45	0.5	0.6	0.7	Av. Error
17	0.316	0.306	0.29	0.285	0.272	0.26	6.70%
Exp.	0.311	0.299	0.26	0.266	0.28	0.227	

Table 16 : Calculated Longitudinal Poisson's Ratio E–Glass/Epoxy

	Fibre Volume Fraction	
Eqn. No.	0.6	Av. Error
17	0.276	2.20%
Exp.	0.27	

Table 17 : Calculated Longitudinal Poisson's Ratio E–Glass/Polyester

Eqn. No.	0.26	0.34	0.45	0.5	0.6	0.7	Ave. Error
	Fibre Volume Fraction						
24	1.7	1.9	2.24	2.45	3	3.88	31%
29	2.5	2.89	3.57	3.96	4.95	6.42	13%
45	2.06	2.39	2.96	3.28	4.07	5.17	12%
46	2.11	2.5	3.22	3.66	4.87	6.9	6%
53	5.78	7.41	9.9	11.1	13.9	17	177%
103	2.08	2.44	3.08	3.45	4.42	5.9	8.70%
105	2.08	2.44	3.09	3.48	4.72	7.6	7%
124	2.13	2.55	3.36	3.89	5.5	-ve	3%
Exp.	2.44	2.44	4.14	4.07	5.5	4	
Note: The average error is calculated without Vf = 0.45 and Vf = 0.7							

Table 18 : Calculated Longitudinal Shear Modulus E–Glass/Epoxy

Eqn. No.	0.43	0.6	Ave. Error
	Fibre Volume Fraction		
24	2.19	3.03	44.40%
29	3.46	4.98	10.60%
45	2.86	4.1	26.20%
46	3.09	4.9	16.50%
53	9.43	13.9	146%
103	2.97	4.45	21.80%
105	2.98	4.75	19.30%
124	3.21	5.52	9.80%
Exp.	3.5	6.23	

Table 19 : Calculated Longitudinal Shear Modulus E–Glass/Polyester

Eqn. No.	0.26	0.34	0.45	0.5	0.6	0.7	Ave. Error
	Fibre Volume Fraction						
21	4.58	5.1	6.04	7.76	8.05	10.3	27.40%
27	6.86	7.61	8.94	11.4	11.76	14.9	10.20%
30	6.7	7.76	9.56	12.3	13.2	17	13.50%
33/34	4.87	5.51	6.56	7.14	8.5	10.3	24.60%
80	6.02	7.18	9.71	10.82	13.4	20	15.80%
92	5.93	7.04	9.25	10.6	14.6	21.5	18%
95	4.97	5.68	6.95	7.69	9.62	12.5	18.40%
100	7.8	8.84	10.71	11.8	14.6	18.8	26.60%
108	6.48	7.8	10.1	11.4	14.76	19.58	20.30%
110	6.47	7.8	10.1	11.5	15.4	23.2	25%
121	5.21	6.1	7.86	9	12.5	-ve	9.10%
Exp.	5.53	6.36	8.27	12.2	12	15	

Table 20 : Calculated Transverse Young's Modulus E–Glass/Epoxy

Results calculated from a modified equation *[30]* with the geometrical factor equal to 1.0 are as follows:–

$V_f =$	0.26	0.34	0.45	0.5	0.6	0.7
$E_{22} =$	5.58	6.53	8.23	9.21	11.74	15.5

This modified equation gives an average error of 5.7%.

Eqn. No.	Vf = 0.6	Error
21	8.16	48.70%
27	12.2	23.30%
30	13.3	16.30%
33/34	8.6	45.90%
80	14.8	6.90%
92	14.9	6.30%
95	9.8	38.40%
100	15.6	1.90%
108	14.9	6.30%
110	15.6	1.90%
121	12.7	20%
Exp.	15.9	

Table 21 : Calculated Transverse Young's Modulus E–Glass/Polyester

Eqn. No.	Vf = 0.5	Error
22	2.9	35.20%
31	4.62	3.10%
51	8.59	91.70%
72	4.43	1.10%
112	2.81	37%
125	3.37	24.80%
Exp.	4.48	

Table 22 : Calculated Transverse Shear Modulus E–Glass/Epoxy

A.7 Summary Of Equations Considered

The following sections summarise the equations considered in Chapter 6 and presented in this Appendix such that they form a quick reference guide to the reader.

A.7.1 Longitudinal Young's Modulus

$$E_{11} = E_m V_m + E_f V_f \qquad [16]$$

$$E_{11} = V_f E_f + V_m E_m' \qquad [26]$$

$$E_{11}^* = m_E\, E_m \qquad [54]$$

$$E_{11+}^* = E_m(m_E v_1 + p v_2) \qquad [62]$$

$$E_{11-}^* = E_m/\{(v_1/m_E)+v_2) \qquad [63]$$

$$E_{11} = E_m V_m + E_f V_f + \frac{4(\nu_f - \nu_m)^2 V_m V_f}{(V_m/K_f) + (V_f/K_m) + (1/G_m)} \qquad [70]$$

$$E_{11} = E_f V_f (1.31 + 0.31\log_{10}(E_m/E_f))^2 + E_m V_m \qquad [94]$$

$$E_{11} = Z(V_f E_f + V_m E_m) \qquad [99]$$

A.7.2 Longitudinal Poisson's Ratio

$$\nu_{12} = \nu_f\, V_f + \nu_m\, V_m \qquad [17]$$

$$\nu_{12}^* = (V_f E_f L_1 + V_m E_m L_2 V_m)/(V_f E_f L_3 + V_m E_m L_2) \qquad [65]$$

$$\nu_{12} = \nu_m\, V_m + \nu_f\, V_f + \frac{(\nu_f - \nu_m)(1/K_m - 1/K_f) V_m V_f}{(V_m/K_f) + (V_f/K_m) + (1/G_m)} \qquad [69]$$

$$\nu_{12} = \frac{M_f V_f \nu_f (2M_m + G_m) + M_m V_m \nu_m (2M_f + G_m)}{M_f(2M_m + G_m) - G_m V_m (M_f - M_m)} \qquad [96]$$

$$\nu_{12} = (1-C)\frac{[K_f \nu_f (2K_m + G_m) V_f + K_m \nu_m (2K_f + G_m) V_m]}{K_f(2K_m + G_m) - G_m(K_f - K_m) V_m} + C\,\frac{[K_m \nu_m (2K_f + G_f) V_m + K_f \nu_f (2K_m + G_f) V_f]}{K_f(2K_m + G_m) + G_f(K_m - K_f) V_m} \qquad [101]$$

A.7.3 Longitudinal Shear Modulus

$$G_{12} = G_m G_f / (V_m G_f + V_f G_m) \qquad [24]$$

$$G_{12} = G_m / \{1 - V_f^{0.5}(1 - G_m/G_f)\} \qquad [29]$$

$$G_{12-}^{*} = G_m/(v_1/m_G + v_2) \qquad [45]$$

$$G_{12+}^{*} = G_m(v_1 m_G + v_2) \qquad [46]$$

$$G_{12}^{*} = G_m \frac{\{\eta(1+V_f) + V_m}{\eta V_m + (1+V_f)\}} \qquad [49]$$

$$G_{12-}^{*} = G_m + V_f/\{1/(G_f-G_m)+V_m/2G_m\} \qquad [52]$$

$$G_{12+}^{*} = G_f + V_m/\{1/(G_m-G_f)+V_f/2G_f\} \qquad [53]$$

$$P = (V_f P_f + \eta V_m P_m) / (V_f + \eta V_m) \qquad [84]$$

$$G_{12} = G_m\{(G_f+G_m)+(G_f-G_m)V_f\}/\{(G_f+G_m)-(G_f-G_m)V_f\} \qquad [89]$$

$$G_{12} = (1-C)G_m \frac{\{2G_f - (G_f-G_m)V_m\}}{2G_m + (G_f-G_m)V_m} + CG_f \frac{\{G_f+G_m)-(G_f-G_m)V_m\}}{(G_f+G_m)+(G_f-G_m)V_m} \qquad [102]$$

$$G_{12} = G_m(1+\xi\eta V_f)/(1-\eta V_f) \quad \text{with } \xi = 1 \qquad [103]$$

$$\xi = 1 + 40V_f^{10} \qquad [105]$$

$$G_{12}/G_m = (1+ABV_f)/(1-B\Psi V_f), \qquad [124]$$

A.7.4 Transverse Young's Modulus

$$E_{22} = E_f E_m / (V_f E_m + V_m E_f) \qquad [21]$$

$$E_{22} = E_f E_m' / \{V_f E_m' + V_m E_f(1-\nu_m^2)\} \qquad [27]$$

$$E_{22} = E_m / \{1 - V_f^{0.5}(1 - E_m/E_f)\} \qquad [30]$$

$V_f < 0.68$:-

$$E_{22} = E_m \frac{\{1 - (1-E_m/E_f)(0.8247V_f^{0.5} - V_f)\}}{\{1 - 0.8247\, V_f^{0.5}(1 - E_m/E_f)\}} \qquad [33]$$

For $V_f > 0.68$ then:-

$$E_{22} = E_m / \{1-V_f(1-E_m/E_f)\} \qquad [34]$$

$$E_{22} = 2K(1-\nu_{12})E_{11} / (E_{11} + 4K\nu_{12}^2) \qquad [78]$$

$$E_{22} = 4KG_{23}/(K+MG_{23}) \qquad [80]$$

$$E_{22} = \{E_m/(1-\nu_m^2)\}(1+0.85V_f^2)/\{(1-V_f)^{1.25}+\{E_mV_f/E_f(1-\nu_m^2)\}\} \qquad [92]$$

$$E_{22} = 2\{1-\nu_f+V_m(\nu_f-\nu_m)\}\frac{\{M_f(2M_m+G_m)-G_mV_m(M_f-M_m)\}}{\{2(M_m+G_m)+2V_m(M_f-M_m)\}} \qquad [95]$$

$$E_{22} = 2[1-\nu_f+(\nu_f-\nu_m)V_m]\{(1-C)\frac{[K_f(2K_m+G_m)-G_m(K_f-K_m)V_m]}{(2K_m+G_m)+2(K_f-K_m)V_m} + C\frac{[K_f(2K_m+G_f)+G_f(K_m-K_f)V_m]\}}{(2K_m+G_f)-2(K_m-K_f)V_m} \qquad [100]$$

$$E_{22} = E_m(1+\xi\eta V_f)/(1-\eta V_f), \quad \text{with } \xi = 2 \qquad [108]$$

$$\xi = 2 + 40V_f^{10} \qquad [110]$$

$$E_{22}/E_m = (1+ABV_f)/(1-B\Psi V_f), \qquad [121]$$

A.7.5 Transverse Poisson's Ratio

$$\nu_{23} = \nu_f\nu_m / (V_f\nu_m + V_m\nu_f) \qquad [23]$$

$$\nu_{23} = \nu_fV_f + \phi\nu_mV_m, \qquad [32]$$

$$\nu_{23} = (K-MG_{23})/(K+MG_{23}) \qquad [81]$$

$$\nu_{23} = (2E_{11}K_{23}-E_{11}E_{22}-4\nu_{12}^2K_{23}E_{22})/2E_{11}K_{23} \qquad [91]$$

A.7.6 Transverse Shear Modulus

$$G_{23} = G_fG_m / (V_fG_m + V_mG_f) \qquad [22]$$

$$G_{23} = G_m / \{1 - V_f^{0.5}(1 - G_m/G_f)\} \qquad [31]$$

$$G_{23-}^* = G_m + V_f/\{1/(G_f-G_m)+V_m(K_m+2G_m)/2G_m(K_m+G_m)\} \qquad [50]$$

$$G_{23+}^* = G_f + V_m/\{1/(G_m-G_f)+V_f(K_f+2G_f)/2G_f(K_f+G_f)\} \qquad [51]$$

$$G_{23} = G_m\{1 + \frac{(1+\beta_m)V_f}{A - V_f[1+(3\beta_m^2V_m^2)/(BV_f^3+1)]}\} \qquad [72]$$

$$G_{23} = G_m \frac{\{K_m(G_m+G_f)+2G_fG_m+K_m(G_f-G_m)V_f\}}{\{K_m(G_m+G_f)+2G_fG_m-(K_m+2G_m)(G_f-G_m)V_f\}} \qquad [90]$$

$$G_{23} = G_m(1+\xi\eta V_f)/(1-\eta V_f) \qquad [112]$$

$$G_{23}/G_m = (1+ABV_f)/(1-B\Psi V_f), \qquad [125]$$

Appendix B : Randomly Oriented Continuous Fibre Composites

The effective properties for a two dimensional randomly oriented continuous fibre composite are given by the following equations and are reported to compare well with experimental data (13):–

$$E_{2D} = (u_1^2 - u_2^2)/u_1 \qquad [129]$$

$$\nu_{2D} = u_2/u_1 \qquad [130]$$

Where,

$$u_1 = (3/8)E_{11} + G_{12}/2 + \{(3+2\nu_{12}+3\nu_{12}^2)G_{23}K_{23}/2(G_{23}+K_{23})\} \qquad [131]$$

$$u_2 = (1/8)E_{11} - G_{12}/2 + \{(1+6\nu_{12}+\nu_{12}^2)G_{23}K_{23}/2(G_{23}+K_{23})\} \qquad [132]$$

The values of E_{11} etc in the above equations are the same as those values calculated for the uni–directional case.

Chou (26) presents the equations of Cox (19) for a two dimensional random distribution of fibres, but reduces the results of Cox to the simple forms:–

$$E_{2D} = E_f V_f / 3 \qquad [133]$$

$$G_{2D} = E_f V_f / 8 \qquad [134]$$

$$\nu_{2D} = 1/3 \qquad [135]$$

A number of authors (20,61,105,277) give the following equations for the case of two dimensional randomly arranged continuous fibres, where:–

$$E_{2D} = 3E_{11}/8 + 5E_{22}/8 \qquad [136]$$

and, $$G_{2D} = E_{11}/8 + E_{22}/4 \qquad [137].$$

The researchers state that E_{11} may be found from the law of mixtures and E_{22} is found from Nielsen's modification of the Halpin Tsai equation, ie, equation *[121]*, for the case of continuous uni–directional fibres. Also, the isotropic relationship – equation *[4]* can be used to determine the in plane Poisson's ratio such that (61,105,277):– $\nu_{2D} = (E_{2D}/2G_{2D})-1$

The equations presented by Manera (17) by which the moduli of randomly oriented discontinuous glass fibre composites may be determined are known as the laminate analogy and are given below. Kardos (24), Smith (61) and Shenoi (105) also state that these equations can be used to determine the moduli of randomly oriented continuous fibre composites. Thus the moduli are given as (17,61,105):–

$$E_{2D} = (U_1+U_4)(U_1-U_4)/U_1 \qquad [138]$$

$$G_{2D} = (U_1-U_4)/2 \qquad [139]$$

$$\nu_{2D} = U_4/U_1 \qquad [140]$$

Where,

$$U_1 = (3Q_{11}+3Q_{22}+2Q_{12}+4Q_{66})/8 \qquad [141]$$

$$U_4 = (Q_{11}+Q_{22}+6Q_{12}-4Q_{66})/8 \qquad [142]$$

And,

$$Q_{11} = E_{11}/(1-\nu_{12}\nu_{21}) \qquad [143]$$

$$Q_{22} = E_{22}/(1-\nu_{12}\nu_{21}) \qquad [144]$$

$$Q_{12} = \nu_{12}E_{22}/(1-\nu_{12}\nu_{21}) \qquad [145]$$

$$Q_{66} = G_{12} \qquad [146]$$

Also, ν_{21} can be found from:–

$$\nu_{12}/E_{11} = \nu_{21}/E_{22} \qquad [147]$$

A paper by Nielsen and Chen (200) presents an equation for obtaining the Young's modulus in the plane of the fibres at any angle of a composite with two dimensionally randomly oriented long fibres directly from the constituent properties. The modulus value can be found by averaging the value of the Young's modulus at any angle, E_θ found from equation *[126]* over all of the values of the angle θ. Nielsen and Chen state that the calculated results agree closely with experimental results. In equation *[126]* the value of E_{11} is found from equation *[16]*, and the value of E_{22} from equation *[100]* – these are the modulus values should the composite be uni–directional. Unfortunately the authors only predict the value of the in–plane Young's modulus in their paper.

B.1 Comparison Of Equations Against Each Other

Note that if the equations used first require the calculation of the uni–directional properties then these have been determined from the equations recommended in Chapter 7. The exception to this is the plain strain bulk modulus, K_{23} as no experimental data is available to check the accuracy of any of the equations quoted in the literature. Thus, when required for the calculation of the in plane properties E_{2D}, and ν_{2D}, K_{23} has been calculated from equation *[88]* as this is the equation given by the author.

In all of the comparisons of equations it is assumed that the composite is an E–glass / epoxy resin composite. The properties of the constituent materials are assumed to be as follows:–

E–Glass – E = 73GPa., ν = 0.22, G = 29.92GPa., K = 53.43GPa.

Epoxy – E = 3.0GPa., ν = 0.415, G = 1.06GPa., K = 6.23GPa.

The graphs of the properties calculated from each equation against the fibre volume fraction can be seen in Chapter 8.

B.1.1 In Plane Poisson's Ratio

Note that E_{2D} and G_{2D} in equation *[4]* are determined from equations *[136]* and *[137]* respectively.

B.2 Comparison Of Equations With Experimental Data

The experimental properties of continuous glass random fibre reinforced composites are:–

1) E–glass / epoxy:

E_{2D} = 11.05GPa., G_{2D} = 2.87GPa., ν_{2D} = 0.3342, Density = 1.38e–6Kg/mm^3, W_f = 0.5, V_f = 0.265

E_{2D} = 14.8GPa., G_{2D} = 4.15GPa., V_f = 0.34

The volume fraction has been determined from equation *[9]*. The properties of the epoxy resin determined experimentally were:–

E = 3.0GPa., ν = 0.415, G = 1.06GPa., K = 6.23GPa.

2) E–glass / Daron XPu2 A2 polyester resin (285):

E_{2D} = 14.86GPa., G_{2D} = 5.766GPa., ν_{2D} = 0.2478, ν_{2T} = 0.262, V_f = 0.36, Density = 1.7e–6Kg/mm^3.

The properties of the hybrid polyester resin – Daron XPu2 A2 are (286):

E = 3.3GPa., ν = 0.3, G = 1.27GPa., K = 3.17GPa., Density = 1.19Kg/mm^3

3) E–glass / polyester:

E_{2D} = 11.88GPa., ν_{2D} = 0.3188, ν_{2T} = 0.32, G_{2D} = 3.1GPa., W_f = 0.435, V_f=0.293

E_{2D} = 9.66GPa., ν_{2D} = 0.3, ν_{2T} = 0.385, G_{2D} = 2.4GPa., W_f = 0.283, V_f = 0.168

The properties of the polyester resin determined experimentally were:–

E = 3.39GPa., ν = 0.44, G = 1.177GPa., K = 9.81GPa., Density =1.1e–6Kg/mm^3

Resin	E2D	v2D	v2T	G2D	Vf
Polyester	9.66	0.3	0.385	2.4	0.168
Epoxy	11.05	0.3342		2.87	0.265
Polyester	11.88	0.3118	0.32	3.1	0.293
Epoxy	14.8			4.15	0.34
Daron	14.86	0.2478	0.262	5.766	0.36

Table 23 : Experimental Properties Of Continuous Random Fibre Composite

The average error shown in the following tables is obtained by comparing the results of the calculations with the experimental values given in the above table. Note that the relevant resin property values have been used in the calculations.

		Fibre Volume Fraction				
Eqn. No.	0.168	0.265	0.293	0.34	0.36	Ave. Error
136	8.53	11.16	12.57	13.63	14.71	5.40%
Exp.	9.66	11.05	11.88	14.8	14.86	

Table 24 : Calculated In Plane Young's Modulus

	Fibre Volume Fraction				
Eqn. No.	0.168	0.265	0.293	0.36	Ave. Error
4	0.407	0.423	0.418	0.422	41%
130	0.397	0.366	0.34	0.31	18%
135	0.33	0.33	0.33	0.33	12.60%
140	0.359	0.311	0.311	0.301	11%
Exp.	0.3	0.3342	0.3118	0.2478	

Table 25 : Calculated In Plane Poisson's Ratio

	Fibre Volume Fraction					
Eqn No.	0.168	0.265	0.293	0.34	0.36	Ave. Error
137	3.03	3.92	4.43	4.78	5.17	26.20%
Exp.	2.4	2.87	3.1	4.15	5.766	

Table 26 : Calculated In Plane Shear Modulus

The in plane shear modulus calculated from the Halpin Tsai equation – equation *[103]* with the geometrical factor as $5+10^5V_f^{10}$ gives the following:–

V_f =	0.168	0.265	0.293	0.34	0.36
G_{2D} =	2.28	2.86	3.47	4.09	5.4

This gives an average error of 5%.

The following equations are for the prediction of the transverse Poisson's ratio of a continuous uni–directional fibre reinforced material – see Appendix A:–

$$\nu_{2T} = \nu_f \nu_m / (V_f \nu_m + V_m \nu_f) \qquad [23]$$

$$\nu_{2T} = \nu_f V_f + \phi \nu_m V_m \qquad [32]$$

$$\nu_{2T} = (K - MG_{23})/(K + MG_{23}) \qquad [81]$$

	Fibre Volume Fraction			
Eqn No.	0.168	0.293	0.36	Ave. Error
23	0.377	0.34	0.265	3.10%
32	0.62	0.584	0.344	50.50%
81	0.663	0.73	0.293	70%
Exp.	0.385	0.32	0.262	

Table 27 : Calculated Transverse Poisson's Ratio.

B.3 Summary Of Equations Considered

The following sections summarise the equation sconsidered in Chapter 8 and presented in this Appendix such that they form a quick reference guide to the reader.

B.3.1 In Plane Young's Modulus

$$E_{2D} = (u_1^2 - u_2^2)/u_1 \qquad \textit{[129]}$$

$$E_{2D} = E_f V_f / 3 \qquad \textit{[133]}$$

$$E_{2D} = 3E_{11}/8 + 5E_{22}/8 \qquad \textit{[136]}$$

$$E_{2D} = (U_1+U_4)(U_1-U_4)/U_1 \qquad \textit{[138]}$$

B.3.2 In Plane Poisson's Ratio

$$\nu_{2D} = (E_{2D}/2G_{2D})-1 \qquad \textit{[4]}$$

$$\nu_{2D} = u_2/u_1 \qquad \textit{[130]}$$

$$\nu_{2D} = 1/3 \qquad \textit{[135]}$$

$$\nu_{2D} = U_4/U_1 \qquad \textit{[140]}$$

B.3.3 In Plane Shear Modulus

$$G_{2D} = E_f V_f / 8 \qquad \textit{[134]}$$

$$G_{2D} = E_{11}/8 + E_{22}/4 \qquad \textit{[137]}$$

$$G_{2D} = (U_1-U_4)/2 \qquad \textit{[139]}$$

Appendix C : Woven Composite Properties

Two publications (30,40) use a modified law of mixtures to predict the longitudinal Young's modulus for a woven material,

$$E_{11} = E_m V_m + \alpha E_f V_f \qquad [148]$$

The parameter 'α' depends on the "efficiency of reinforcement", or allows for "slight fibre misalignment". For woven rovings the value of α is approximately 0.5 (40). It is reported by Johnson (30) that for a woven roving polyester laminate the experimental data is approximated well when $\alpha = 0.59$ for tensile modulus, $\alpha = 0.7$ in compression and $\alpha = 0.52$ for the flexural modulus.

C.1 Fibre Undulation Model

In one publication Chou and Ishikawa (84) report that the elastic constants of the fill yarn with respect to the global x,y and z axes can be defined by including parameters related to the weave angle as:–

$$E_x = \frac{1}{\mathrm{Cos}^4\theta/E_{11} + (1/G_{13} - 2\nu_{31}/E_{11})\mathrm{Cos}^2\theta\mathrm{Sin}^2\theta + \mathrm{Sin}^4\theta/E_{33}} \qquad [149]$$

$$E_y = E_{22} = E_{33} \qquad [150]$$

$$G_{xy} = 1 / (\mathrm{Cos}^2\theta/G_{12} + \mathrm{Sin}^2\theta/G_{23}) \qquad [151]$$

$$\nu_{xy} = \nu_{31}\mathrm{Cos}^2\theta + \nu_{23}\mathrm{Sin}^2\theta \qquad [152]$$

Where, θ is the angle of the weave.

C.2 Laminate Analogy

The effective elastic properties of the fill thread for the slice array model presented in Chapter 9 are (205):–

$$E_{fL} = E_{11} / \{1 + (\theta^2/3)(E_{11}/G_{12} - 2\nu_{12})\} \qquad [153]$$

$$E_{fT} = E_{22} \qquad [154]$$

$$\nu_{f21} = \nu_{21} + (\theta^2/3)(\nu_{23} - \nu_{21}) \qquad [155]$$

$$G_{f12} = G_{12} / \{1 + (\theta^2/3)(G_{12}/G_{23} - 1)\} \qquad [156]$$

Where, 'θ' is the maximum off axis angle as the undulated thread and is given for the warp and fill as (205):–

$$\theta_f = \pi h_f/2a_f \qquad [157]$$

$$\theta_w = \pi h_w/2a_w \qquad [158].$$

See Figure 24 in Chapter 9 for definitions of 'a' and 'h'.

C.3 Classical Lamination Theory

Classical lamination theory (CLT) is stated by a number of authors (38,47, 287) to be the most widely used technique for analysing composites. The theory assumes that the laminate is in a state of plane stress hence ignoring the interlaminar effects, and the out of plane stress components. Griffin (38) states thet the CLT solutions have a high degree of accuracy away from the edges of the laminate, but are not applicable at a 'boundary layer' which "extends inward from the edge to a distance approximately equal to the laminate thickness". However, Chou (26) and Enie and Rizzo (287) report that when the properties through the thickness of the laminate significantly contribute to its response to a loading condition the CLT breaks down, and hence cannot be used to analyse thick laminates. Sun and Li (288) report that the high order plate theories that are used to analyse moderately thick laminates are usually more mathematically involved than the CLT, and cannot be used to account for three dimensional characteristics.

The procedures to evaluate the stresses and deflections of laminates is dependent on the fact that the thickness of the laminates is much smaller than its in–plane dimensions (140). If this is so then the laminate can be analysed on the basis of the assumptions of thin plate theory. The classical assumptions of thin plate theory are (33,140):–

1. The thickness of the plate is much smaller than the in–plane dimensions.

2. The strains in the deformed plate are small.

3. Normals to the undeformed plate surface remain normal to the deformed plate surface.

4. Vertical deflection does not vary through the thickness.

5. Stress normal to the plate surface is negligible, also shear strains in planes perpendicular to the surface are assumed to be zero.

The laminate is assumed to be made up of a number of perfectly bonded laminae, with the bond assumed to be infinitesimally thin (28). The laminate is thus assumed to be a single layer of material.

The laminate analogy would determine the laminate stiffnesses from the following equations such that (205,277):–

$$E_x = (A_{11}A_{22} - A_{12}^2)/hA_{22} \quad [159]$$

$$E_y = (A_{11}A_{22} - A_{12}^2)/hA_{11} \quad [160]$$

$$\nu_{xy} = A_{12}/A_{22} \quad [161]$$

$$G_{xy} = A_{66}/h \quad [162]$$

Where, 'h' is the overall laminate thickness.

The extensional stiffnesses 'A' are given by:–

$$A_{ij} = \Sigma\, Q_{ij}\, h_k \quad [163]$$

Where, h_k is the thickness of each ply. Note that for two plies of material, such as in a typical woven composite, where each ply has the same thickness 'h_k', then the terms 'h' and 'h_k' in the above equations cancel one another out such that the thickness of the plies is not required.

The stiffnesses 'Q' are given by equations *[143]* to *[146]* given previously in Appendix B:–

$$Q_{11} = E_{11}/(1-\nu_{12}\nu_{21})$$

$$Q_{22} = E_{22}/(1-\nu_{12}\nu_{21})$$

$$Q_{12} = \nu_{12}E_{22}/(1-\nu_{12}\nu_{21})$$

$$Q_{66} = G_{12}.$$

The properties E_{11}, E_{22}, G_{12}, ν_{12} and ν_{21} are the properties of a uni-directional ply and may be determined from the equations recommended in Chapter 7.

The through thickness shear moduli for a balanced woven material can be obtained by averaging the ply shear moduli G_{13} and G_{23}, according to Smith (61), such that:-

$$G_{xz} = G_{yz} = 0.5(G_{13} + G_{23}) \qquad [164]$$

For a 4:1 biased laminate, based on the 'x' direction then:-

$$G_{xz} = 0.2(4G_{13} + G_{23}) \qquad [165]$$

and, $$G_{yz} = 0.2(G_{13} + 4G_{23}). \qquad [166]$$

Similar expressions may be obtained for the Poisson's ratios ν_{xz} and ν_{yz}.

C.4 Mechanical Property Calculations

The composite materials studied in this thesis consist of an E-glass reinforcement together with a matrix of either epoxy or polyester resin. The material properties of these constituent materials are given in Appendix A. The units used for the Young's modulus, shear modulus and the plane strain bulk modulus are GPa., whilst the units used for the density are Kg/mm^3.

C.4.1 Experimental Data

The experimental data given here is that presented in the literature for a balanced plain weave composite.

i). E-glass / epoxy (205)

$E_x = 19.3$, $E_y = 19.3$, Overall $V_f = 0.42$, Thread or yarn $V_f = 0.7$

ii). E-Glass / Epoxy (82)

The following experimental properties are given in a paper by Paumelle et al (82) for a plain weave glass/epoxy composite, note that more than one value is quoted as different values were determined by different experiments by Paumelle et al:-

Thread fibre volume fraction = 0.81

Warp and fill fibre volume fraction = 0.5

$E_x = E_y = 24.9\ (27.3),\ E_z = 14$

$G_{xy} = 5.9\ (7.8),\ G_{xz} = G_{yz} = 4.0\ (5.0)$

$\nu_{xy} = 0.275\ (0.15),\ \nu_{xz} = \nu_{yz} = 0.36\ (0.22)$

iii). E–Glass / Epoxy (217)

The FE model developed by Paumelle et al in the paper referenced above is also used in another publication (217) to analyse a glass/epoxy balanced weave composite. The FE analysis is used to determine the following properties given a thread $V_f = 0.81$, and an overall $V_f = 0.51$:–

$E_x = E_y = 28.88,\ E_z = 12.105$

$G_{xy} = 6.236,\ G_{xz} = G_{yz} = 3.783$

$\nu_{xy} = 0.15,\ \nu_{xz} = \nu_{yz} = 0.32$

Although the above properties are not experimentally determined they are included here as they may be useful as there appears to be little experimental data available.

C.4.2 Calculated Values

The properties of the E–glass are taken to be:–

E–Glass – E = 73GPa., ν = 0.22, G = 29.92GPa., K = 53.43GPa.

The properties of the epoxy resin are taken to be:–

E = 3.45GPa., ν = 0.35, G = 1.28GPa., K = 4.2GPa.

C.4.2.1 Laminate Analogy

Note that only the value of the overall V_f has been used and that details of the thread or yarn V_f have not been considered for the reasons detailed in Chapter 9. The properties determined from the laminate analogy equations are summarised in the following tables.

	Fibre Volume Fraction			
	0.42	0.5	0.51	Ave. Error
	32.6	38.2	38.9	52%
Exp.	19.3	24.9	28.88	

Table 28 : Calculated E_x

	Fibre Volume Fraction		
	0.5	0.51	Ave. Error
	3.5	3.57	42%
Exp.	5.9	6.236	

Table 29 : Calculated G_{xy}

	Fibre Volume Fraction		
	0.5	0.51	Ave. Error
	0.284	0.284	89%
Exp.	0.15	0.15	

Table 30 : Calculated ν_{xy}

C.4.2.2 Empirical Method

The properties determined from the empirical formulae given in Chapter 7 are summarised in the following tables.

	Fibre Volume Fraction			
	0.42	0.5	0.51	Ave. Error
	21.5	25.2	25.7	7.80%
Exp.	19.3	24.9	28.88	

Table 31 : Calculated E_x

	Fibre Volume Fraction		
	0.5	0.51	Ave. Error
	13.95	14.28	9.20%
Exp.	14	12.1	

Table 32 : Calculated E_z

	Fibre Volume Fraction		
	0.5	0.51	Ave. Error
	5.83	5.95	2.80%
Exp.	5.9	6.236	

Table 33 : Calculated G_{xy}

	Fibre Volume Fraction		
	0.5	0.51	Ave. Error
	3.75	3.83	3.70%
Exp.	4	3.783	

Table 34 : Calculated G_{yz}

	Fibre Volume Fraction		
	0.5	0.51	Ave. Error
	0.15	0.15	0%
Exp.	0.15	0.15	

Table 35 : Calculated ν_{xy}

	Fibre Volume Fraction		
	0.5	0.51	Ave. Error
	0.3663	0.364	7.70%
Exp.	0.36	0.32	

Table 36 : Calculated ν_{yz}

Appendix D : Variation In Mechanical Properties

D.1 Overall Variation In Mechanical Properties

The following graphs show how the calculated mechanical properties vary due to the epoxy matrix properties varying between the two extremes highlighted in Appendix A.

D.1.1 Uni–Directional Material

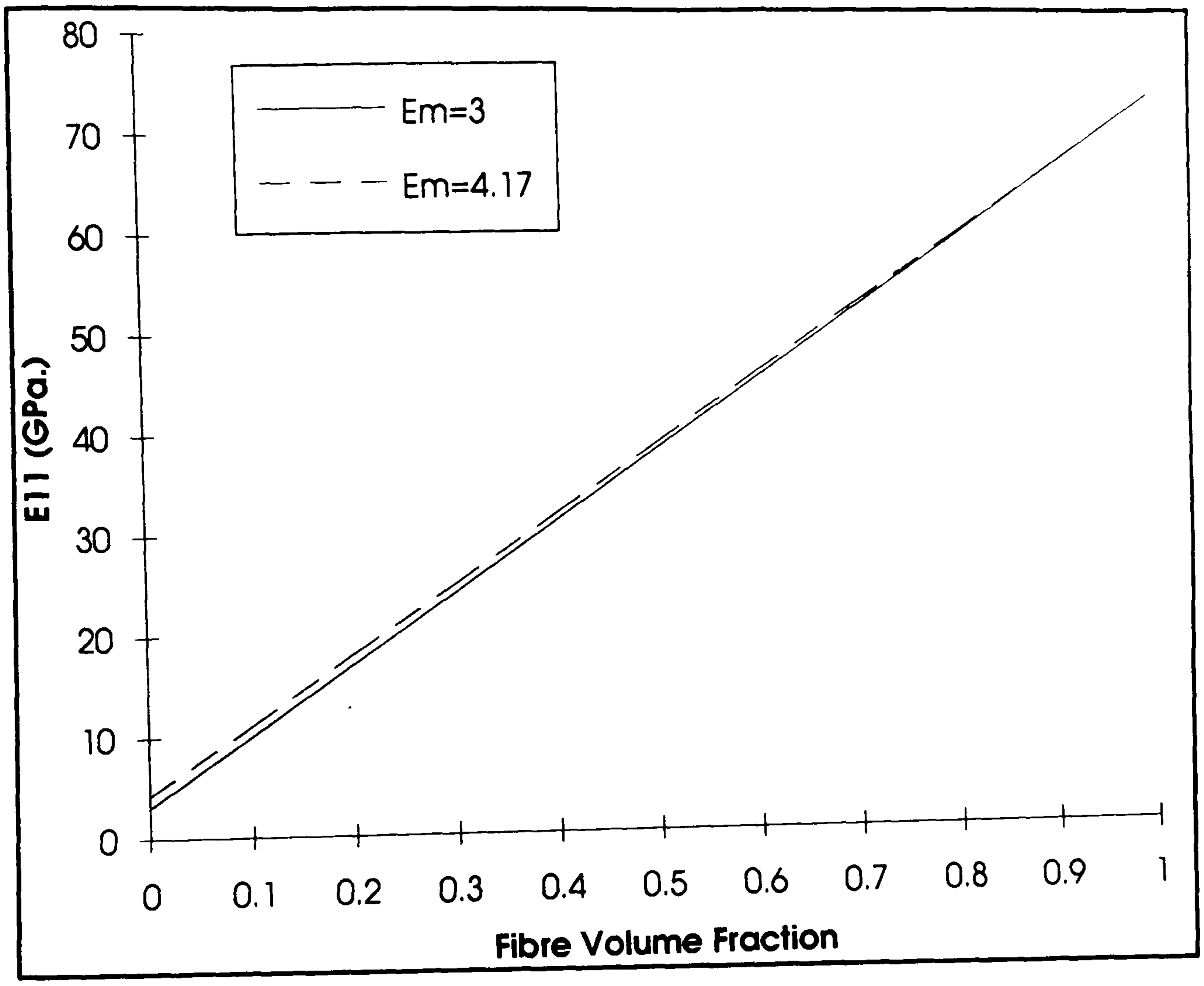

Figure 49 : Longitudinal Young's Modulus.

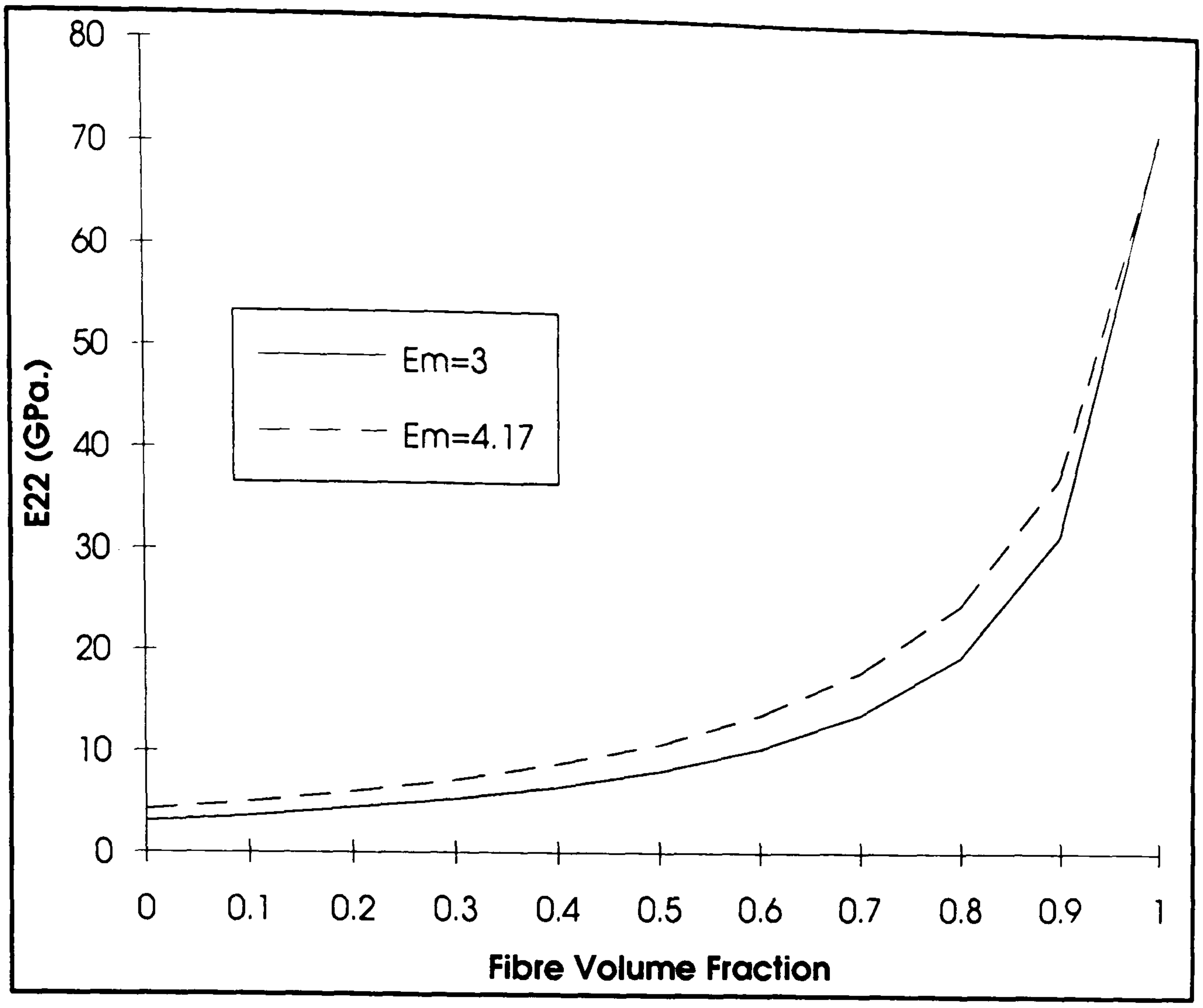

Figure 50 : Transverse Young's Modulus.

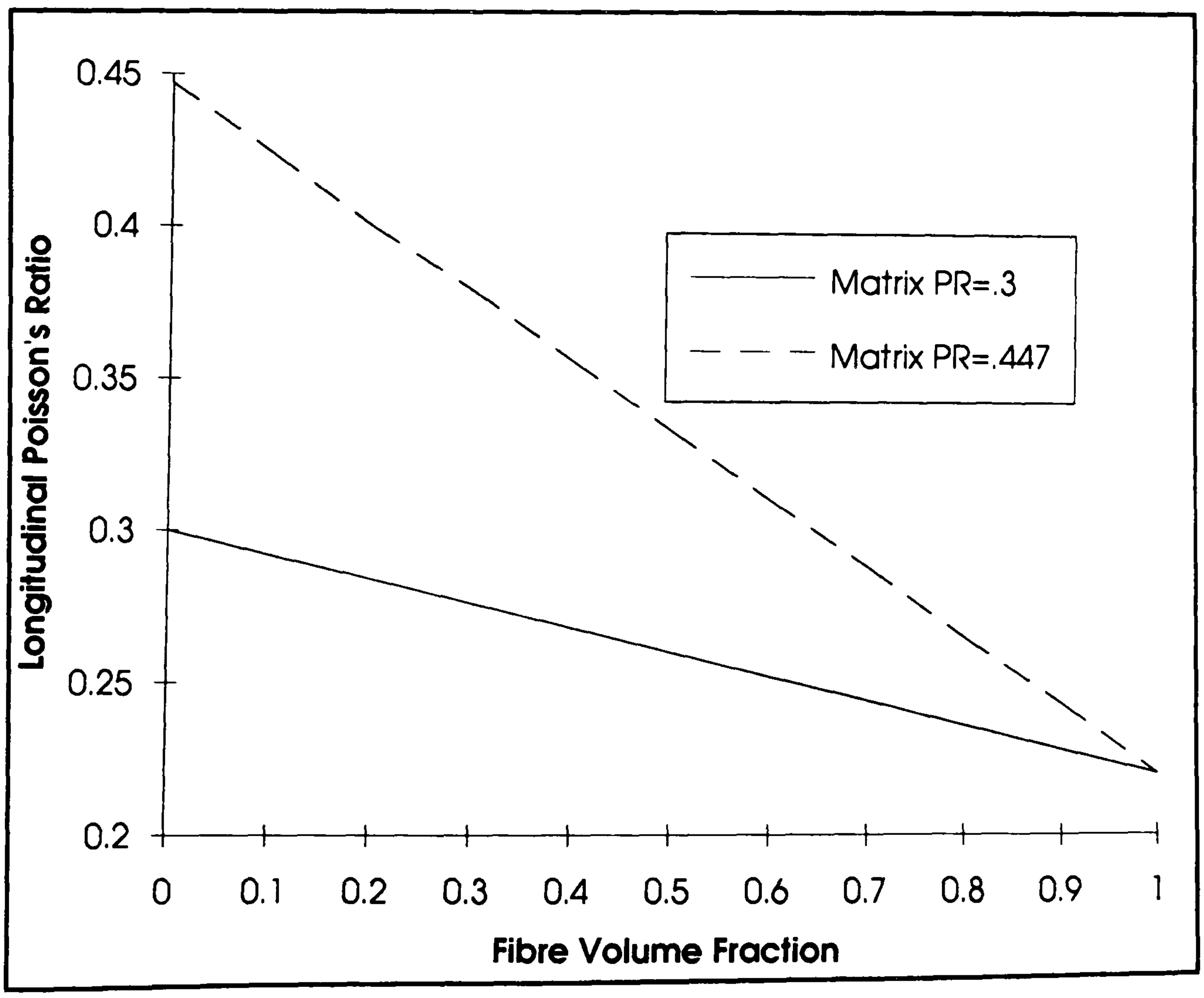

Figure 51 : Longitudinal Poisson's Ratio.

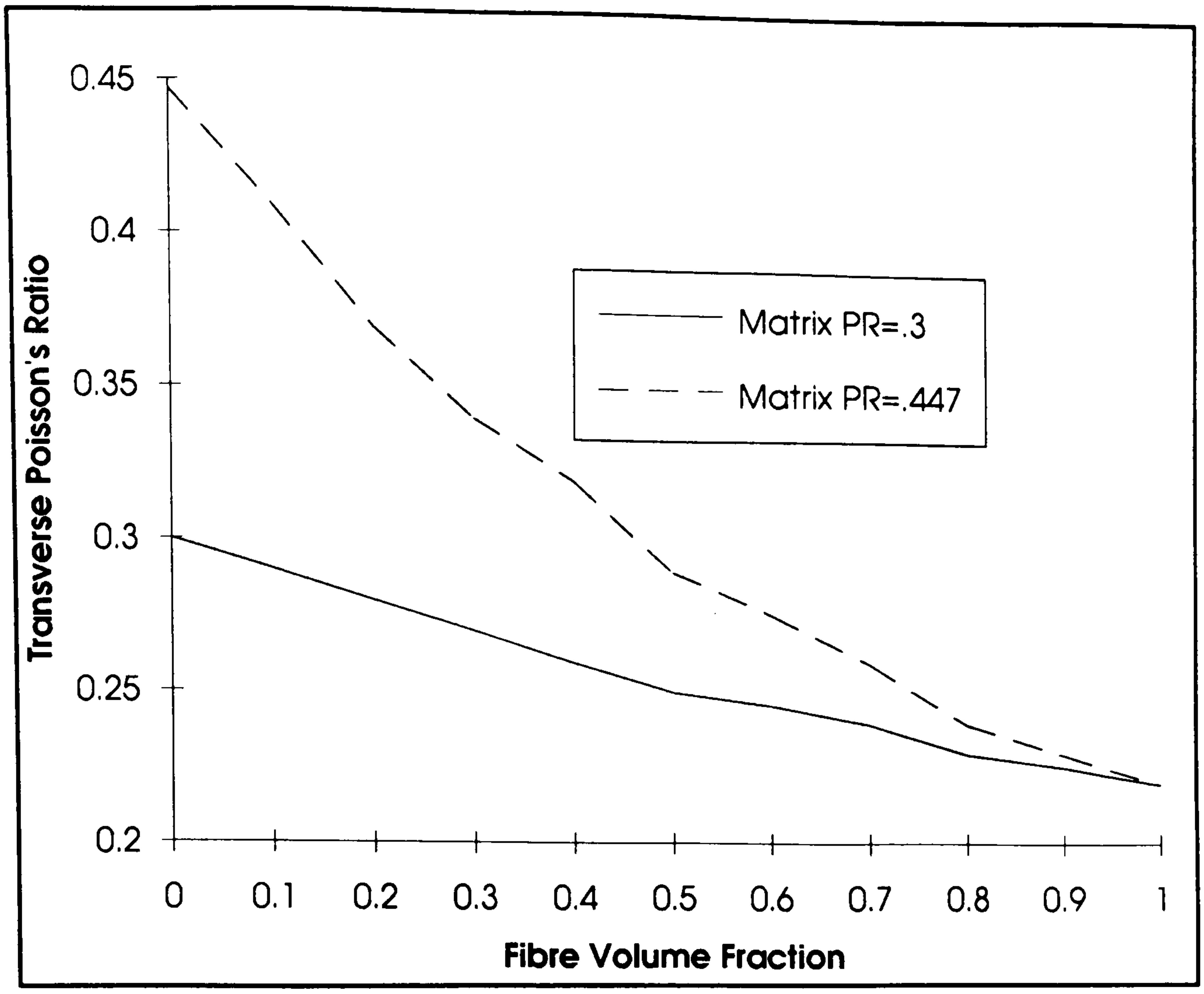

Figure 52 : Transverse Poisson's Ratio.

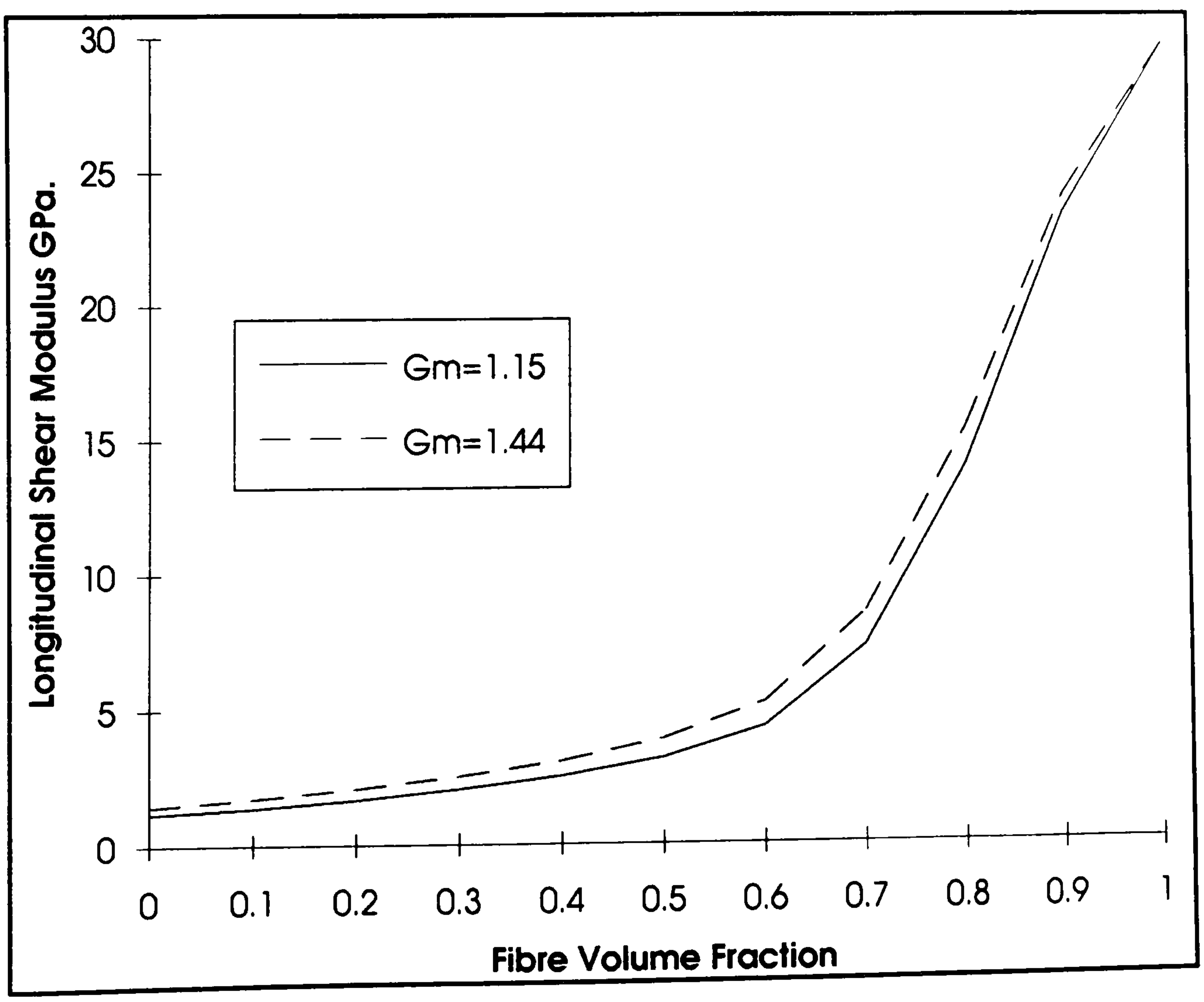

Figure 53 : Longitudinal Shear Modulus.

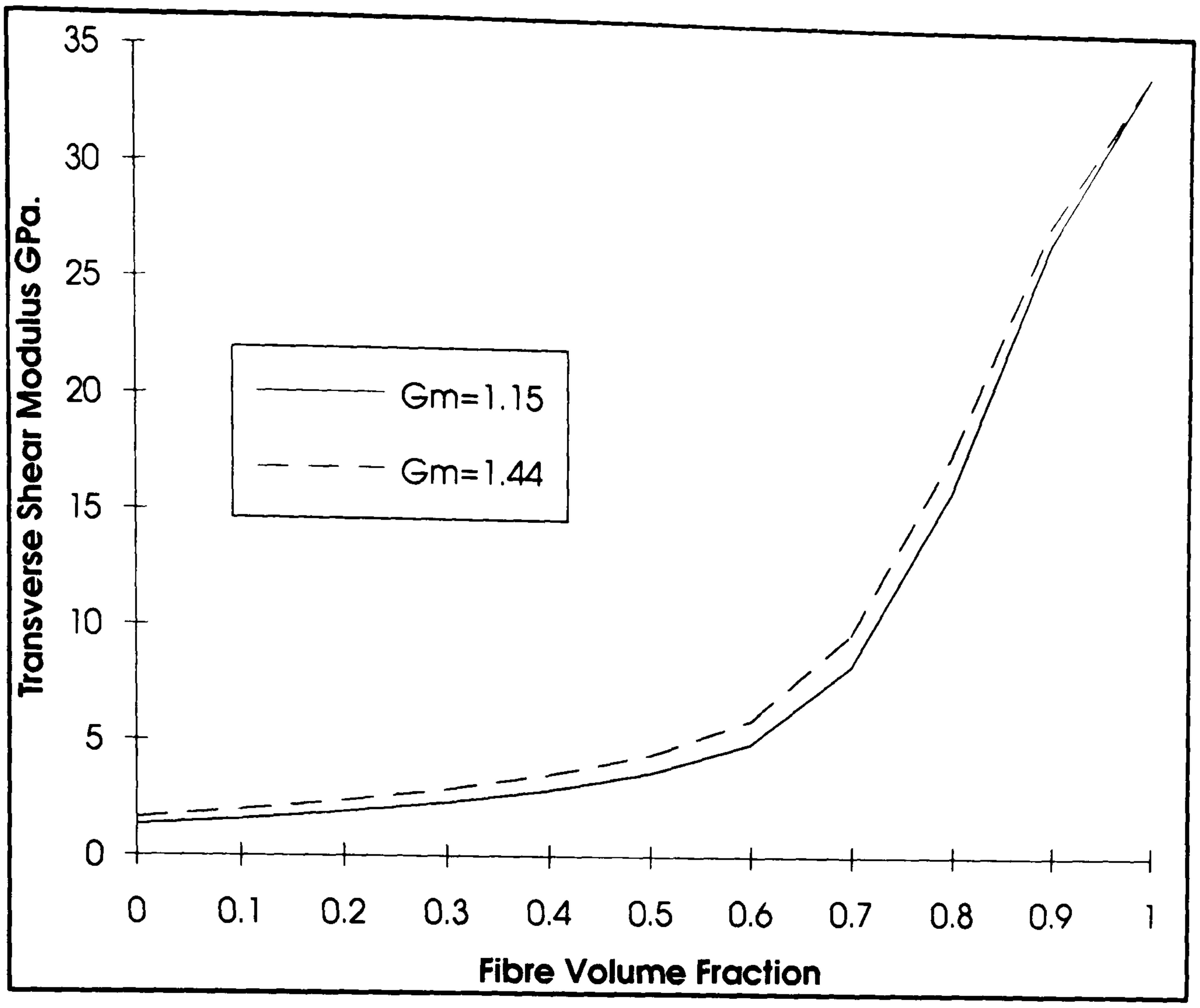

Figure 54 Transverse Shear Modulus.

D.1.2 Random Fibre Material

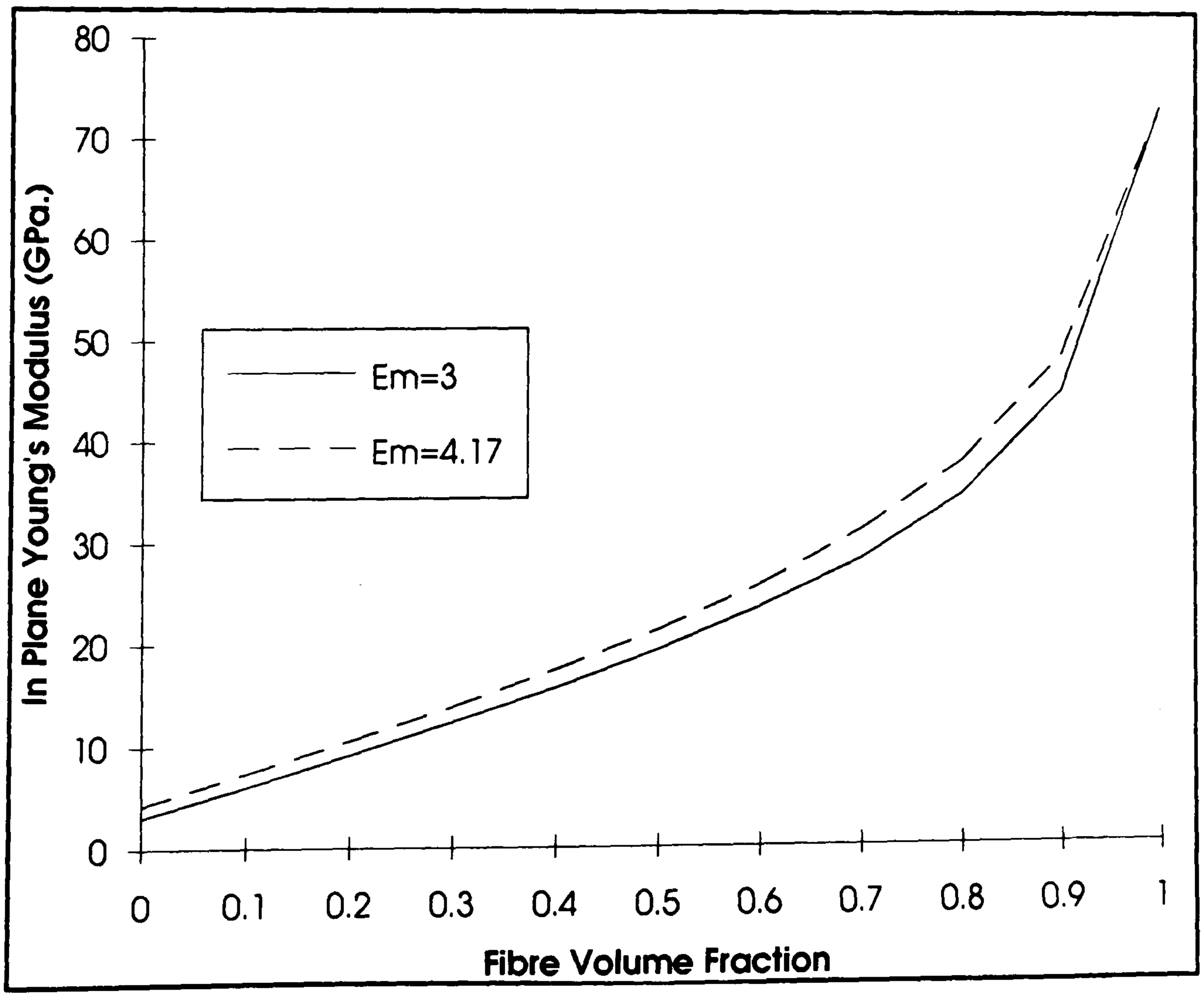

Figure 55 : In Plane Young's Modulus.

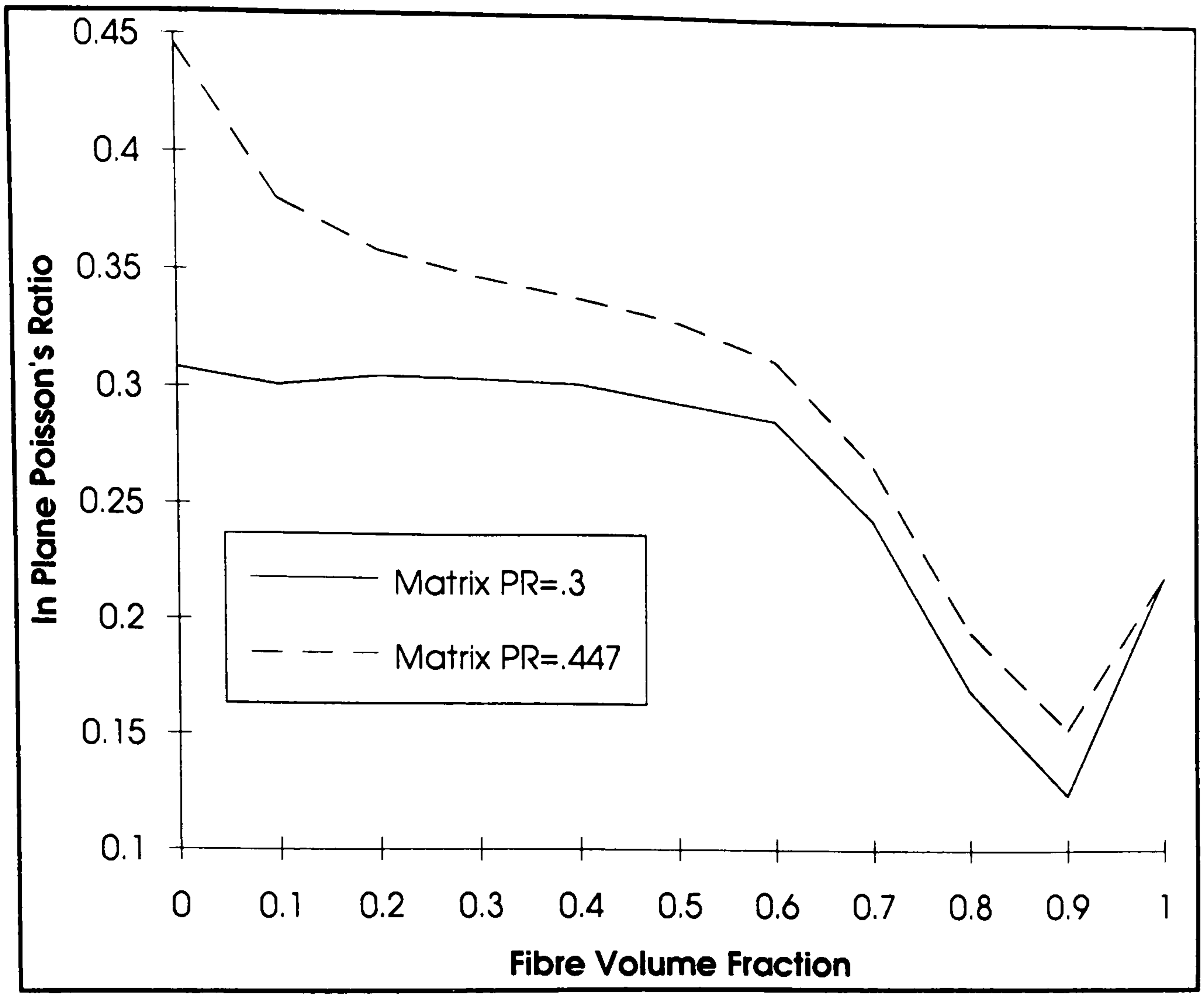

Figure 56 : In Plane Poisson's Ratio.

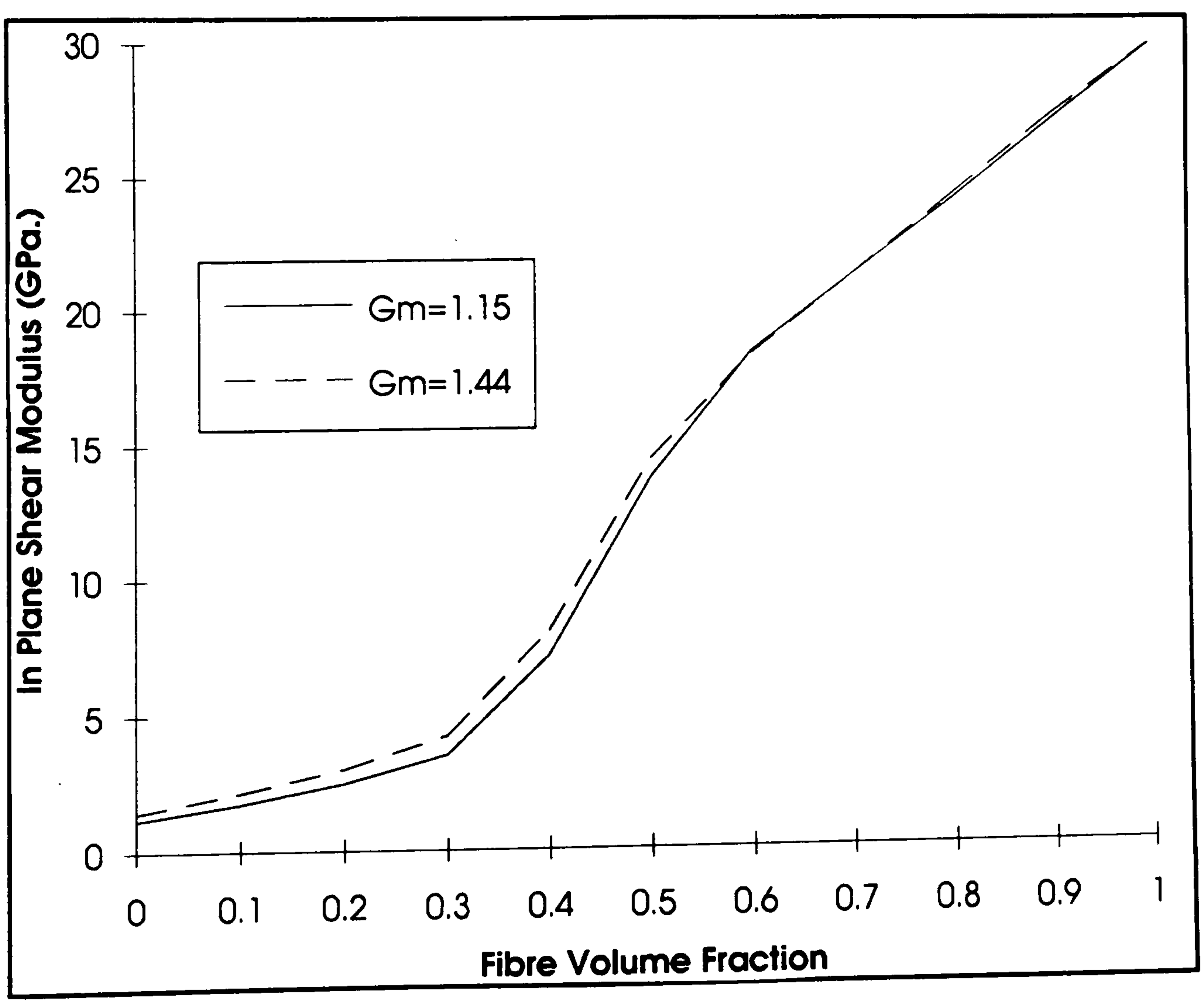

Figure 57 : In Plane Shear Modulus.

D.1.3 Woven Fibre Material

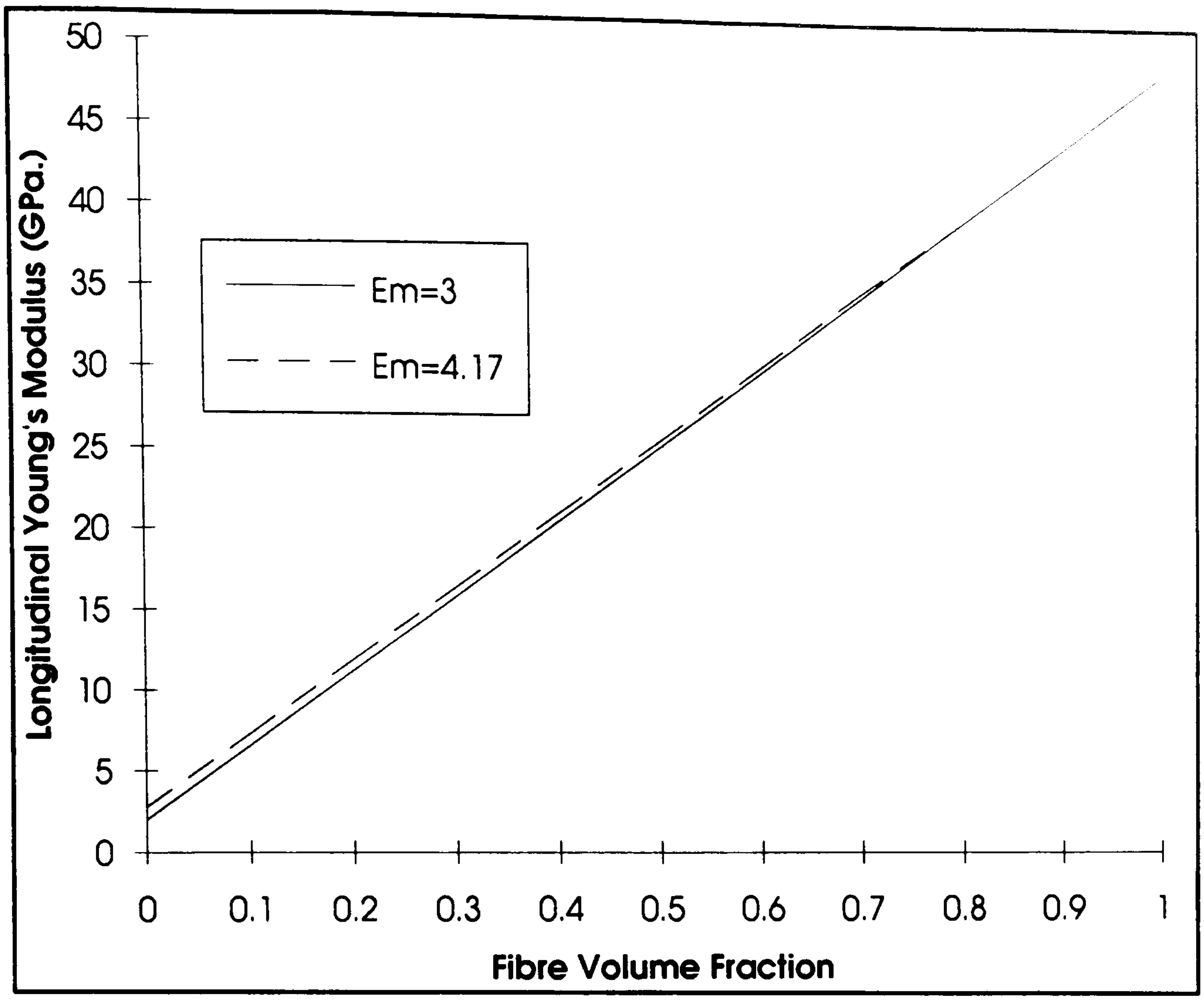

Figure 58 : Longitudinal Young's Modulus.

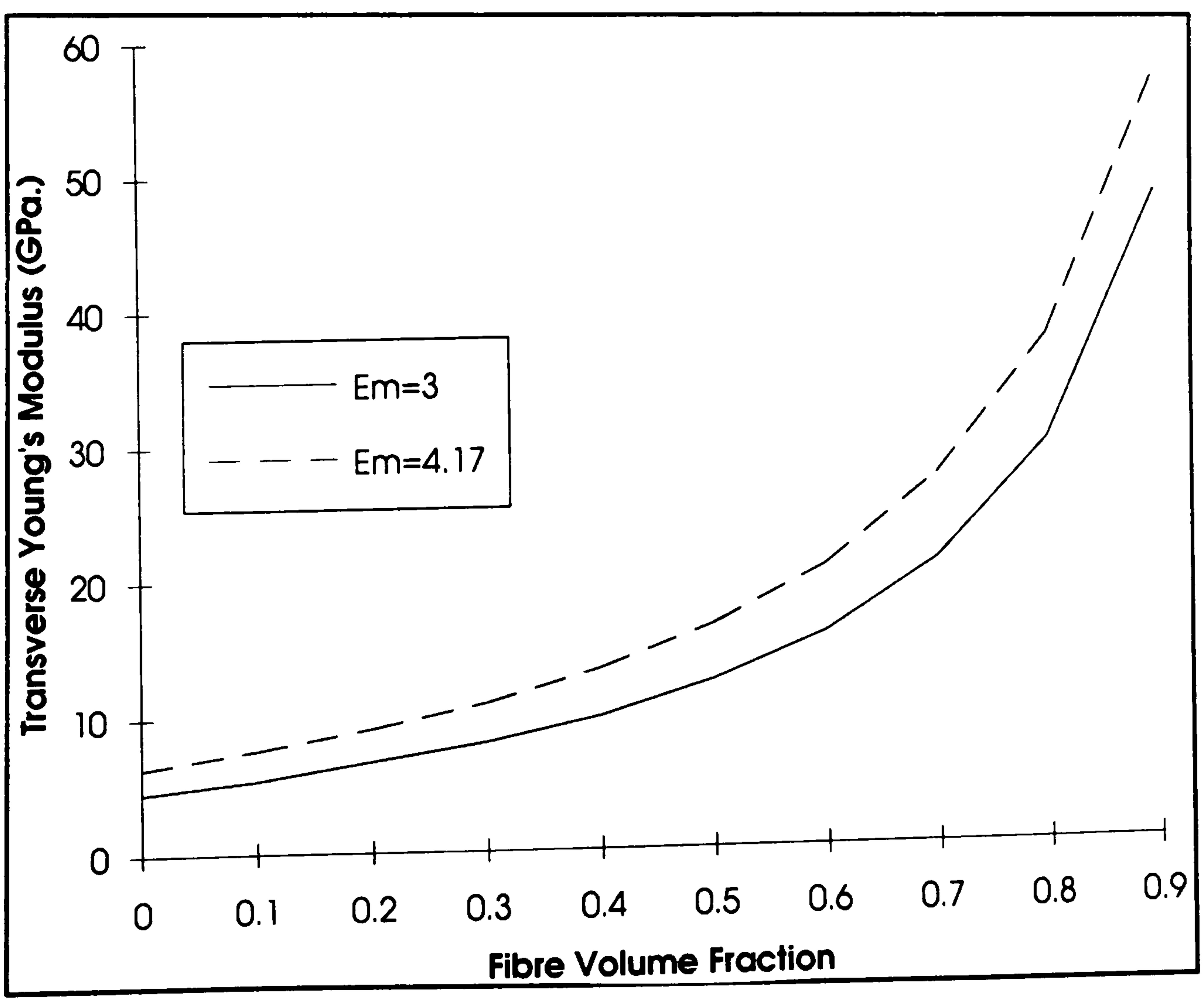

Figure 59 : Transverse Young's Modulus.

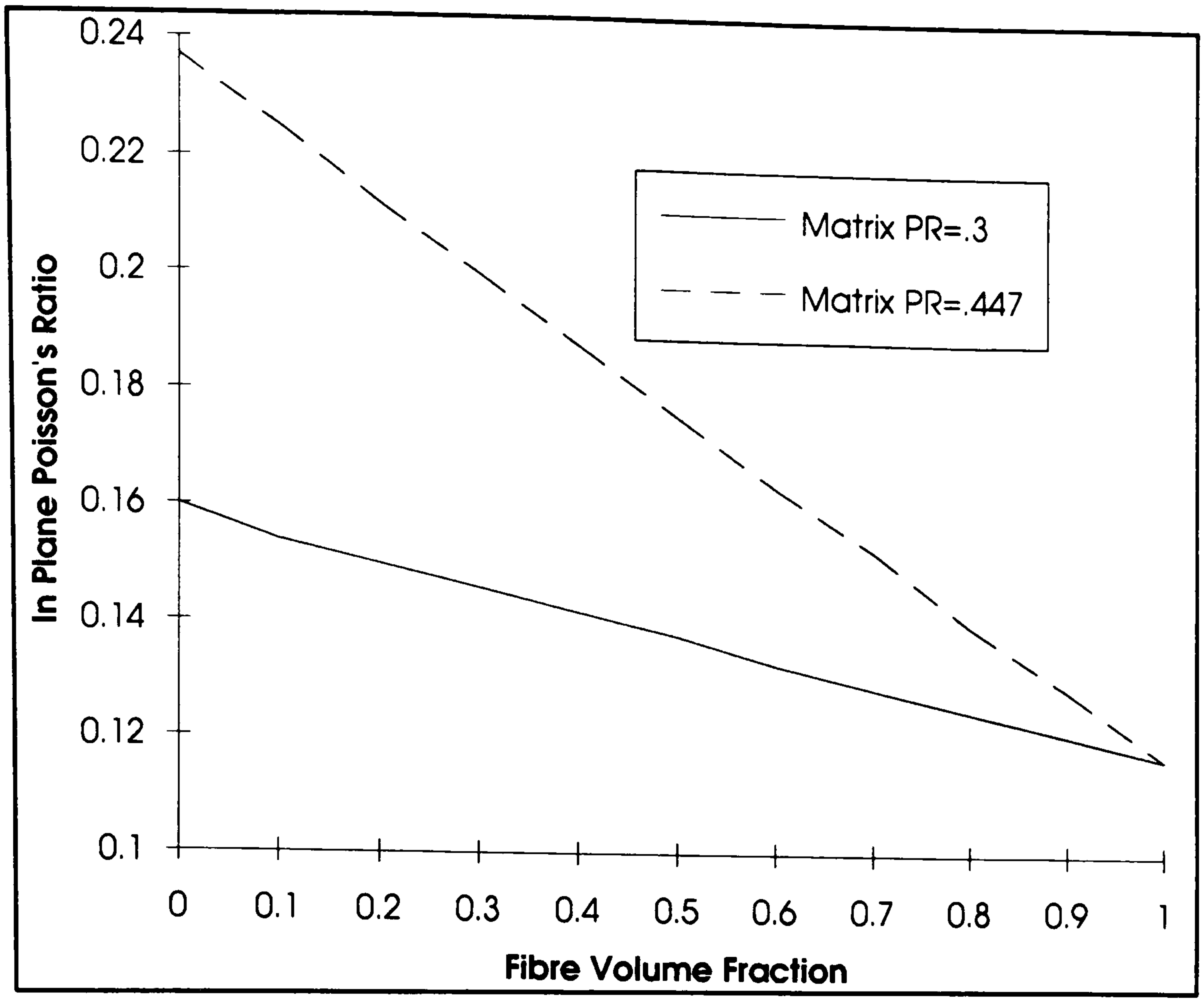

Figure 60 : Longitudinal Poisson's Ratio.

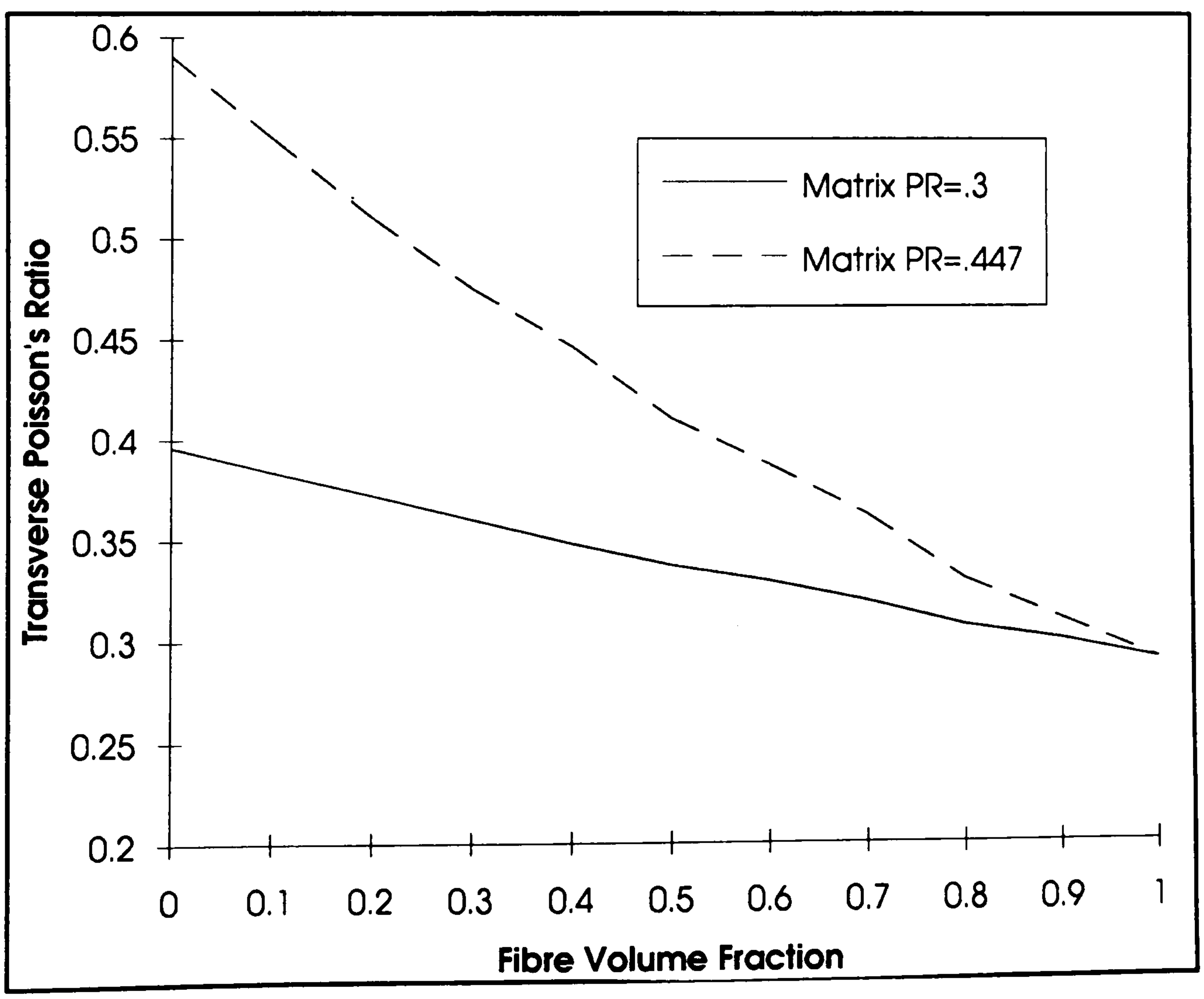

Figure 61 : Transverse Poisson's Ratio.

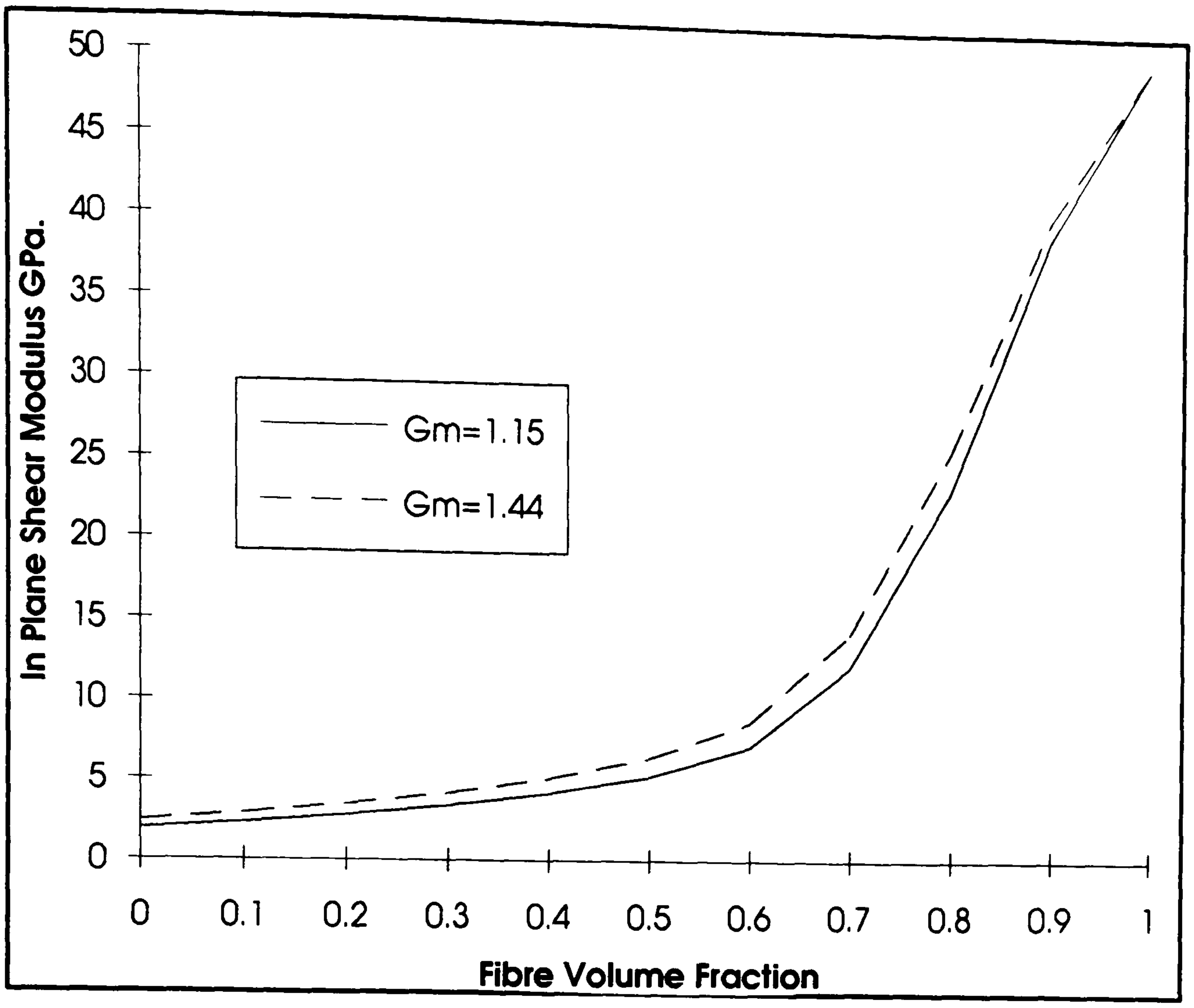

Figure 62 : Longitudinal Shear Modulus.

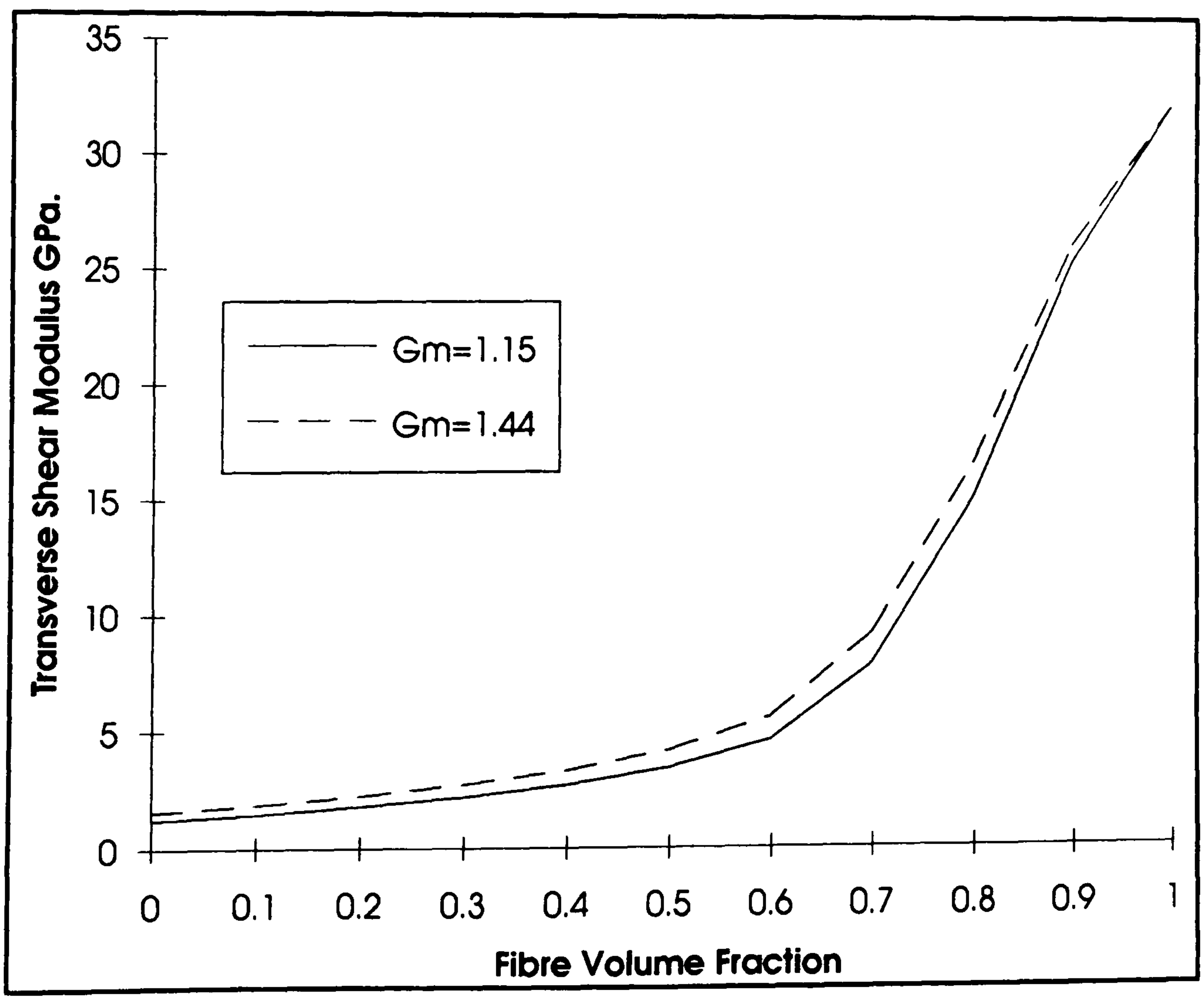

Figure 63 Transverse Shear Modulus.

D.2 SMC Ribbed Plate

Other work that the author has been involved with has led to two analyses of a ribbed and notched SMC plate manufactured from charges of material in two different positions, ie a lateral charge and a central charge. The plate dimensions and charge positions are shown in Figures 64 and 65 (289).

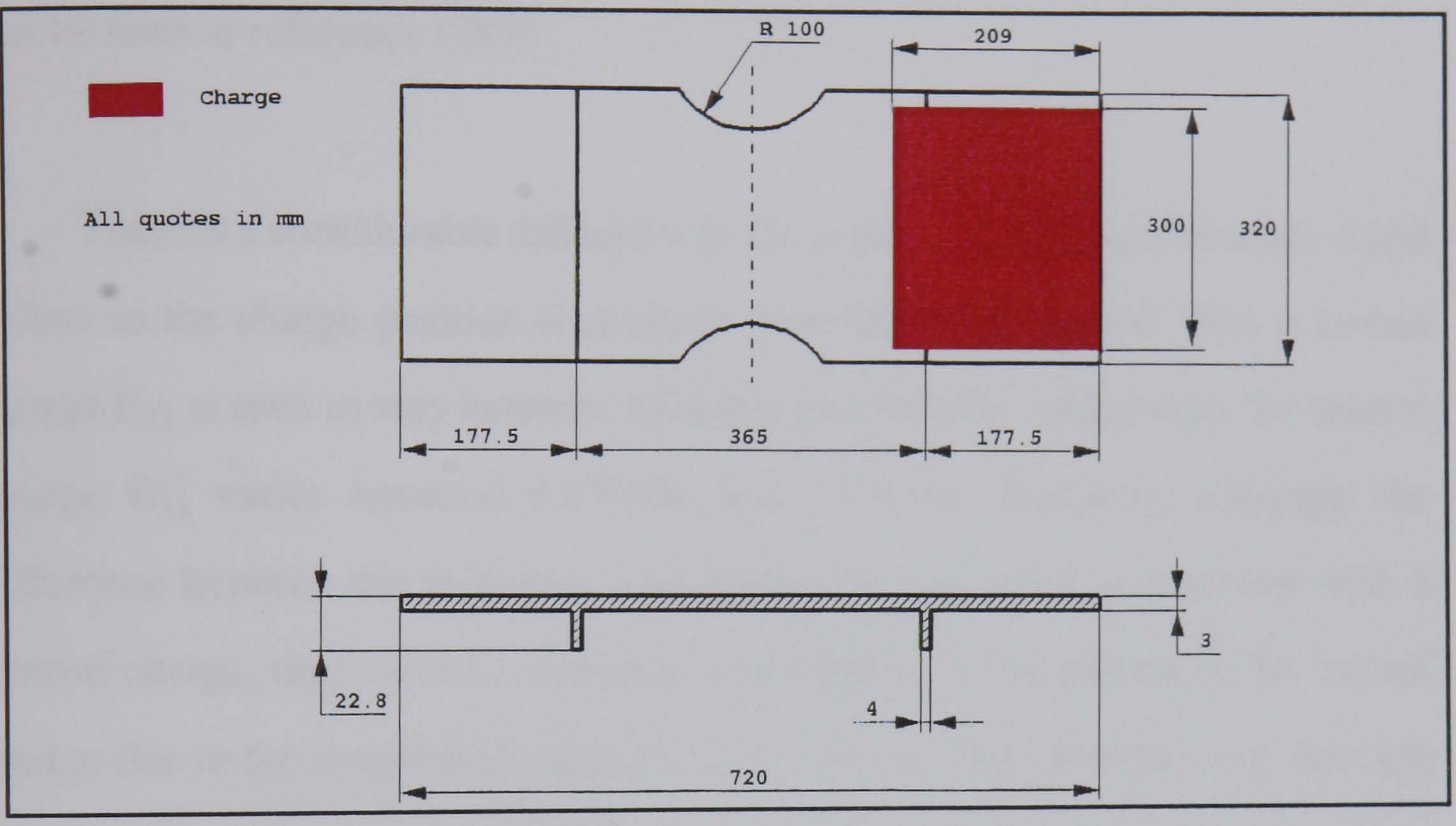

Figure 64 : Dimensions Of The Ribbed SMC Plate With Lateral Charge (289).

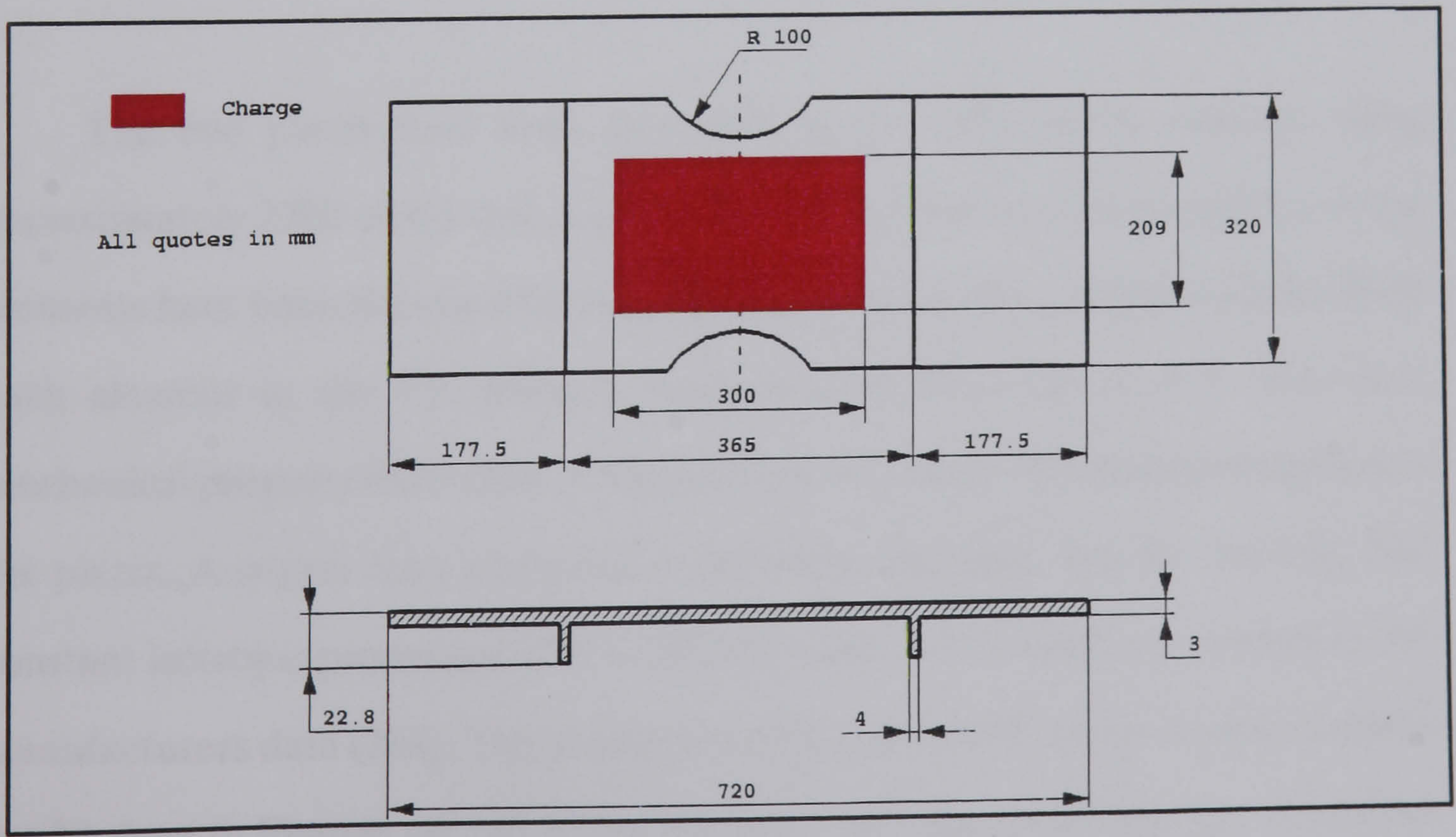

Figure 65 : Dimensions Of The Ribbed SMC Plate With Central Charge (289).

A simulation of the compression moulding process for both plates has been undertaken by Fiat (264) using the PLASTEC flow modelling software and the resultant mechanical properties used here for structural analysis. The mechanical properties determined from the simulation were E_{11}, E_{22}, ν_{12}. G_{12} was assumed to remain constant. The FE mesh and mechanical properties were provided in ANSYS format and were used here for analysis. Details of the flow analysis results can be seen in reference (289).

There is a considerable difference in the predicted modulus distribution and values as the charge position is changed from lateral to central. With a lateral charge E_{11} is seen to vary between 8.69GPa. and 9.3GPa., whilst with the central charge E_{11} varies between 9.57GPa. and 15.2GPa. However, although the difference between the minimum and maximum values of E_{11} is greater with a central charge, there is more variation seen overall in the plate with the lateral charge due to the longer flow path. Thus, the initial charge position and resulting flow pattern of the material in the compression moulding process can be seen to effect the resultant mechanical properties (289).

The two plates have been modelled in the ANSYS FE software using approximately 2300 stif63 thin shell elements. The mechanical properties of the elements have been obtained from the flow simulation and applied individually to each element in the FE analysis. Each element thus has its own individual mechanical property data table. A nominal tensile load of 1KN has been applied to the plates. Analyses have also been undertaken assuming that the material had constant isotropic properties of $E = 10$GPa., and $\nu = 0.3$ which correspond to the manufacturers data (264). The results from the anisotropic and isotropic analyses can be seen in Figures 66 and 67 for the plate with the lateral charge. The stress results plotted are for the Von Mises stress.

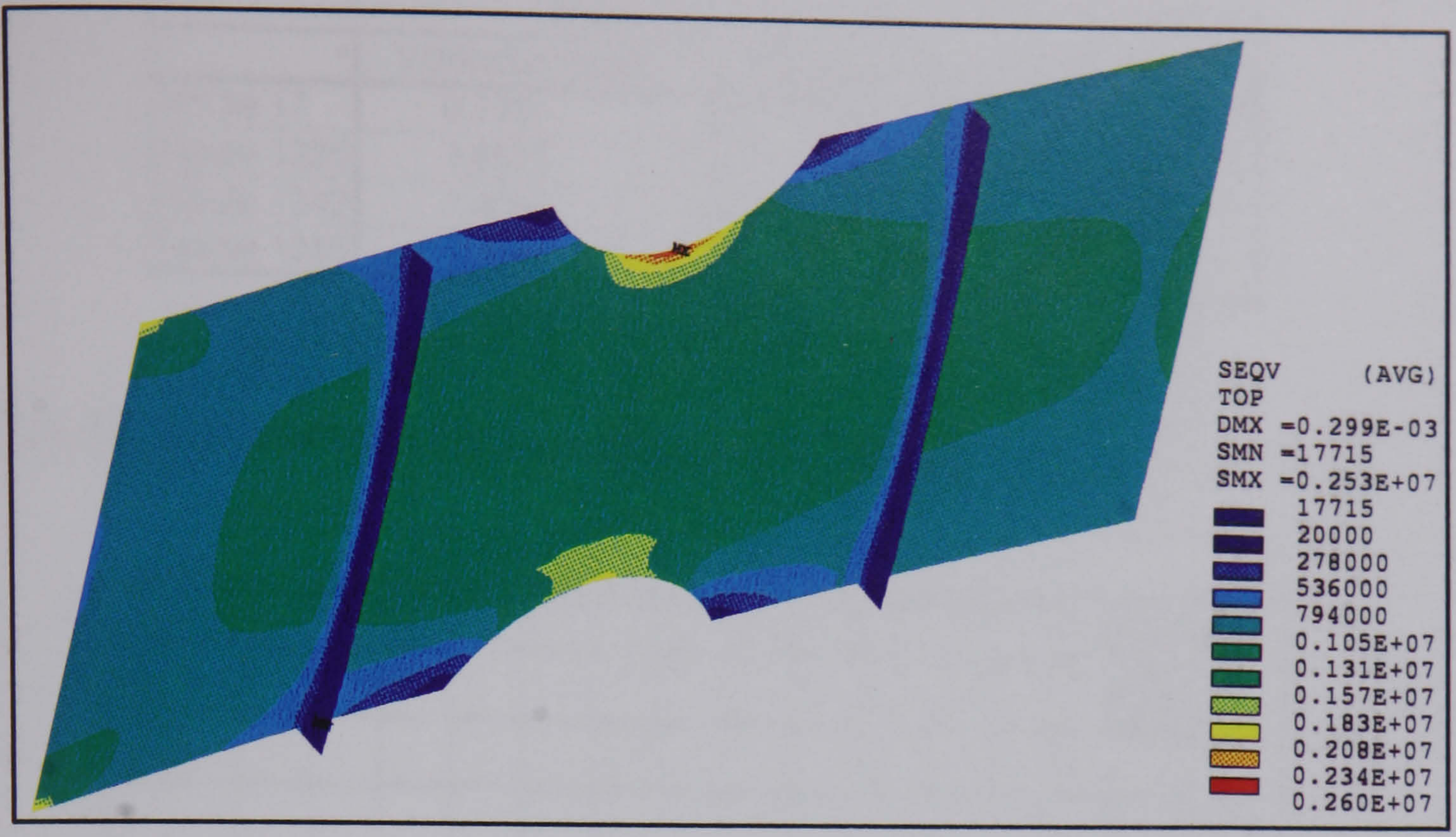

Figure 66 : Stress Results For Anisotropic Data For Plate With Lateral Charge.

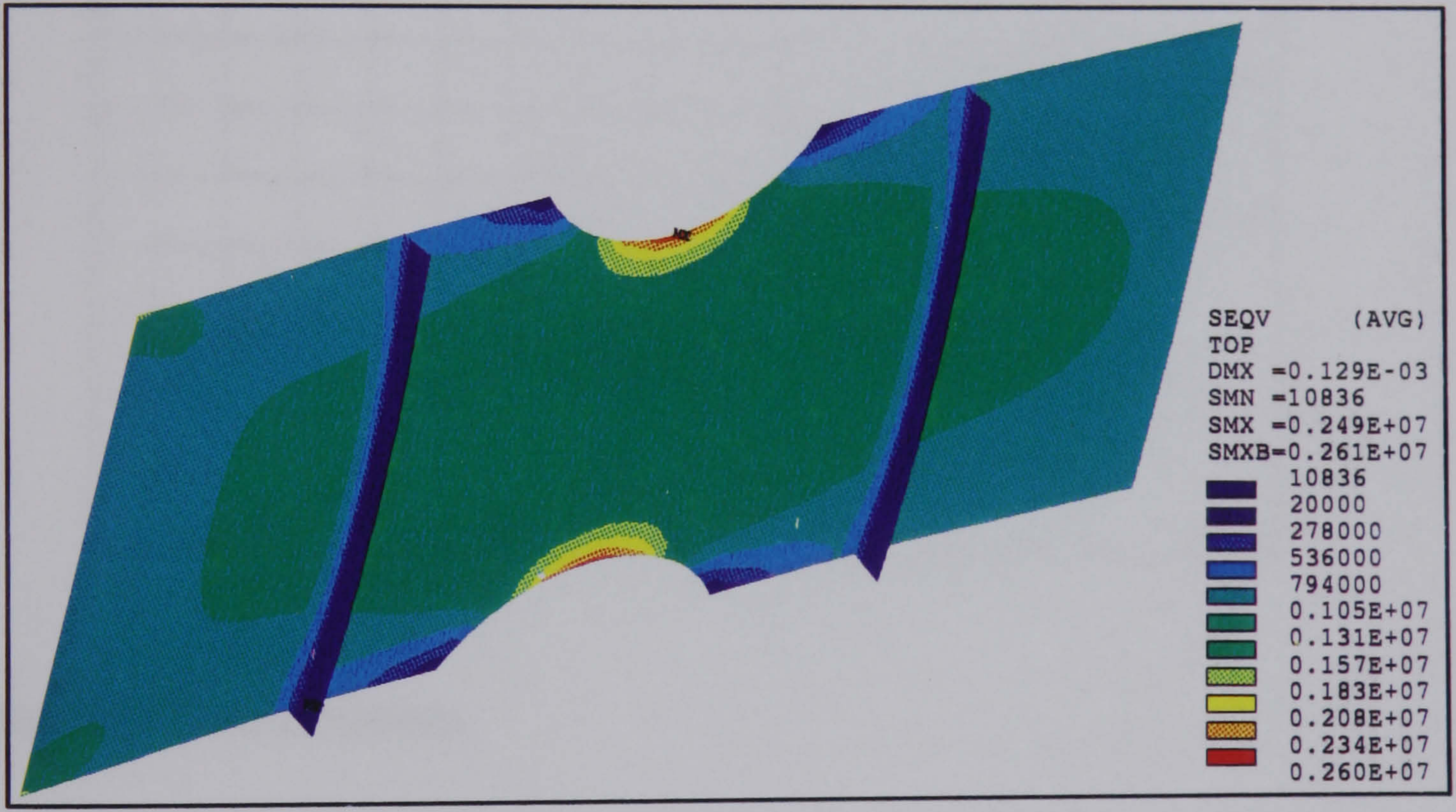

Figure 67 : Stress Results For Isotropic Data For Plate With Lateral Charge.

The difference between the two sets of results can be seen in Table 37 for the nodal locations identified in Figure 68.

	Von Mises Stress MPa.		
	Variable Props	Constant Props	Difference
Node 17	0.779	0.844	8%
Node 1225	1.857	2.49	25%
Node 1342	1.406	1.35	4%
Node 1345	1.306	1.53	15%

Table 37 : Nodal Stresses For Plate With Lateral Charge.

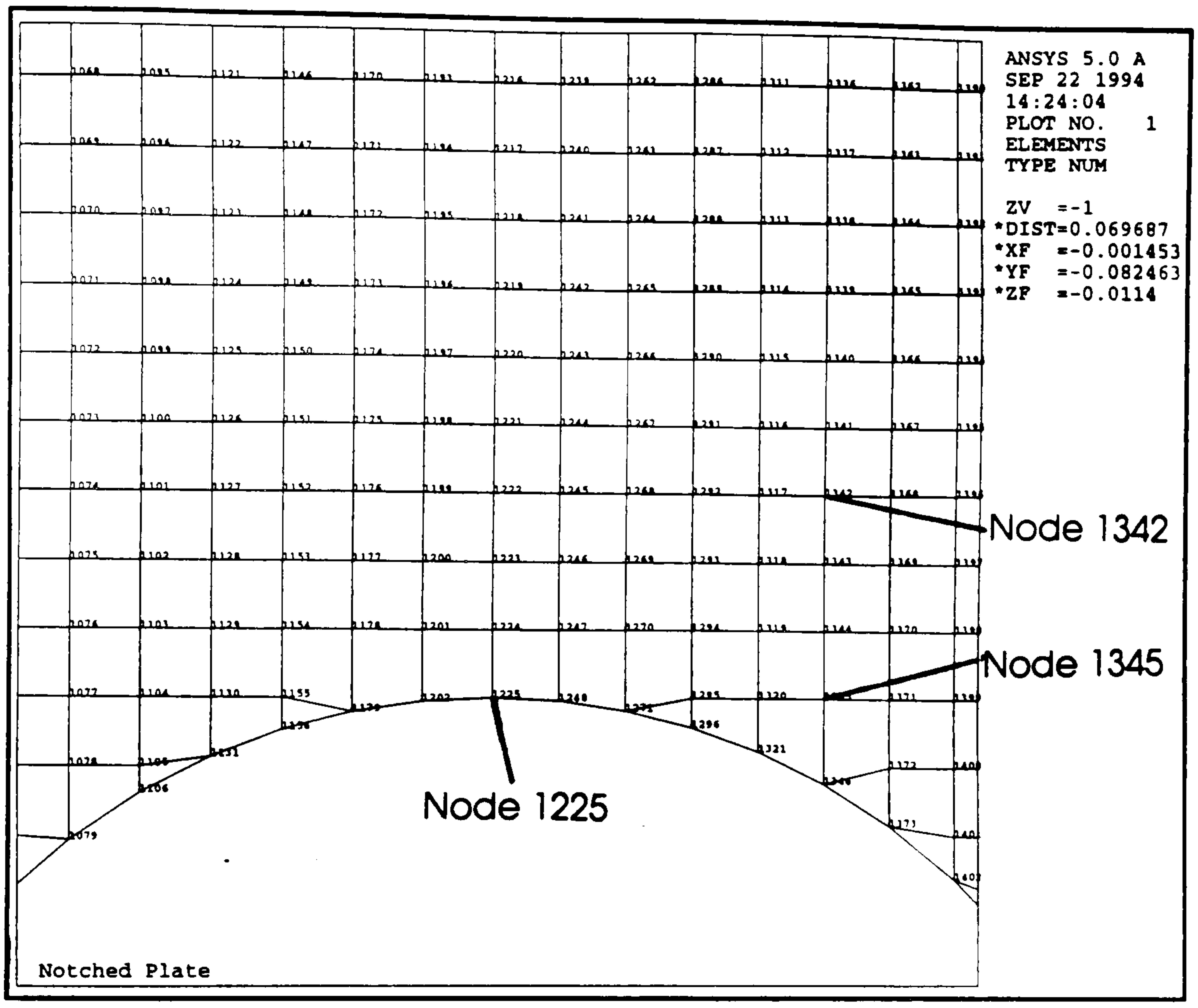

Figure 68 : Nodal Locations.

The most pronounced difference between the two sets of results is at the lower cut out at node 1225. At this location the difference between using the varying properties and the constant isotropic properties results in a reduction in the Von Mises stress of 25%. However, this difference is seen to vary across the plate with an increase of the Von Mises stress observed in some areas.

The results from the anisotropic and isotropic analyses can be seen in Figures 69 and 70 for the plate with the central charge.

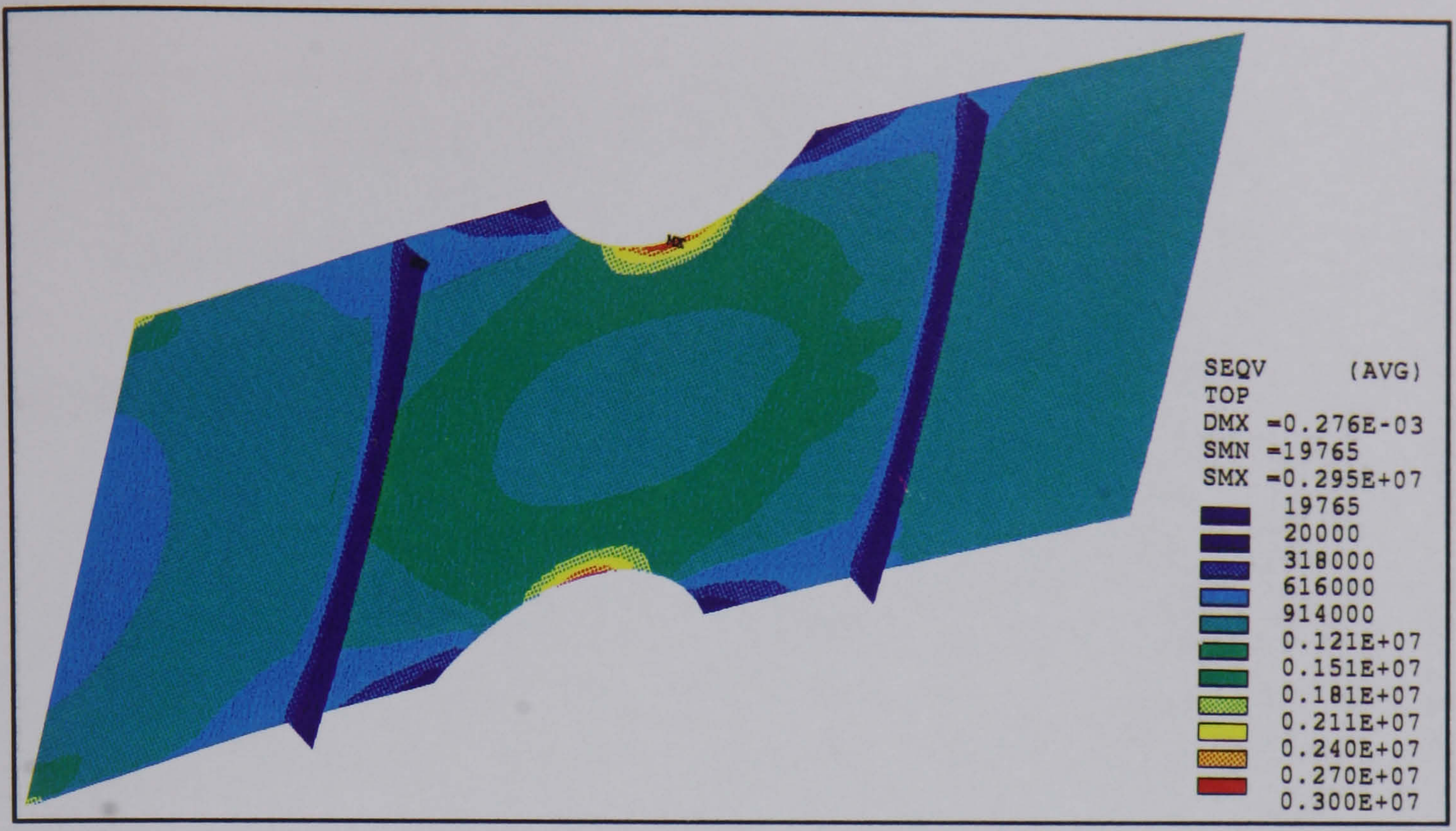

Figure 69 : Stress Results For Anisotropic Data For Plate With Central Charge.

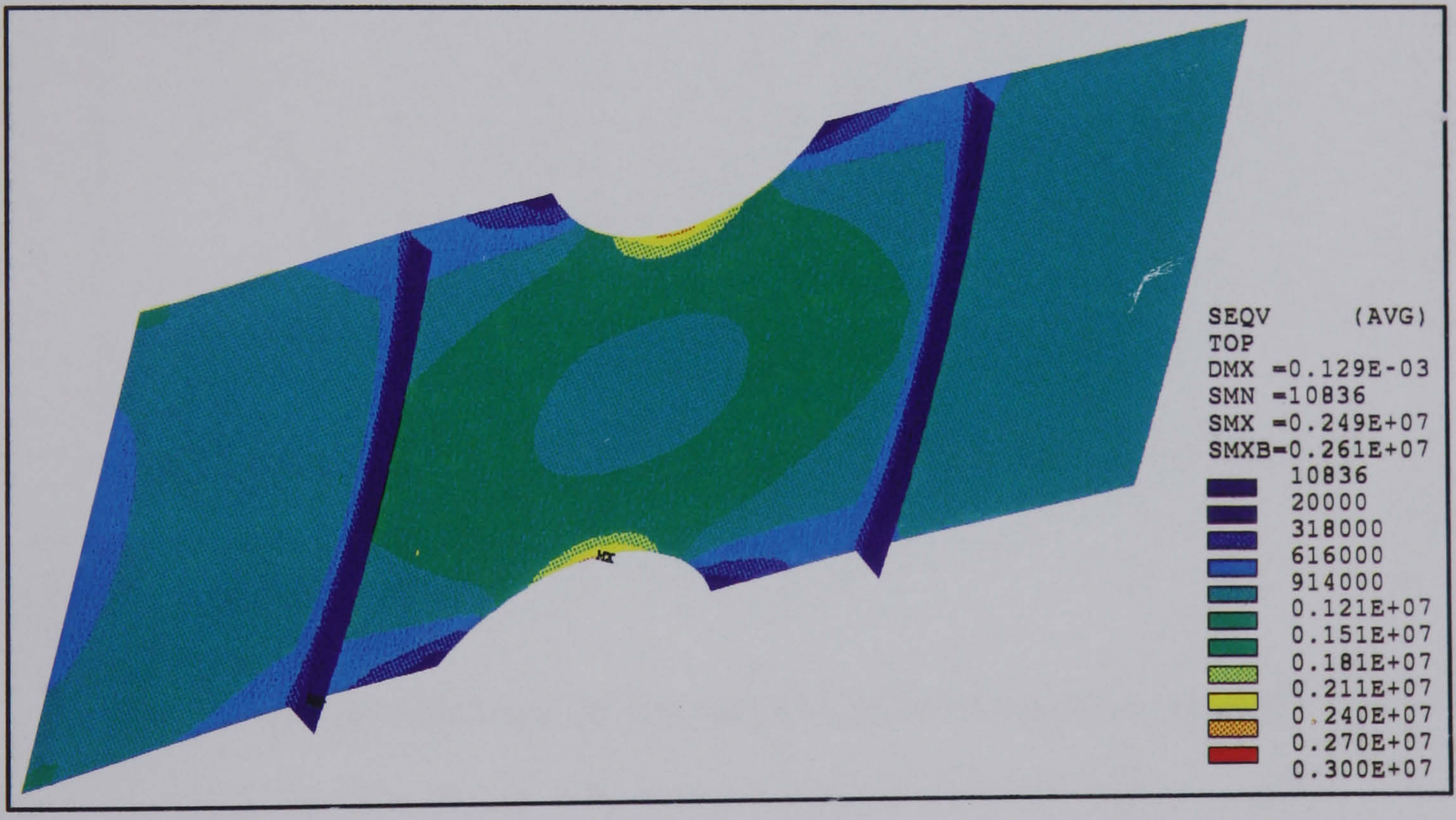

Figure 70 : Stress Results For Isotropic Data For Plate With Central Charge.

The difference between the two sets of results can be seen in Table 38 for the nodal locations identified in Figure 71.

	Von Mises Stress MPa.		
	Variable Props	Constant Props	Difference
Node 145	0.882	0.948	7%
Node 1225	2.795	2.49	12%

Table 38 : Nodal Stresses For Plate With Central Charge.

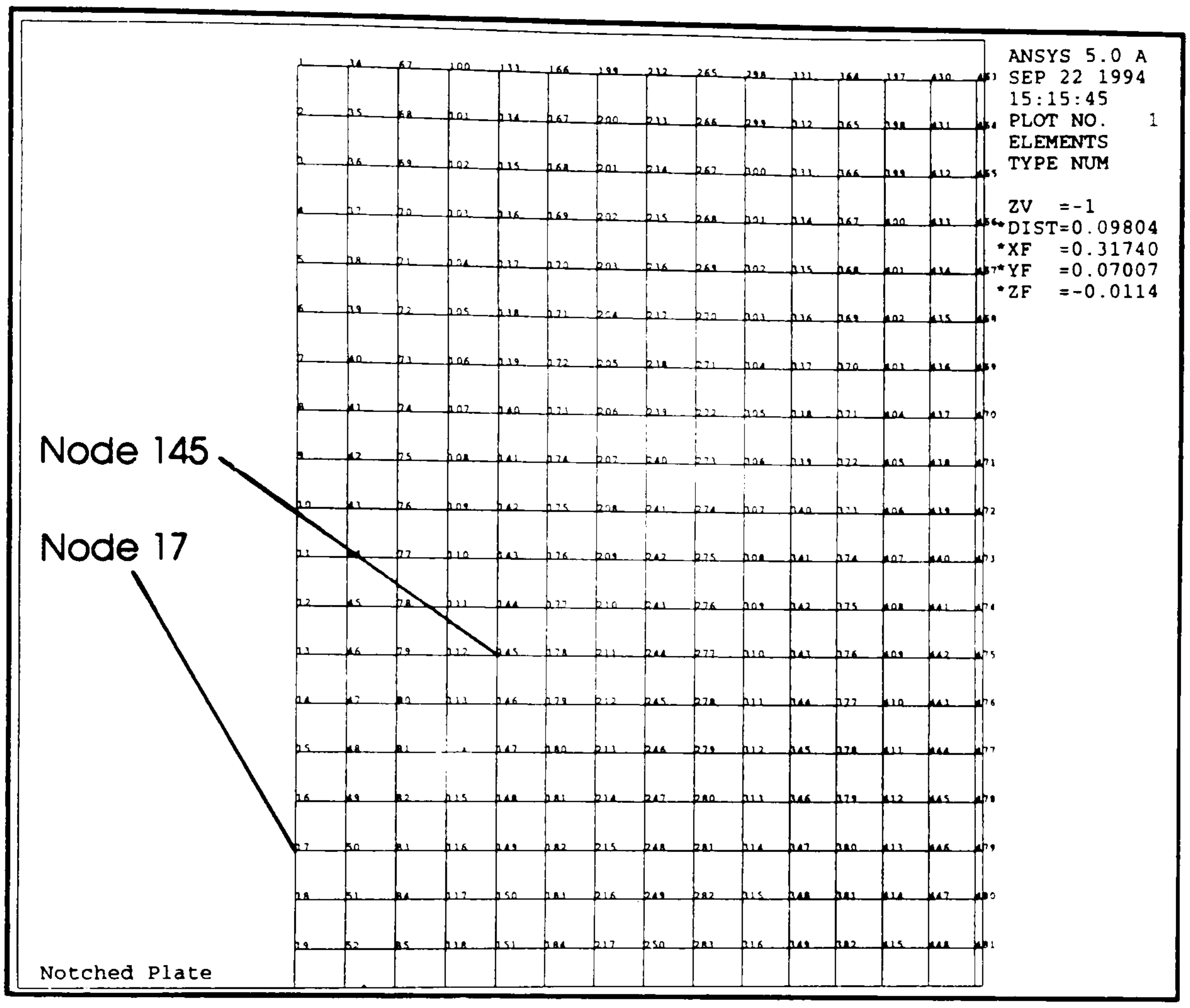

Figure 71 : Nodal Locations.

With the charge material in the central position there is less flow of the material which results in less difference between the two sets of results. The maximum difference observed is again at the lower cut out and using the varying properties results in an increase in the Von Mises stress of 12%.